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NATIONAL

ARITHMETIC;

ORAL AND WRITTEN.

FOR COMMON AND GRADED SCHOOLS, ACADEMIES, ETC.

BY

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PREFACE

THE order in which the different parts of the subject are presented to the mind in this book is natural and logical; and the principles of each successive topic are carefully developed by appropriate exercises, so graded that the mind of the student must inevitably grasp the relations of the whole subject, and when the work is completed, comprehend it, not as a mass of loosely connected details, but as a unified whole.

Oral and written arithmetic are supplementary to each other, since the principles and processes are the same in both; and as the former is more easily understood, because of the smaller numbers used in the operations, the exercises, analysis, and reasoning applied to the Oral are made the means of elucidating the principles and processes of the Written. In this way *formal rules* have been, in a great measure, dispensed with, and FORMULAS addressed to the eye as well as to the mind have been made to take their place. These will be found more efficacious, both as guides to practice and helps to the memory, since the principles are so sharply defined, and so clearly developed and illustrated by practical examples, that the mind of the student will necessarily acquire the habit of depending upon his own reasoning powers rather than upon the clumsy guide-posts of verbal rules.

Many new features have been introduced in the working out of this plan. Only the most salient are here referred to.

In the natural and logical *arrangement,* and in the *gradation of topics and exercises,* as well as in the statement of principles, rules, and explanations, the greatest possible simplicity has been aimed at.

The numerous and varied *examples for practice* have been constructed with great care, illustrating the multifarious applications of arithmetical science to business operations, the object being to prepare the student for the exigencies of daily life.

Since *good methods* in arithmetic are a vital element of success, because they economize both time and energy, by securing rapidity and accuracy in calculation, several new and important methods have been introduced.

The treatment of FRACTIONS will be found to embody several valuable improvements. By a close regard to fundamental principles, the multiplicity of cases, usually so confusing to the student, has been avoided. This will be especially apparent in *Multiplication* and *Division*, which are treated at first in two cases each, afterward reduced to one; while, finally, *all the processes* of multiplication and division of fractions are brought under ONE GENERAL RULE.

CANCELLATION has been more generally applied than in other arithmetical text-books, and in such a way as not only to abridge the processes, but to render the *rationale* of the methods more obvious. Its application to the calculation of *Interest*, presents some features of considerable importance in practical work.

In this connection, attention is also called to the brief, simple, and eminently practical method of working examples in *Partial Payments*, by means of the "Time Table" and Cancellation.

MENSURATION, the METRIC SYSTEM, and some of the more *theoretical* topics have been placed in the Appendix, so that the course may be completed independently of them, at the option of the student or teacher.

The NATIONAL ARITHMETIC is a comprehensive work in *two parts*, each issued in a small, compact volume, or both bound together in a single volume, comprising both the "Practical" and the "Advanced" books of the series. The first part or book is made to cover all the ground of elementary arithmetic, Fractions and Denominate Numbers; and to this has been added a brief treatment of what are usually considered advanced topics,—Percentage, Proportion, Involution, etc.,—the design being to supply a convenient book, at a low price, sufficiently comprehensive for the wants of the great mass of pupils in the lower grades of Grammar Schools and in rural district schools, whose limited opportunities require the largest possible amount of information and training in a short space of time.

The *Written Arithmetic* has been brought down to the capacity of advanced *primary* grades, so that pupils taught *orally* in the lower grades may be able to take up the study of this part of the work without using the TABLE BOOK AND PRIMARY ARITHMETIC.

Each book is complete in itself, or may be used consecutively in the series; while the entire work, in one volume, will serve for a complete course. By a close economy of every line and space, excluding useless matter and avoiding all needless repetitions of analysis and explanation in the advanced portions, the author has been able to present a large amount of material in a very small compass, the object being to economize *time* and *labor*, as well as *money*, in this branch of school study.

The author desires to acknowledge his indebtedness to several eminent teachers and educators of experience for valuable assistance in the preparation of this work, as also for many important methods and suggestions embodied therein.

COLUMBIA, MO., Aug., 1881.

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NATIONAL ARITHMETIC

DEFINITIONS

1. A **Unit** is *one* thing, or a group of things considered as *one;* as *one, one* book, *one* dozen, *one* ten, *one* hundred.

2. A **Number** is a unit, or a collection of units; as *one, four, six* pens, and *nine* books.

A number answers to the question, "How many?"

3. The **Unit of a Number** is *one* of the units forming that number. Thus, the unit of six is *one;* of ten cents is *one* cent.

4. An **Abstract Number** is a number that consists of units that are not named; as *one, four, eight, twenty-five.*

5. A **Concrete** or **Denominate Number** is a number the unit of which is named; as one *rod,* five *cows,* ten *acres.*

6. **Like Numbers** are numbers that have the same units, or express the same kind of quantity, and may be either abstract or denominate; as *five* and *seven,* eight *miles* and ten *miles.*

7. **Unlike Numbers** are numbers that have different units, or express different kinds of quantity; as three *pounds* and five *rods,* six *men* and nine *horses.*

Notation and Numeration

8. Notation is a method of *writing* numbers.

9. Numeration is the method of *reading* numbers expressed by figures.

10. Numbers may be expressed as follows :

1st. By *words*, as one, two, three, five, twenty, etc.

2d. By the Arabic Method.

3d. By the Roman Method.

11. In the **Arabic Notation**, *ten* characters called figures are used to represent numbers.

FIGURES.	0	1	2	3	4	5	6	7	8	9
NAMES.	*Naught,*	*One,*	*Two,*	*Three,*	*Four,*	*Five,*	*Six,*	*Seven,*	*Eight,*	*Nine.*

The figure 0, called *naught, cipher,* or *zero,* has no value. The other nine figures are called *digits,* or *significant figures,* and each represents the number written under it. Any number can be expressed by these ten characters, when combined according to a certain method or principle.

12. To establish a uniform method in representing numbers, objects are supposed to be arranged in *groups* of *tens,* each group being *ten times* as great as the next lower group and having a different name. Hence, we have single things or *units ;* next, *ten* single things are regarded as forming a group called *ten ;* next, *ten* of the groups called ten, as forming a group of greater value, called *one hundred ;* then, *ten* of the groups called hundred as forming a group of still greater value, called *one thousand,* etc.

13. Each group is called an **Order of Units.**

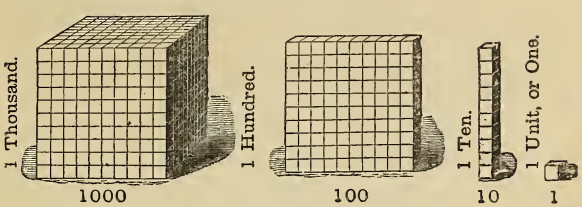


TABLE.

10 Units (1)	make	1 Ten, - - -	10.
10 Tens (10)	"	1 Hundred, - -	100.
10 Hundreds (100)	"	1 Thousand, -	1000.
10 Thousands (1000)	"	1 Ten-thousand,	10000.

Any figure standing alone, or in the first place at the right of a number, represents simply *units*, or units of the *first order;* standing in the second place from the right, it represents *tens*, or units of the *second order;* in the third place from the right it represents *hundreds*, or units of the *third order;* in the fourth place, *thousands*, or units of the *fourth order*, etc.

Thus, 4, standing alone, represents four *units;* in the number 40, it represents four *tens;* in 400, it represents four *hundreds;* in 4000, it represents four *thousands;* etc.

14. PRINCIPLES. I. *The successive orders of units increase in value tenfold from right to left.*

II. *Ten units of any order in a number equal one unit of the next higher order.*

III. *Each removal of a figure one place to the left increases its representative value tenfold.*

IV. *Each removal of a figure one place to the right diminishes its value tenfold.*

EXERCISES.

15. 1. How many tens and units in 45? 68? 84? 76? 97? 89? 77? 93? 86?

2. How many hundreds, tens, and units in 243? 461? 385? 197? 614? 805? 746? 570? 964?

3. How many thousands, hundreds, tens, and units in 1345? 3762? 5084? 6712? 2968? 9407?

Copy and read the following:

(1.)	(2.)	(3.)	(4.)	(5.)
75	250	893	2344	7319
49	475	905	4608	8175
126	681	760	6314	5084

Express in figures the following:

1. 6 tens 8 units.
2. 7 tens 9 units.
3. 9 tens 3 units.
4. 6 hundreds 4 tens 5 units.
5. Four hundreds six tens.
6. Seven hundreds five units.
7. 9 hundreds 6 tens 7 units.
8. 8 hundreds 8 tens.
9. 6 thous. 9 hunds. 4 tens.
10. Three thous. six hunds.
11. Seven thous. eight tens.
12. Eight thous. six hunds.
13. 9 thous. 3 hunds. 7 tens.
14. Nine thous. fourteen.
15. Five thous. eighty-one.
16. Six thous. seven hunds.
17. Seven thousand one hundred seventy-three.
18. One thousand nine hundred eighty-seven.
19. Five thousand seven hundred nineteen.
20. Eight thousand seven hundred fifty-nine.
21. What number is composed of 5 hundreds 6 tens 3 units?
22. What number is composed of 7 thousands 5 tens?
23. Of 8 hundreds 4 tens 2 units? Of 4 thousands 9 hundreds 7 units?
24. Write in figures and read, five units of the fourth order, six units of the third order, one unit of the second order, and nine units of the first order.

16. A **Scale** in Arithmetic is the relation between the successive orders of units.

In the Arabic system of notation, the scale is *ten;* that is, the value of the unit in any order is *ten* times as great as the unit in the next lower order; hence, it is called the *Decimal Scale,* from the Latin word *decem,* meaning *ten.*

17. Every successive group of *three orders* of units, counting from the right, is called a **Period.**

The *first period,* named the period of *units,* is formed of the first, second, and third orders of units, or units, tens, and hundreds; the *second period,* named the period of *thousands,* is formed of the second group of three orders from the right, and comprises thousands, ten-thousands, and hundred-thousands; the *third period,* named millions, is formed of the third group of three orders from the right; the *fourth period,* named *billions,* is formed of the fourth group of three orders from the right, etc., illustrated as follows:

18. NUMERATION TABLE.

NAMES OF PERIODS.	of Quintillions.	of Quadrillions.	of Trillions.	of Billions.	of Millions.	of Thousands.	of Units.
ORDERS OF UNITS.	Hundreds Tens Units	Hundreds Tens Units	Hundreds Tens Units	Hundreds Tens Units	Hundreds Tens Units	Hundreds Tens Units	Hundreds Tens Units
NUMBER.	6 6 6 ,	6 6 6 ,	6 6 6 ,	6 6 6 ,	6 6 6 ,	6 6 6 ,	6 6 6
PLACES.	21st. 20th. 19th.	18th. 17th. 16th.	15th. 14th. 13th.	12th. 11th. 10th.	9th. 8th. 7th.	6th. 5th. 4th.	3d. 2d. 1st.
PERIODS.	7th.	6th.	5th.	4th.	3d.	2d.	1st.

19. There must be three figures in every period, except the one at the left, which may have one, two, or three. Every order of a number not occupied by a significant figure must be filled with a cipher.

EXERCISES.

20. Write upon the board any number, as the following:

555,555,555,555,555,555,555.

1. Let the pupil name the *order* of units in the following places: First, Third, Fifth, Second, Fourth, Sixth, Ninth, Eighth, etc.

2. Then name the *places* occupied by the following orders: Hundreds, Millions, Thousands, Billions, Ten-thousands, etc.

3. Then name the following *periods:* First, Third, Second, Fourth, Sixth, Fifth, Seventh.

4. And then the *period* and the *place* of the following: Thousands, Millions, Hundred-thousands, Ten-millions, etc.

5. Then practice on the above number or some other, as follows: Five units; five tens, or fifty, which is ten times five units; five hundreds, or five hundred, which is ten times five tens; five thousands, or five thousand, which is ten times five hundreds; five ten-thousands, or fifty thousand, which is ten times five thousand, etc.

6. Next reverse the process, commencing with any order, thus, five million; five hundreds of thousands, or five hundred thousand, which is one-tenth of five million; five tens of thousands, or fifty thousand, which is one-tenth of five hundred thousand; five thousands, or five thousand, which is one-tenth of fifty thousand, etc.

Practice the last two exercises until the *decimal scale* of increase and decrease is thoroughly understood.

7. Next write on the blackboard several numbers, each consisting of six or more periods. Thus,

300,300,300,300,300,300.
666,666,666,666,666,666.

Exercise the pupils on the *periods* in the same manner as on the *orders*, until it is well fixed in the mind that each period represents a value one *thousand* times as large as the period on its right, and one *thousandth* as large as the period on its left.

Express in figures *seventy-five thousand eighty-six.*

ANALYSIS. Write 7 ten-thousands in the 5th place, 5 thousands in the 4th place, a cipher 0 in the 3d place, there being no hundreds, 8 tens in the 2d place, and 6 units in the 1st place, and we have 75086.

21. RULE FOR NOTATION. *Beginning at the left, write the hundreds, tens, and units of each successive period in their proper order, filling all vacant orders and periods with ciphers.*

Read the number expressed by 24567384.

ANALYSIS. Separate the number into periods of three figures each; thus, 24,567,384. The third period is *millions*, the second is *thousands*, and the first is *units*; hence the number is 24 million 567 thousand 384.

In the same manner read

1. 56037.	3. 320765.	5. 5760040.
2. 80140.	4. 1270084.	6. 25003075.

22. RULE FOR NUMERATION. I. *Begin at the right and separate the number into periods of three figures each.*

II. *Then begin at the left and read each successive period as if it were units, giving each its name except the period of units.*

23. Express in figures and read the following numbers:

1. Three hundred seven thousand sixty.

2. Seventy-seven thousand two hundred eight.

3. Four hundred thousand six hundred forty.

4. Two million forty-eight thousand four hundred twenty.

5. Sixty-three million fifty-two thousand eight hundred.

6. One hundred five million seven hundred eighty thousand six hundred fifteen.

7. Six units of the 6th order, eight of the 3d, and five of the 2d.

8. Three units of the 7th order, seven of the 6th, one of the 4th, four of the 3d, and nine of the 1st.

9. Write fifty-six units in the 3d period, one hundred fifty in the 2d period, and fifteen in the 1st.

10. Write nine units in the 5th period, and twenty-five in each of the lower periods.

11. Write twelve trillion 3 hundred thirty billion 205 million 7 hundred thousand ninety.

12. Write 3 in the units place of five periods and read the number. Write 33 in the tens and unit places.

13. Write successively in each of six periods 205, and read the numbers.

14. Write 420 in every alternate period of seven periods, beginning with the first, and read the number.

15. Three hundred twenty million seventy-five thousand nine hundred eighty-nine.

16. Seven billion twenty-five million four hundred thousand six hundred fourteen.

17. Fifty-six trillion one hundred seven billion five hundred sixty million three hundred fifty-two thousand.

18. Write two billion ten million forty thousand seventy-five.

19. Eight quadrillion three hundred sixty trillion thirty-five billion seven hundred forty thousand.

Read the following numbers:

1. 48075.	6. 960307.	11. 245006245.
2. 80016.	7. 3762081.	12. 5820157086.
3. 73240.	8. 5007163.	13. 30064205000.
4. 260450.	9. 63100075.	14. 812000762153.
5. 508200.	10. 40250903.	15. 27360450008074.

24. The **Roman Notation** employs seven capital letters to express numbers.

LETTERS.	I,	V,	X,	L,	C,	D,	M.
VALUES.	1,	5,	10,	50,	100,	500,	1000.

When used *alone*, each letter has a fixed value.

Numbers may be expressed by combining or repeating these seven capital letters according to the following principles:

1. *Repeating* a letter repeats its value.

Thus, III represents 3; XXX, 30; CCC, 300, etc.

2. A letter of less value placed *before* one of greater *takes* its value from that of the greater.

Thus, IV represents 4; IX, 9; XL, 40, etc.

3. A letter of less value placed *after* one of greater *adds* its value to that of the greater.

Thus, XI represents 11; XV, 15; LX, 60; DC, 600, etc.

4. A letter placed *between* two letters, each of greater value, is *taken from* the *sum* of the other two.

Thus, XIX represents 19; LIV, 54; CXL, 140.

5. A *bar* placed over a letter increases its value a *thousand times.*

Thus, $\overline{\text{X}}$ denotes 10,000; $\overline{\text{L}}$, 50,000; $\overline{\text{C}}$, 100,000, etc.

Roman Notation is used principally in marking dials, numbering chapters and sections in books, etc.

25. Table of Roman Notation.

Roman.	*Script.*	*Roman.*	*Script.*	*Roman.*	*Script.*
I	*1*	XVII	*17*	LX	*60*
II	*2*	XVIII	*18*	LXX	*70*
III	*3*	XIX	*19*	LXXX	*80*
IV	*4*	XX	*20*	XC	*90*
V	*5*	XXI	*21*	C	*100*
VI	*6*	XXII	*22*	CC	*200*
VII	*7*	XXIII	*23*	CCC	*300*
VIII	*8*	XXIV	*24*	D	*500*
IX	*9*	XXV	*25*	DC	*600*
X	*10*	XXVI	*26*	CM	*900*
XI	*11*	XXVII	*27*	M	*1000*
XII	*12*	XXVIII	*28*	MD	*1500*
XIII	*13*	XXIX	*29*	$\overline{\text{X}}$	*10000*
XIV	*14*	XXX	*30*	$\overline{\text{C}}$	*100000*
XV	*15*	XL	*40*	$\overline{\text{M}}$	*1000000*
XVI	*16*	L	*50*		

MDCCCLXXXI = 1881, one thousand eight hundred eighty-one.

Exercises.

26. Express by Roman Notation:

1. Sixteen.
2. Forty-two.
3. Thirty-five.
4. Eighty-seven.
5. Fifty-nine.
6. Sixty-four.
7. Ninety-six.
8. Seventy-eight.
9. Two hundred forty.
10. Five hundred sixty-four.
11. Seven hundred seventy.
12. Ten thousand twenty-six.
13. 695.
14. 888.
15. 1529.
16. 2730.
17. 3897.
18. 40000.

27. Express by Arabic Notation:

1. LXVI.
2. XCIX.
3. CXXXI.
4. CCLX.
5. DCXIV.
6. $\overline{\text{CCC}}$XCV.
7. $\overline{\text{D}}$LXXV.
8. MCLXVIII.
9. MDCCCXCIX.
10. $\overline{\text{X}}$CXLIX.
11. $\overline{\text{L}}$DCCXIX.
12. MMDCCXCI.

ADDITION.

INDUCTIVE EXERCISES.

28. 1. How many men are 5 men and 9 men?

2. What number is obtained by uniting 6 units and 7 units?

3. What number contains as many units as 8 and 6 united?

4. How many cents are 12 cents and 8 cents? 7 tons and 9 tons? 6 miles and 10 miles?

5. How many hats are 9 hats and 8 hats? 11 pens and 6 pens? 5 figs and 9 figs?

6. How many tens are 3 tens and 6 tens? 4 tens and 7 tens? 8 tens and 9 tens?

7. How many hundreds are 5 hundreds and 4 hundreds? 9 hundreds and 6 hundreds? 4 thousands and 6 thousands?

8. How many are 6 and 9? 12 and 8? 9 and 11?

9. What sum of money is equal to 5 dollars and 14 dollars?

10. What is the *unit* of their sum?

11. What is the *unit* of 8 books and 7 books? What is the unit of their *sum?*

12. What is the unit of 12 bushels? Of 10 feet? Are their units *like* or *unlike?* Can they be united? Why not?

13. What is the unit of 7 tens? Of 6 hundreds? Can their units be united? Why not?

14. What kind of *numbers* only can be united? What *orders* of units?

15. How many are 7, 4, and 6? 4, 7, and 6? 6, 4, and 7?

16. When the same numbers are united in a different order, is the result changed?

29. Uniting two or more numbers or groups of objects of the same kind into *one* is called *Addition;* and the number obtained by adding is called the *Sum.*

30. DRILL TABLE NO. 1.

	A	B	C	D	E	F	G	H	I
1.	3	4	2	7	5	1	6	8	9
2.	2	3	8	6	7	9	4	6	2
3.	4	7	3	9	6	5	8	3	8
4.	1	5	8	3	4	6	2	5	6
5.	6	1	7	4	8	5	9	4	3
6.	7	6	4	2	9	3	3	2	9
7.	5	4	9	7	6	4	6	7	5
8.	2	8	6	8	8	7	4	6	9
9.	8	3	5	6	2	2	7	8	4
10.	6	7	8	7	5	7	2	7	3
11.	9	6	6	4	3	5	8	9	8
12.	4	9	4	3	4	6	3	5	9

By columns, *at sight*, give the sum of each number and the one *next below* it; then, by lines, give the sum of each number and the one next on the *right* of it.

In the same manner, by columns and by lines, practice with sets of *three* figures, then of *four*, etc., until long columns can be added rapidly and accurately.

31. A **Sign**, in Arithmetic, is a character used to indicate an operation, or relation.

32. The **Sign of Addition** is +. It is read *plus,* which signifies *more.*

Thus, 8 + 4 shows that 8 and 4 are to be *added,* and is read, 8 *plus* 4.

33. **The Sign of Equality** is =. It is read, *equals*, or *is equal to.*

Thus, $8+4=12$ denotes that the *sum* of 8 and 4 *equals* 12, and is read 8 *plus* 4 *equals* 12. It may also be read, 8 *and* 4 *are* 12.

DRILL EXERCISES.

34. 1. Add or count by 2's from 2 to 60.

* *Written,*	2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2, etc.
Read,	2, 4, 6, 8, 10, 12, 14, 16, 18, etc.

2. Then commence with 1 and read, thus:

1, 3, 5, 7, 9, 11, 13, 15, 17, 19, etc.

3. In the same manner, add by 3's from 3 to 90.

4. Next form a new combination, commencing with 1 and then with 2, thus,

2, 5, 8, 11, 14, 17, 20, 23, 26, 29, etc.

5. Treat the remaining digits, 4, 5, 6, 7, 8, and 9 in the same manner, forming as many combinations, *less one*, with each, as there are units in the leading digit, extending the process as far as may be desirable.

6. Add alternately by 2's and 3's to 50.

Written,	2 + 3 + 2 + 3 + 2 + 3 + 2 + 3 + 2 + 3 + 2 + 3, etc.
Read,	2, 5, 7, 10, 12, 15, 17, 20, 22, 25, 27, 30, etc.

In the same manner add:

7. By 2's and 4's to 60.
8. By 3's and 4's to 70.
9. By 4's and 5's to 72.
10. By 2's, 3's, and 4's to 81.
11. By 2's, 5's, and 3's to 97.
12. By 2's and 5's to 56.
13. By 5's and 3's to 69.
14. By 10's and 5's to 90.
15. By 2's and 6's to 64.
16. By 6's and 7's to 91.

* At first these exercises may be written before adding; then the writing may be omitted entirely.

Additional exercise may be had upon Drill Table No. 1, as follows: Add each *column* from bottom to top rapidly, pronouncing partial results; thus, in column A, 4, 13, 19, 27, 29, 34, etc. Then from top to bottom; thus, 3, 5, 9, 10, 16, 23, 28, etc.

In the same manner add each *line* from left to right, and then from right to left.

Then form a new combination, commencing to add with 1 each column and line; then, successively, commence with 2, 3, 4, and so on to 9.

35. When the question-mark ? is placed after the sign =, thus, = ?, the two together are read *equals how many?* or *what?* and signifies that the *answer* is to be found.

The following exercises may be copied on the slate, or the teacher may read the *parts*, and the pupil promptly give the *sum*.

10+ 4+ 6 = ?	7+12+ 3 = ?	14+ 6+ 7 = ?
13+ 8+ 7 = ?	18+ 0+10 = ?	4+ 2+15 = ?
6+ 7+10 = ?	12+10+ 8 = ?	12+10+ 5 = ?
8+11+ 5 = ?	20+ 8+ 6 = ?	23+ 5+ 9 = ?
15+ 3+10 = ?	12+ 7+ 9 = ?	10+25+ 7 = ?
16+ 5+ 7 = ?	21+10+ 7 = ?	6+14+12 = ?
7+23+ 5 = ?	5+25+12 = ?	5+ 6+13 = ?
19+10+ 4 = ?	22+ 8+10 = ?	6+ 8+ 9 = ?
9+13+ 8 = ?	14+ 5+ 9 = ?	10+12+10 = ?
24+ 6+ 9 = ?	7+15+ 8 = ?	21+11+10 = ?
26+ 0+10 = ?	18+ 0+12 = ?	14+10+12 = ?
15+ 9+ 7 = ?	9+12+11 = ?	20+40+10 = ?

A few minutes of every recitation in written arithmetic should be appropriated to the drill tables and mental exercises preceding each division of the subject.

MENTAL EXERCISES.

36. 1. How many units are 8, 6, and 7? 9, 5, and 8?

2. How many *tens* are 90, 40, and 20? 60, 30, and 50?

3. How many *tens* and *units* are 20, 12, and 9?

4. Numbers composed only of *tens* and *units* may be added *at sight* by adding first the *tens*, then the *units*, and writing the results. Thus, $45 + 23 = 68$.

Observe that 4 *tens* and 2 *tens* are 6 *tens*, and 5 *units* and 3 *units* are 8 *units*, which added to 60 make 68.

5. *At sight*, give the sum of each of the following:

$40+10$	$20+64$	$36+22$	$90+14$	$16+75$
$50+9$	$70+28$	$80+17$	$28+37$	$49+31$

6. What is the sum of 7, 5, and 9? Of 70, 50, and 90? Of 700, 500, and 900? Of 7000, 5000, and 9000?

Observe that the *sum* of 7, 5, and 9 is the same, whether they represent units, tens, hundreds, or thousands.

7. Give the *sum* of each of the following, *at sight*. Thus, $200 + 60 + 5 = ?$ Read, 2 *hundreds* + 6 *tens* + 5 *units* = *what?* *Ans.* *Two hundred sixty-five*, 265.

8. $300+20+9=?$	12. $1000+500+80=?$
9. $600+70+6=?$	13. $3600+200+36=?$
10. $800+40+25=?$	14. $6000+3000+90=?$
11. $500+100+75=?$	15. $600+1000+42=?$

Also when written in columns, as follows:

(16.)	(17.)	(18.)	(19.)	(20.)
60	400	5000	1500	6000 tons.
40	700	700	650	1040 "
30	240	90	36	805 "

In all the drill and mental exercises, little more than *models* can be given. The teacher is expected to give such additional examples as he may deem proper.

21. John gave 20 cents for a slate, 10 cents for a book, and had 15 cents left. How many cents had he at first?

ANALYSIS. He had the sum of 20 cents, 10 cents, and 15 cents, which is 45 cents.

22. A pole is 15 feet in the air, 9 feet in the water, and 5 feet in the earth. How long is the pole?

23. Belle gave 15 cents for some thread, 10 cents for needles, and 8 cents for pins. How much did she spend?

24. An orchard contains 40 peach trees, 30 pear trees, and 25 plum trees. How many trees in the orchard?

25. There are 45 boys and 30 girls in a school. How many pupils in all?

26. I paid 25 dollars for a coat, 10 dollars for a pair of boots, and 7 dollars for a hat. What did I pay for all?

27. James' father was 40 years old 10 years since. How old will he be 12 years hence?

28. How many acres of land in three fields containing 40, 15, and 25 acres respectively?

29. A lady paid 45 dollars for a dress, 20 dollars for a shawl, and 10 dollars for a bonnet. What did all cost?

30. What is the cost of a pound of tea at 50 cents, a pound of coffee at 30 cents, and a quart of vinegar at 18 cents?

31. For how much must I sell a horse that cost me 120 dollars to gain 25 dollars?

32. George has 75 cents and Clarence has 60 cents; if each earns 25 cents more, how many cents will each then have?

33. A horse cost 250 dollars, a carriage 100 dollars, and a harness 50 dollars. What did all cost?

34. A carpet was sold for fifty dollars, which was fifteen dollars less than it cost. What did it cost?

37. PRINCIPLES. I. *Only like numbers can be added.*

II. *Only like orders of units can be added.*

III. *The sum and the parts added are like numbers.*

DEFINITIONS.

38. The **Sum** or **Amount** of two or more numbers is the number equivalent to all the units of the given numbers.

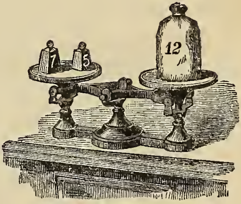
39. Addition is the process of finding the sum of two or more numbers.

40. An **Equation** consists of two equal numbers or sets of numbers connected by the sign of *equality.*

Thus, $7+5=12$ and $15=8+7$ are *equations.*

The expression on the *left* of the sign is called the *first member* of the equation, that on the *right,* the *second member.*

Thus, $7+5=12$ is an *equation,* and is read, 7 plus 5 equals 12, $7+5$ being the *first member,* and 12 the *second member* of the equation; and 7, 5, and 12 are called the *terms* of the equation.



EQUATION.

Name the *members* and *terms* of each of the following:

1. $12+6=18.$	3. $20+10=30.$	5. $9+21=26+4.$
2. $24+8=32.$	4. $40+20=60.$	6. $15+30=25+20.$

41. 1. The **Sign of Dollars** is \$. It is read, *dollars.*
2. The character ¢, or the letters *cts.* are used to denote *cents.*

Thus, 65¢, or 65 *cts.,* is read, 65 *cents.*

3. To distinguish *dollars* from *cents,* when written as *one number,* a point (.) is placed between them.

Thus, \$84.56 is read 84 *dollars* 56 *cents.*

4. Since one dollar is 100 cents, *cents* always occupy *two* places at the right of the point.

Thus, 28 *cents* may be written \$.28; 40 *cents,* \$.40.

5. When the number of cents is less than 10, a *cipher* must occupy the first place at the right of the point.

Thus, 9 *cents* is written \$.09; 7 *dollars* 8 *cents*, \$7.08.

6. Neither the sign (\$) nor the point (.) should be omitted.

Read the following equations:

1. \$9 + \$7 = \$16.
2. \$14 + \$21 = \$35.
3. \$45 + \$32 = \$77.
4. \$.25 + \$.50 = \$.75.
5. \$3.10 + \$.75 = \$3.85.
6. \$1.05 + \$.08 = \$1.13.

Express by the proper figures and signs:

7. Ten dollars fifty cents.
8. Twelve dollars five cents.
9. Ninety-seven cents.
10. Five dollars forty cents.
11. Eighty-eight cents.
12. Three dollars fifteen cents.
13. One dollar sixty cents.
14. One hundred dol. six cts.
15. Seventy dollars twelve cts.
16. Ten dollars ten cents.

Written Exercises.

42. ***To find the sum of any two or more like numbers.***

1. Find the sum of 537, 365, and 849.

OPERATION.	
537	
365	
849	
1751	Sum.

Explanation. Write the numbers so that units of the same order stand in the same column.

When the sum in any column is 10, or more than 10, it contains one or more units of a *higher* order, which must be added to the next column.

Beginning with the lowest order of units, add each column separately; thus, 9, 14, 21, the sum of the *units*, equal to 2 tens 1 unit. Write the 1 unit under the units' column, and add the 2 tens to the tens' column; thus, 2, 6, 12, 15, the sum of the *tens*, equal to 1 hundred 5 tens. Write the 5 tens under the tens' column, and add the one hundred to the hundreds' column thus, 1, 9, 12, 17, the sum of the *hundreds*, equal to 1 thousand 7 hundreds, which write in the hundreds' and thousands' places. Hence the entire sum is 1751.

In like manner copy and add the following:

(2.)	(3.)	(4.)	(5.)	(6.)	(7.)
347	195	607	480	298	472
683	736	825	673	765	638
928	548	779	309	77	65

RULE. 1. *Write the numbers to be added so that like orders of units stand in the same column.*

2. *Commencing with the lowest order, add each column separately, and if the sum can be expressed by one figure, write it under the column added.*

3. *If the sum of any column contains more than one figure, write the unit figure under the column added, and add the remaining figure or figures to the next column.*

To test the correctness of the result, perform the addition in the reverse direction, from top to bottom, and if the results agree the work is probably correct.

A great number of written exercises may be made from the Drill Table on page 18.

To illustrate, copy six examples of three numbers each from any three consecutive columns, as A B C, thus:

(8.)	(9.)	(10.)	(11.)	(12.)	(13.)
342	238	473	158	617	764
238	473	158	617	764	549
473	158	617	764	549	286

Observe that the *first number* of the first example is composed of the first three figures in line 1; the *second*, of the first three in line 2; and the *third*, of the first three in line 3.

Also, that the second example commences with the first three figures in line 2; the third example with the first three figures in line 3, and so on.

In the same manner copy examples of *four, five,* or more numbers each. Then use in the same way any other three consecutive columns, as B C D, C D E, etc.

Then in the same manner from *four* consecutive columns, as B C D E, C D E F, etc., copy examples composed of *four* numbers each, then of *five,* and so on.

These exercises may be extended, if desired, to *eight* columns, and to *eight* or *ten* numbers in each example.

43. The following is a good exercise for the slate or blackboard:

EXPLANATION. Write two numbers, as 275, 463; add them without drawing a line, and use their sum 738 as a *third* number; then add the three numbers and use their sum 1476 as a *fourth* number, and so on as far as desired.

275
463
738
1476

In like manner add the following, extending each to the *sixth* number:

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)
472	647	308	1306	2670	8907
335	285	705	972	1247	5062

44. When numbers composed of dollars and cents are to be added, dollars are written under dollars, and cents under cents, so that the points stand in the same *vertical* line.

Copy, add, and test the following eleven examples:

	(1.)	(2.)	(3.)	(4.)	(5.)
6.	$36.27	$50.04	$200.00	$100.07	$304.00
7.	5.96	7.80	2.50	40.50	75.75
8.	12.08	102.10	30.04	10.10	12.05
9.	120.40	15.08	9.28	9.68	27.54
10.	75.00	208.00	16.75	.87	5.81
11.	.94	3.43	94.56	250.00	63.02

For examples 6 to 11, read across the page.

Find the sum

12. Of 2500 acres, 1715 acres, 3007 acres, 510 acres.

13. Of 1420 yards, 672 yards, 1200 yards, 700 yards, and 1040 yards.

14. Of 1850 men, 1650 men, 2000 men, 480 men, and 96 men.

15. Of \$84.70, \$175.05, \$300, \$15.75, \$28, \$9.87, and \$.99.

16. I bought at different times wheat as follows: 150 bushels, for \$320.50; 410 bushels, for \$927.80; 1000 bushels, for \$2120; and 605 bushels, for \$1310.40. How much wheat did I purchase, and what did it cost me?

17. By selling a farm for \$13050, I lost \$960.50. What did it cost me?

18. Four loads of hay weigh respectively 1976, 2048, 2137, and 1896 pounds. What is the weight of all?

19. If a father gives to each of six sons \$2514.25, what sum does he give to all?

20. A merchant engaged in trade with \$10875. The first year he gained \$2516.75; the second, \$1809.40; and the third year, \$3500. How much capital had he then?

21. A man bought a house for \$7500. He paid \$1271.40 for repairs, and \$375.75 for painting. For what must he sell it to gain \$540?

22. A railroad company bought of A, 750 cords of wood; of B, 910 cords; of C, 325 cords; and of D, as many cords as of B and C both. How many cords did the company buy of all?

23. Three persons enter into partnership. A puts in \$4500; B, \$3275; and C puts in as much as A and B both. What did C put in, and what sum did all put in?

24. A drover bought 450 sheep of one farmer, 384 of a second, 510 of a third, 600 of a fourth, and of a fifth as many as he bought of the first and second. How many did he buy of all?

25. A man owns a farm worth \$10500, a mill worth \$9000, mining stock worth \$12575, and \$7650 in city bonds. What is he worth?

26. The distance from New York to Gibraltar by sea is 3300 miles; from Gibraltar to Malta, 1025 miles; from Malta to Alexandria, 750 miles; and from Alexandria to Bombay, 6499 miles. What is the distance from New York to Bombay?

27. A man gave to each of his three sons, \$5650; to each of his two daughters, \$3825; and to his wife, \$6375. What sum did he give to all?

28. The receipts of the Centennial Exhibition from May 10 to November, 1876, were nearly as follows: for admission fees, \$3813724.49; for rents, \$290000; for royalties and percentages, \$205010.75. What was the total?

For examples 29 to 41 inclusive, read across the page; for 42 to 51, in columns as arranged.

	(42.)	(43.)	(44.)	(45.)	(46.)
29.	\$290.57	\$10.90	\$257.25	\$18.86	\$784.65
30.	87.81	7.67	27.25	29.42	300.50
31.	57.49	14.39	247.65	172.64	79.62
32.	122.67	17.30	489.42	86.09	150.40
33.	118.44	20.21	193.36	57.50	638.79
34.	501.76	23.31	290.52	98.79	47.09
35.	114.52	26.04	79.35	8.09	500.00

	(47.)	(48.)	(49.)	(50.)	(51.)
36.	3541386	2464580	708061	60389	400300
37.	962000	8763042	3187519	57063	808967
38.	4093176	931876	48000	40938	912739
39.	86040	650489	307908	91872	297346
40.	109371	76132	80356	6307	730825
41.	2163000	5875	4891605	819	47961

SUBTRACTION.

INDUCTIVE EXERCISES.

45. 1. How many cents are 12 cents and 8 cents?

2. Twenty cents are how many more than 12 cents? Than 9 cents? Than 8 cents? Than 11 cents?

3. How many less are 12 cents than 20 cents? 8 cents than 20 cents? 11 cents than 20 cents?

4. If Mary has 18 cents, how many more must her father give her, that she may have 25 cents?

5. James had 25 cents and spent 7; how many cents had he left?

6. 25 is how many units greater than 18? 18 is how many units less than 25?

7. From a piece of cloth containing 21 yards, 10 yards were cut; how many yards remained?

8. How many more are 20 bushels than 16 bushels?

9. How many tens are 9 tens less 6 tens?

10. How many hundreds greater are 7 hundreds than 4 hundreds? 10 hundreds than 2 hundreds?

11. How many thousands less are 5 thousands than 9 thousands?

12. What is the unit of 16 rods and 12 rods? How many rods less are 12 rods than 16 rods? The unit of 4 rods?

13. What is the unit of 26 days? Of 15 men? Are the units like, or unlike?

14. Can 15 men be taken from 26 days? Why not?

15. Only what kind of a number can be taken from another?

46. Comparing two numbers by finding how many units the one is greater or less than the other is called *Subtraction*, and the result obtained is called the *Difference*.

47. The **Sign of Subtraction** is —. It is read *minus*, which signifies *less*.

Thus, $14 - 9 = 5$ is read 14 *minus* 9 equals 5, or 14 *less* 9 is 5.

When a part is taken from the whole, the difference is sometimes called the *Remainder*. Thus, the *difference* between 14 and 5 is 9, or if 5 yards of cloth are cut from 14 yards, the *remainder* is 9 yards.

48. The **Parenthesis**, ( ), denotes that the enclosed expression is to be considered as one number. The **Vinculum**, $\overline{\quad}$, has the same signification.

Thus, $25 - (9 + 6)$, or $25 - \overline{9 + 6}$, shows that the *sum* of 9 and 6 is to be taken from 25.

Drill Exercises.

49. 1. Write on the blackboard any number, as 5. The teacher may name *one part*, and the pupil the *other*, until all its parts are named. Thus, the teacher says "1 and," the pupil adds "4 are 5;" again, "2 and," the pupil adds "3 are 5," etc.

2. In like manner, name the *parts* of each number from 6 to 25.

3. Again write on the slate or blackboard a line of figures, thus,

2, 5, 3, 7, 6, 8, 4, 9.

Give promptly *at sight* the difference between each two consecutive numbers; as, 5 less 2, 5 less 3, 7 less 3, etc.

4. Next, the difference between the *sum* of each two consecutive numbers and the next one on the right; thus, 2 and 5 less 3, 5 and 3 less 7, etc.

5. Subtract each number in succession from 10; then from 11, 12, 13, etc., to 25.

6. Subtract by 2's from 24 to 0.

Written. 24—2—2—2—2—2—2—2—2—2—2—2.
Read. 24, 22, 20, 18, 16, 14, 12, 10, 8, 6, 4, 2, 0.

7. Add by 3's from 0 to 48, and subtract by 3's back to 0.
8. From 1 to 49, and back. From 4 to 52, and back.
9. In like manner, begin at different numbers, and add forward and subtract back by 4's, 5's, 6's, 7's, 8's, 9's, and 10's.
10. Dictation exercises, to be treated as on page 20:

19— 6=?	7+12—9=?	13+ 5—7=?	19—4+ 7=?
21—10=?	16+ 6—8=?	20+ 7—8=?	12—5+13=?
24— 8=?	12+ 9—7=?	9+11—6=?	21—8+12=?
20—12=?	15+10—8=?	14+ 5—9=?	25—9— 6=?

11. Give the difference between

18 and 7+5.	20 and 7+6.	6+10 and 9+ 5.
21 and 9+2.	24 and 9+3.	12+ 7 and 6+ 3.
17 and 10+5.	19 and 11+2.	14+ 6 and 10+10.

12. For rapid exercise:

Write. 12+6—5+7—3—8+10+6—7+9+3, etc.
Read. 12, 18, 13, 20, 17, 9, 19, 25, 18, 27, 30, etc.

9+10+6—7+2—8=? 25—9+8+6—5—10+7=?
16—12+20—4—3+9=? 6+30—10—5+12—3+4=?

13. Again, write on the board combinations as follows:

25—9, 20+6, 21—12, 14+10, 9+7—3, etc.

Then require the pupil to make *applied examples* for the same. Thus, for 25 — 9: "I had 25 cents, and gave 9 cents for a slate; how many cents had I left?" Another pupil answers: "The difference between 25 cents and 9 cents, which is 16 cents." Again, for 9+7—3, "James had 9 marbles he bought 7 more and lost 3; how many had he then?" etc.

Exercises similar to the above should be used for occasional drill, and may be extended and varied, at the option of the teacher.

14. Any number above 20 and less than 100 can be separated at sight into two parts, one of which shall contain 1 *ten* and the *units*, if any. Thus, $56 = 40 + 16$, $60 = 50 + 10$, $47 = 30 + 17$, $88 = 70 + 18$, $95 = 80 + 15$, etc.

15. Let the pupil practice upon all the numbers from 20 to 99, separating each into parts, as in example 14.

16. Then find the difference between any *digit* and a number composed of *two figures*, when the unit figure is less than the digit, as 8 and 43. Thus,

Separate 43 into the parts $30 + 13$, and at once we know that the difference between 13 and 8 is 5, which united with 30 makes 35, the difference between 43 and 8.

17. Write on the slate or board a series of numbers between 20 and 99, arranged in any order, and under each number write the same digit. Thus,

21	27	30	24	36	32	23	25	33	41	etc.
3	3	3	3	3	3	3	3	3	3	

18. After subtracting 3 from each number and writing the difference, erase the same and repeat, until the difference can be given *at sight*.

19. In the same manner practice with 4, 5, 6, 7, 8, and 9.

20. Then write a new set of numbers, and practice upon *them* in the same manner, and so continue until the difference between any *digit* and any number composed of but *two figures* can promptly be given at sight.

21. What is the difference between

74 and 6?	61 and 8?	22 and 3?	93 and 9?
37 and 9?	53 and 4?	75 and 8?	87 and 8?
85 and 7?	47 and 9?	31 and 5?	66 and 7?
61 and 3?	84 and 5?	46 and 9?	52 and 6?
55 and 6?	92 and 8?	75 and 7?	34 and 9?
26 and 9?	63 and 7?	91 and 3?	87 and 8?

22. Find the difference between 73 and 48?

Since the 8 *units* in the smaller number cannot be taken from the 3 *units* in the larger number, separate the larger number into 60+13, and take the 8 units from 13 units, and the 4 tens or 40 from the 6 tens or 60, and the difference between 73 and 48 is 2 tens and 5 units, or 25.

$$\begin{array}{rcrcr} 73 & = & 60 & + & 13 \\ \underline{48} & = & \underline{40} & + & \underline{8} \\ 25 & = & 20 & + & 5 \end{array}$$

23. In like manner, find the difference between

62 and 27.	85 and 39.	42 and 29.	77 and 68.
48 and 71.	56 and 82.	93 and 38.	80 and 44.
19 and 36.	60 and 31.	57 and 18.	46 and 72.

24. What number added to 7 makes 23? 31? 25? 34?
25. What number taken from 32 leaves 9? 7? 6? 8? 10?
26. What number added to 26 makes 31? 42? 54? 60?
27. What number taken from 54 leaves 15? 26? 45? 37?
28. How many units are 8 units less 5 units? $8-5=?$
29. How many tens are 8 tens less 5 tens? $80-50=?$
30. How many hundreds are 8 hundreds less 5 hundreds? $800-500=?$
31. How many thousands are 8 thousands less 5 thousands? $8000-5000=?$

Observe that the *difference*, abstractly considered, between 8 and 5 is the same, whether it represents units, tens, hundreds, or thousands, etc.

32. Give the *difference* of each of the following *at sight:*

$40-20=?$	$500-300=?$	$1200-800=?$	$250-50=?$
$60-30=?$	$700-200=?$	$5000-2000=?$	$480-400=?$
$90-40=?$	$600-400=?$	$2700-900=?$	$300-80=?$

Also when arranged in columns:

825	1100	4000	950	3600	2400	3200
600	500	1000	700	800	1200	900

MENTAL EXERCISES.

50. 1. George had 50 cents, and gave 20 cents for a slate. How many cents had he left?

2. If 9 gallons of vinegar are drawn from a cask containing 42 gallons, how many gallons remain?

3. From a bin containing 60 bushels of wheat, 12 bushels were taken at one time, and 8 at another. How many bushels remained?

4. If I live 16 years longer, I shall be 72 years old. How old am I now? How old was I 13 years since?

5. A lady went shopping with 2 ten-dollar bills and 1 five-dollar bill. She paid $13 for a shawl, $4 for a pair of gaiters, and $2 for a pair of gloves; how much had she left?

6. What number must be subtracted from 70 to make the number 12 more than 36? 9 less than 50?

7. If I had $25 more than I now have, I should have $63. What have I now?

8. A laborer receiving $64 as a month's wages, paid $10 for a barrel of flour, $15 for coal, and $20 for an overcoat. How much had he left?

9. A man having 83 acres of land, sold 25 acres. How many acres had he left?

10. A man having $500 in bank, drew out $300. How much was left in bank?

11. The sum of two numbers is 450, and the less is 200; what is the greater number? If the greater is 350, what is the less?

12. A street car starts with 20 passengers. If 12 leave, and 23 get in, how many passengers are then aboard? How many more than at starting?

13. A man earned $36 one week, and $25 the next; after paying $35 expenses, what had he left?

14. How many less than 45 tons are 25 tons plus 12 tons?

15. A lady bought a muff for $16, a fur collar for $18, and a pair of fur gloves for $7, and gave in payment a 50-dollar bill. How much change should she receive?

16. How many less than 70 miles are 26 plus 32 miles?

17. Forty-four plus 21 equals 70 minus how many?

18. What number must be added to 24, to make the sum 9 less than 45?

19. $26 - (9 + 8) = ?$

20. $42 - 17 + 5 = ?$

21. $22 + 40 - 30 = ?$

22. $36 - 9 = 12 + ?$

23. $8 + 17 = 30 - ?$

24. $8 + 20 - 12 = 30 - ?$

25. $11 + 19 - 7 = 20 + ?$

26. $16 + 8 - (7 + 12) = ?$

27. $47 - 13 - ? = 10 + 15.$

28. $60 - \overline{20 + 15} = ? + 5.$

51. Principles. I. *Any number, or any order of units, can be subtracted only from a like number or a like order of units.*

II. *The minuend, subtrahend, and remainder must be like numbers.*

III. *The sum of the remainder and subtrahend must equal the minuend.*

DEFINITIONS

52. The **Difference** between two numbers is a number which added to the less gives a sum equal to the greater.

When a part is taken from the whole, the *difference* is called the **Remainder.**

53. Subtraction is the process of finding the difference between two like numbers.

54. The **Minuend** is the greater of two numbers whose difference is required.

55. The **Subtrahend** is the smaller of two numbers whose difference is required.

WRITTEN EXERCISES.

56. ***To find the difference between any two like numbers.***

1. Find the difference between 745 and 478.

OPERATION.

	6 13 15		6 0 0		1 3 0		15
Min.	7 4 5	=	7 0 0	+	4 0	+	5
Sub.	4 7 8	=	4 0 0	+	7 0	+	8
Diff.	2 6 7	=	2 0 0	+	6 0	+	7

EXPLANATION. Write the numbers as in Addition (**42**), and subtract each order of units separately.

Since 8 units cannot be subtracted from 5 units, add 10 units (1 *ten*) taken from the next higher order, making 15 units; 8 units from 15 units leave 7 units.

As 1 ten (10 units) has been taken from the 4 tens, there are 3 tens left, and since 7 tens cannot be subtracted from 3 tens, add 10 tens (1 hundred) taken from the next higher order, making 13 tens; 7 tens from 13 tens leave 6 tens.

As 1 hundred (10 tens) has been taken from the 7 hundreds, there are 6 hundreds left, and 4 hundreds from 6 hundreds leave 2 hundreds. Hence, the *difference* between 745 and 478 is 267, since $267+478=745$ (Prin. III).

Instead of taking 1 from the next higher order of units in the *minuend*, we may add 1 to the next higher order of units in the *subtrahend*. Thus, in the above example, we may say, 8 units from 15 units leave 7 units, 8 tens from 14 tens leave 6 tens, and 5 hundreds from 7 hundreds leave 2 hundreds, the difference 267 being the same in both operations.

In like manner, solve and prove the following:

	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)
From	475	613	327	841	760	915
Subtract	148	247	165	573	482	358

	(8.)	(9.)	(10.)	(11.)	(12.)	(13.)
From	524	363	837	603	700	401
Subtract	342	94	420	240	184	96

Rule. 1. *Write the subtrahend under the minuend, so that units of the same order stand in the same column.*

2. *Beginning at the right, subtract the units of each order of the subtrahend from the units of the corresponding order of the minuend, and write the remainder underneath.*

3. *When the number of units of any order of the subtrahend is greater than the number of units of the corresponding order of the minuend, add 10 to the latter and subtract. Then diminish by 1 the units of the next higher order of the minuend, or increase by 1 the units of the next higher order of the subtrahend, and proceed as before.*

Proof. *Add the difference or remainder to the subtrahend, and if the sum is equal to the minuend, the work is correct.*

For practice with abstract numbers, copy examples from the Drill Table on page 18.

To illustrate: Use the first three columns A, B, and C. For the first example, copy the numbers, each composed of three figures, opposite **1** and **2**, writing the less number under the greater. For the second example, copy the numbers opposite **2** and **3**; and so on, as follows:

342	473	473	617	764	764	549
238	238	158	158	617	549	296

In like manner, copy examples from any other three consecutive columns, as B C D, C D E, etc. Then examples of *four* figures each from *four* consecutive columns, as A B C D, B C D E, etc.; then of *five, six,* or more figures.

Examples with numbers of *four* figures each from columns B C D E:

4275	7396	7396	5834	6429
3867	3867	5834	1748	1748

How many years from the date of each of the following events to the year 1884?

14. Herculaneum and Pompeii destroyed in the year 79.
15. Mariner's compass invented in 1302.
16. Columbus discovered America in 1492.
17. Figures first introduced into Europe in 900.
18. Decimals were invented in 1464.
19. Algebra first known in Europe in 1496.
20. The telescope invented by Jansen in 1598.
21. Printing by cut type invented in 1441.
22. First newspaper published in America in 1704.
23. Robert Fulton built the first steamboat in 1808.
24. First passage of the Atlantic by steam in 1839.
25. First railway in the United States built in 1827.
26. The electric telegraph first used in the U. S. in 1844.
27. Watches invented at Nuremberg in 1477.
28. The pilgrims landed at Plymouth in 1620.
29. The Declaration of Independence was made in 1776. Which is the longer period, from the landing of the Pilgrims to the Declaration of Independence, or from the Declaration to 1884?

Numbers composed of *dollars* and *cents* must be written, as in addition, in such order that the points will stand in the same *vertical* line.

When one of the given numbers contains *cents* and the other does not, fill the vacant places with two ciphers.

	(30.)	(31.)	(32.)	(33.)	(34.)
From	\$750.08	\$306.10	\$534.27	\$809.00	\$71.40
Subtract	261.50	129.05	362.00	327.18	5.65

(35.)	(36.)	(37.)	(38.)
4030 men.	1807 acres.	30040 tons.	\$4000.00
1426 "	540 "	2056 "	760.07

Find the difference between

39. 37106 and 24004.	44. \$258.37 and \$77.90.
40. 706000 and 84023.	45. \$50400 and \$3156.50.
41. 125060 and 34202.	46. \$1570.26 and \$2104.08.
42. 207610 and 43570.	47. 24680 feet and 9764 feet.
43. 700090 and 60017.	48. 306086 votes and 276005 votes.

49. From 53000 plus 4072 subtract 30420 plus 5200.

50. From \$672.45 plus \$510.50 subtract \$1200 minus \$475.84.

57. ***When the sum of two or more numbers is to be subtracted from another.***

1. From 4672 subtract 1541 + 1062 + 450 + 76.

Min.	4672
Sub.	1541
	1062
	450
	76
Rem.	1543

Explanation. Write the numbers as in addition, the minuend at the top. Then add and subtract thus: 6, 8, 9, and 3 more make 12; write 3 in the remainder and carry 1 to the next column. 1, 8, 13, 19, 23, and 4 more make 27; write 4 in the remainder, and carry 2 to the next column. 2, 6, 11, and 5 more make 16; write 5 in the remainder and carry 1. 1, 2, 3, and 1 more make 4; write 1 in the remainder. Hence the entire remainder is 1543.

Prove the work by adding the remainder and the parts of the subtrahend together, and if correct the sum will equal the minuend.

In solving the applied examples, this method should be used when applicable.

In like manner, solve and prove the following:

	(2.)	(3.)	(4.)	(5.)	(6.)
Min.	8709	24784	87632	\$709.50	20000
Sub.	3416	7063	42306	\$310.09	7040
	1042	486	13459	117.36	3609
	764	1273	7243	27.48	1764
	37	95	407	5.80	857
Rem.	3450				

Written Exercises.

58. 1. The sum of three numbers is 16832, and two of them are 5760 and 7325. What is the other number?

2. Find the final remainder in subtracting 3416 as many times in succession as possible from 18633.

3. From the difference between \$4567.50 and \$984, subtract the difference between \$2500 and \$450.75.

4. The area of the United States is 3026504 square miles, and of Brazil 3956000 square miles. How much does Brazil exceed the United States?

5. Canada contains 686353 square miles, and Mexico, 829916 square miles. How much larger is Mexico than Canada?

6. The yearly income of Mr. A is \$7000. If he pays \$2800 general expenses, \$1250 rent, and \$372.75 taxes, how much can he save?

7. A man bought a pair of horses and a carriage for \$1600. He afterwards sold the horses for \$750, and the carriage for \$560.75. What did he lose by the sale?

8. Texas contains 274356 square miles, California 188981 square miles, and New York 47156 square miles. How much larger is Texas than California? Than New York and California?

9. I bought a house and lot in the city for \$21800, giving in exchange a farm worth \$10500, a mill worth \$5700, a note for \$1292.60, and the balance in cash. How much cash did I pay?

10. Mr. Smith bought a city lot for \$2000. He paid \$125.60 taxes, \$235.75 for paving, and then sold it at a loss of \$150. What did he receive for the lot?

11. A man worth \$10000 received a legacy of \$5640. He spent \$2500 in traveling, and lost \$1750 by a bad investment. How much had he left?

12. A merchant engaged in trade with $12500, and the first year he lost $2750.80, but the second year he gained $6315.50. What was he then worth?

13. A grain dealer had in store 12650 bushels of corn, and received 5700 bushels more. He filled one order for 6845 bushels, and another for 4090 bushels. How many bushels were left in store?

14. Two men bought a piece of land for $6816, and made improvements that cost $1173. They then sold it at a loss of $300 to each. What was the selling price?

15. A farmer raised 3750 bushels of grain, of which 1500 bushels was wheat, 1037 bushels corn, 416 bushels barley, and the remainder oats. How many bushels of oats had he?

16. Three men bought a hotel. A paid $3876.50, B paid $736.75 more than A, and C paid as much as A and B minus $687.25. What did C pay? How much more did B and C pay than A?

For examples 17 to 46 inclusive, find the difference between each set of two numbers, first in columns A and B, then in the columns B and C, and then in C and D.

A		B		C		D
17.	509345.	27.	601405.	37.	3735091.	40087021.
18.	445136.	28.	163071.	38.	642500.	7300134.
19.	427643.	29.	297300.	39.	730145.	16060700.
20.	513007.	30.	580082.	40.	750249.	8413629.
21.	640908.	31.	870406.	41.	984006.	51006073.
22.	$4710.36.	32.	$1206.14.	42.	$84570.60.	$20000.75.
23.	2561.05.	33.	941.87.	43.	93400.45.	36084.09.
24.	296.74.	34.	400.50.	44.	7036.87.	7129.63.
25.	278.41.	35.	1720.07.	45.	15600.18.	18407.40.
26.	3000.00.	36.	5710.10.	46.	22080.90.	70050.28.

MULTIPLICATION.

INDUCTIVE EXERCISES.

59. 1. If 8 cents a quart are paid for 3 quarts of milk, how many times 8 cents are paid?

2. How many cents are 8 cents + 8 cents + 8 cents? What is the *sum* of 8 cents taken *three* times?

3. Add by 8's to 24. By 3's to 24. By 2's to 24.

4. If 4 is written 5 times in a column and added, what is the *sum?* $4+4+4+4+4=?$ $5+5+5+5=?$

5. How many are five 4's, or 5 times 4? Four 5's, or 4 times 5?

6. Do the results differ? Why not?

7. Add by 4's to 20; by 5's. How many 4's in 20? 5's?

8. What is the difference between four 7's and seven 4's?

9. John bought 8 pencils, at 5 cents each; how many times 5 cents did he pay? How many cents?

10. What is the sum of 5 cents repeated 8 times? 8 times $5=?$

11. What sum is produced by taking $9 four times?

12. What is produced by taking 7 as many times as there are units in 3? In 4? In 5? In 6?

13. At 6 dollars a barrel, how many times 6 dollars will 7 barrels of apples cost?

14. What is the unit of 6 dollars? Of 7? Of the number produced by taking 6 dollars 7 times?

15. Should we repeat $6 7 times, or 7 six times? Why?

16. What will the unit of the number produced always be like? *Ans.* Like the unit of the number repeated.

60. The process of repeating any number a given number of times is called *Multiplication;* and the result or sum found is called the *Product.*

61. The number to be repeated is called the *Multiplicand* and the number that shows *how many times* the multiplicand is to be repeated or added is called the *Multiplier.*

62. The Multiplicand and Multiplier are called the *Factors* of the product.

Thus, in the statement, 6 times 9 miles are 54 miles, 54 miles is the *product,* 6 and 9 are the *factors,* 9 being the *multiplicand* and 6 the *multiplier.*

The difference between *factors* and *parts* of a number should be carefully taught.

The *factors* are *multiplied,* but the *parts* are *added* to produce a number.

Thus, 2 and 4 are *factors* of 8; the *parts* of 8 are 5 and 3, 4 and 4, 6 and 2, etc.

63. The **Sign of Multiplication** is $\times$. It is read *times,* or *multiplied by.*

When placed between two numbers, it shows that the number before it is to be multiplied by the number after it.

Thus, 8×7 is read, 8 *multiplied by* 7, or 7 *times* 8. The expression 8×7 may be read 7 *times* 8, or 8 *times* 7, since changing the order of the factors does not change the product.

64. Nothing is more essential in business transactions than the ability to reckon with *rapidity* and *accuracy.*

The pupil should therefore be able to multiply by any integral number less than 13, without the least hesitation. In order to do this, he must know the multiplication table perfectly, carried as far at least as 12.

65. Multiplication Table.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

Drill Exercises.

66. The following exercises will furnish easy and attractive methods of memorizing the Table:

1. Write on the slate or blackboard a series of 2's, and under them write the numbers from 1 to 12; thus,

2	2	2	2	2	2	2	2	2	2	2	2
1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24

Then say, 1 two is 2; 2 twos are 4; 3 twos are 6; 4 twos are 8, and so on to 12 twos are 24. Write the *products* under each example, and observe that each succeeding product is obtained by *adding* 2 to the last one.

2. Next erase the *products*, and rewrite them from memory until they can be given *orally* and *at sight*.

3. Then arrange the numbers in the lower line in any order, and give the *products* orally at sight; thus,

2	2	2	2	2	2	2	2	2	2	2	2
3	5	6	2	7	4	1	8	11	9	10	12

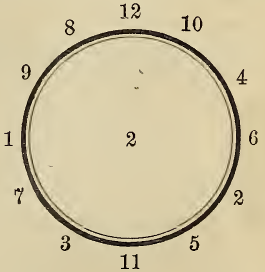
4. Then vary the exercise as follows:

1	2	3	4	5	6	7	8	9	10	11	12
2	2	2	2	2	2	2	2	2	2	2	2

Write and erase the *products* under each example, until from memory the result can be given *at sight*.

5. In the same manner memorize and practice upon the products by 3's, 4's, 5's, 6's, to 12's, inclusive.

67. The little device here presented may be used to facilitate the preceding operations:



Make a small circle on the slate or board, and around the outside write the numbers from 1 to 12 inclusive, either in order or irregularly.

Write one of the numbers in the centre for a multiplier, as 2, and commencing at any number in the margin, as at 12, and going around the circle to the *right*, read thus: 2 times 12 are 24, 2 times 10 are 20, etc.; then in like manner read to the *left*. Then use in rapid succession the figures in the margin for multipliers, and the central figure for the multiplicand, thus: 12 times 2 are 24, 10 times 2 are 20, etc.

The teacher may now point in rapid succession to the numbers in the margin, and the pupil promptly give the *products;* thus, the teacher points to 12, the pupil instantly gives the product 24; to 10, he responds, 20; to 4, he says 8, etc.

Then, by erasing the 2 and inserting any other number, we have a new table or set of products, to be treated in the same manner; and so for the entire tables.

68. Write or pronounce the second member in each of the following equations:

$4\times 8=?$	$8\times 5=?$	$7\times 8=?$	$9\times 9=?$
$3\times 5=?$	$5\times 7=?$	$4\times 6=?$	$10\times 4=?$
$6\times 4=?$	$9\times 3=?$	$9\times 8=?$	$11\times 5=?$
$9\times 6=?$	$6\times 7=?$	$7\times 7=?$	$12\times 3=?$

The pupil may make *applied examples* for the above. Thus, for 4×8: "What will 8 oranges cost, at 4 cents each?" Another pupil may solve it; thus, "8 oranges will cost 8 times 4 cents, or 32 cents." For 3×5: "If 3 bushels of apples fill 1 barrel, how many bushels will fill 5 barrels?" etc.

For *dictation:*

6 times 5, plus $4=?$	$9\times 7-12=?$	$95-7\times 12=?$
3 times 8, plus $9=?$	$9\times 9+14=?$	$24-10\times 0=?$
5 times 9, less $7=?$	$7+3\times 10=?$	$0\times 11+20=?$
8 times 6, plus $5=?$	$46-5\times 9=?$	$8\times 0\times 9=?$

The above exercises may be extended, at the option of the teacher.

Mental Exercises.

69. 1. What do we mean when we say, multiply 9 by 6?

2. In the statement, 8 times 9 are 72, which number is the multiplicand? The multiplier? The product?

3. Of what number are 7 and 5 factors? 8 and 9?

4. What are the two factors of 15? 18? 25? 30? 36?

5. If 9 is one of the factors of 45, what is the other?

6. How many are 2 times 4 times 6? Of what are 2, 4, and 6 factors?

7. What is the cost of 9 tons of coal, at $6 a ton?

Analysis. Nine tons of coal, at $6 a ton, costs 9 times $6, or $54.

Find the cost

8. Of 10 oranges, at 4 cents each. At 5 cents.

9. Of 11 yards of cloth, at $3 a yard. At $5. At $7.

10. What is the cost of 9 ploughs, at $11 each? At $12?

11. What is the cost of 12 hats, at $5 each? At $7?

12. At $12 each, what is the cost of 7 sheep? 10 sheep?

13. What will be the cost of 4 barrels of flour at $10 a barrel, and 1 ton of coal at $6?

14. At $2 a bushel, what will be the cost of 6 bags of wheat, each bag containing 3 bushels?

15. George had 9 marbles, and Charles had 5 times as many, less 5. How many had both?

16. Annie gave 6 cents each for 7 oranges; how much change should she receive for 50 cents?

17. If one man earns $19 a week, another $12, what will both earn in 3 weeks? In 6 weeks?

18. What is the difference in the cost of 12 cords of wood, at $4 a cord, and 9 tons of coal, at $6 a ton?

19. In how many days will 1 horse eat as many oats as 8 horses will eat in 12 days?

20. A tailor cut 16 yards of cloth from a piece containing 25 yards. What was the remainder worth, at $4 a yard?

21. How much less than $75 will 6 barrels of flour cost, at $9 a barrel? How much more than $47?

What is the result:

22. Of 5 times 7 units? Of 7×5? Of 9×6? Of 8×7?

23. Of 5 times 7 tens? Of 70×5? Of 80×5?

24. Of 5 times 7 hundreds? Of 700×5? Of 600×3?

25. Of 5 times 7 thousands? Of 7000×5? Of 6000×6?

26. Of 6×5? 60×5? 600×5? 6000×5? 60000×5?

Observe that each order of units is independent of every other order in a number, and that to find the product of a given number taken any number of times, each order must be multiplied separately.

Thus, to find 5 times 73, separate 73 into 7 tens 3 units, or $70 + 3$, and taking each of these parts 5 times, the sum of the products will be 5 times 73; or 5 times 70 plus 5 times 3 equals $350 + 15 = 365$.

27. How many are 6 times 41 ? 45? 54? 62? 75?

28. How many are 7 times 63? 8 times 64? 9 times 23?

29. At 50 cents a pound, what will 4 pounds of tea cost? 6 pounds? 8 pounds? 10 pounds? 12 pounds?

30. At $45 each, what must be paid for 6 cows? 5 cows? 7 cows? 8 cows? 9 cows?

31. If an express train runs 53 miles an hour, how far will it run in 7 hours? In 9 hours? In 12 hours?

32. What will be the cost of 32 pairs of boots, at $5 a pair? At $6? At $7? At $9?

33. If a clerk earns $90 a month, and spends $50, how much can he save in 6 months? In 11 months?

34. If a vessel sails 200 miles in 1 day, how far will she sail in 4 days? In 6 days? In 8 days?

35. A tailor has a piece of cloth containing 63 yards; he cuts from it 13 suits, each containing 4 yards. How many yards remain?

36. A man bought 7 tons of hay, at $15 a ton, and sold the same for $20 more than he gave for it; what did he receive?

37. How much more will 10 pounds of beef cost, at 16 cents a pound, than 10 pounds of pork, at 12 cts. a pound?

38. A man owing $100, gave a cow worth $30, 15 cords of wood, at $3 a cord, and the remainder in money; how much money did he pay?

39. How much less is 8 times 14 than 3 times 44?

For *dictation:*

40. $7 + 10, -12, \times 6, +5, -15, -10, \times 3, -8, +8, -5, +2 = ?$

41. $18 - 8, \times 3, -15, +10, \times 2, +20, -50, +4, -10 = ?$

42. $24 + 30, -50, \times 9, +4, -10, \times 2, +10, +5, -25, \times 10 = ?$

43. $35-18, \ -7, \ \times 4, \ +6, \ -26, \ -12, \ \times 8, \ +6, \ -30, \ +7 = ?$

44. $15 \times 3, \ +5, \ -25, \ +9, \ -17, \ -10, \ \times 12, \ +6, \ +10, \ -50 = ?$

45. $12 \times 12, \ -100, \ +6, \ -40, \ \times 2, \ \times 3, \ +12, \ -60, \ \times 10, \ +8 = ?$

46. $200-150, \ \times 2, \ -60, \ +10, \ -30, \ \times 4, \ +15, \ -5, \ \times 10, \ -40 = ?$

47. $40+23, \ +7, \ -50, \ \times 2, \ +5, \ -15, \ -20, \ \times 3, \ +25, \ -9, \ +4, \ -18 = ?$

70. PRINCIPLES. I. *The multiplicand may be either an abstract, or a denominate number.*

II. *The multiplier is always an abstract number.*

III. *The product and multiplicand are like numbers.*

IV. *The product of any number of factors in continued multiplication will be the same, in whatever order they are taken.*

When one of the factors is a concrete number, it is the *true multiplicand;* but when it is the smaller, it is often used for convenience, abstractly, as the multiplier.

DEFINITIONS

71. The **Product** of two or more numbers is the result obtained by repeating one number as many times as there are units in the other. Or,

It is the *sum* of the same number taken a given number of times.

72. Multiplication is the process of finding the product of two numbers.

73. The **Multiplicand** is the number to be multiplied.

74. The **Multiplier** is the number by which we multiply, and shows *how many times* the multiplicand is to be taken.

3

WRITTEN EXERCISES.

75. ***When the multiplier consists of but one order of units.***

1. Multiply 286 by 4.

FIRST OPERATION.

286	Equal Parts.
286	
286	
286	
1144	Sum.

EXPLANATION. Since the units, tens, and hundreds in 286 are each required to be taken 4 times, the result may be found by writing 286 four times in a column and finding the *sum*.

SECOND OPERATION.

Multiplicand,	286	=	200 +	80 +	6
Multiplier,	4		4	4	4
Product,	1144	=	800 +	320 +	24

EXPLANATION. The preceding process can be very much shortened by writing the multiplicand 286 but once; and since its parts, 2 hundreds, 8 tens, and 6 units, are each to be taken 4 times, write the multiplier 4 under the units and multiply, thus: 4 times 6 units are 24 *units*, or 2 tens 4 units. Write the 4 units in units' place, and reserve the 2 tens to be added to the product of the tens.

Then 4 times 8 tens are 32 tens, and 2 tens reserved added make 34 *tens*, or 3 hundreds 4 tens. Write the 4 tens in tens' place, and reserve the 3 hundreds to be added to the product of the hundreds.

Lastly, 4 times 2 hundreds are 8 hundreds, and 3 hundreds reserved added make 11 *hundreds*, or 1 thousand 1 hundred, which write in the hundreds' and thousands' places. Hence, the *product* 1144 is the same as the *sum* of its equal parts obtained by addition.

	(2.)	(3.)	(4.)	(5.)
Multiply	475	3264	1807	46251
by	6	5	7	8
Product,	2850	16320		

6. Multiply 1341 feet by 6; by 4; by 7; by 8; by 9.
7. Multiply $3245 by 3; by 6; by 5; by 9; by 8.
8. What will be the cost of 8 horses, at $326 each?
9. What cost 9 lots of land, at $1248 each? at $950 each?

10. What is the cost of 12 acres of land, at $206 an acre?

11. At $8463 each, what is the cost of 9 palace cars?

12. What is the cost of building 11 houses, at $2560 each?

13. A grocer bought 9 barrels of sugar, at $23 a barrel, and sold the whole for $245.75. What was his gain?

14. A farmer sold 5 horses, at $216 each, and 137 sheep, at $6 each; what did he receive for all?

15. What will 384 barrels of flour cost, at $8 a barrel?

We may use 384 for the multiplicand, reasoning as follows: If the flour was at $1 a barrel, the cost would be $384; hence, at $8, it would be 8 *times* $384. (See Note, p. 49.)

16. If a man travels 9 miles an hour, how far will he travel in 684 hours? In 1256 hours?

17. What cost 420 yards of cloth, at $5 a yard? at $7?

18. Find the cost of 596 barrels of flour, at $8 a barrel?

19. At 9 cts. a pound, what is the cost of 125 pounds of sugar? Of 256 pounds?

When either factor contains cents, the *product* is *cents*, which may be changed to dollars and cents by inserting the point (.) *two* places from the *right*, and placing the sign ($) at the left. Thus, 9 cts. $\times$ 125 = 1125 cts. = $11.25.

What will be the cost

20. Of 796 pounds of grapes, at 5 cts. a pound? at 7 cts.?

21. Of 1248 pounds of rice, at 7 cts. a pound? at 9 cts.?

22. Of 9 barrels of potatoes, at $4.75 a barrel? at $3.87?

23. Of 8 rods of fence, at $12.25 a rod? at $17.68?

Multiply each of the following numbers by each of the numbers from 2 to 12 inclusive:

24. 237.	28. $42.63.	32. 32014.	36. 180624.
25. 438.	29. $84.31.	33. 40370.	37. 246030.
26. 620.	30. $65.74.	34. 56781.	38. 637819.
27. 734.	31. $75.24.	35. 81009.	39. 708063.

76. DRILL TABLE No. 2.

	A	B	C	D	E	F	G	H
1.	4	9	2	6	5	7	12	40
2.	1	3	4	8	2	5	9	200
3.	7	6	8	4	4	3	10	12
4.	3	8	2	5	7	9	11	40
5.	9	3	6	7	3	5	30	400
6.	5	7	3	6	8	4	8	350
7.	6	4	5	2	7	6	50	80
8.	8	5	4	3	6	7	15	120
9.	2	9	6	7	5	8	9	500
10.	5	2	8	3	4	3	20	640
11.	9	3	2	6	9	5	12	300
12.	7	8	5	9	3	6	26	700
13.	4	2	3	5	7	9	40	70
14.	6	5	7	3	4	2	90	200
15.	8	1	6	4	5	7	10	910
16.	2	8	4	7	3	5	9	800

Drill Table No. 2 will furnish all the additional exercises for the slate and board that will be required.

When the multiplicand contains *three* figures, and the multiplier, *one* figure, copy the figures opposite **1**, **2**, **3**, etc., in columns A B C for multiplicands, and for multipliers use the figure directly under or over the right-hand figure of each multiplicand. Thus,

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)
492	134	768	382	936	573
4	8	2	6	3	5

In the same manner, copy examples from columns B C D, C D E, D E F, etc.

In like manner, copy from the several columns, commencing opposite **1**, multiplicands containing *four*, *five*, or more figures, using the figure directly under or over the right-hand figure of each multiplicand for a multiplier.

77. ***When the multiplier consists of two or more orders of units.***

1. Multiply 673 by 235.

	OPERATION.	
Multiplicand,	673	
Multiplier,	235	
1st Partial Prod.,	3365	$= 673 \times 5$
2d Partial Prod.,	2019	$= 673 \times 30$
3d Partial Prod.,	1346	$= 673 \times 200$
Entire Product,	158155	$= 673 \times 235$

EXPLANATION. Write the multiplier under the multiplicand, like orders of units in the same column (**42**).

Since the multiplier consists of 5 *units*, 3 *tens*, 2 *hundreds*, the multiplicand is to be taken 5 times + 30 times + 200 times, or 235 times.

5 times 673 is 3365, the *first* partial product; 3 tens or 30 times 673 is 20190, the *second* partial product; and 2 hundreds or 200 times 673 is 134600, the *third* partial product. The *sum* of these partial products is the entire product 158155.

In practice we omit the ciphers at the right of the partial products.

2. Multiply 876 by 56; by 75; by 127; by 243.

RULE. 1. *Write the multiplier under the multiplicand, placing units of the same order in the same column.*

2. *Multiply each order of units of the multiplicand by each order of units of the multiplier successively, writing the unit figure of each partial product under the order of the multiplier used. The sum of the partial products will be the product required.*

When ciphers occur *between* the significant figures of the *multiplier*, pass over them, and multiply by the significant figures only.

3. Multiply 2468 by 215; by 324; by 146; by 266.
4. Multiply 36072 by 205; by 276; by 308; by 436.
5. Multiply $645.36 by 72; by 108; by 66; by 263.

Find the value

6. Of 64 railway coaches, at $9045.75 each.
7. Of 456 acres of land, at $108.50 an acre.
8. Of 2625 bushels of wheat, at $1.75 a bushel.
9. Of 277 hogsheads of molasses, at $57.25 a hogshead.
10. Bought two farms; one containing 217 acres at $57 an acre, the other 326 acres at $48 an acre. What was the cost of both? What the difference in their cost?

Multiply

11. 30742 feet by 178.
12. 3407 days by 406.
13. $762.84 by 562.
14. 81794 cts. by 208.
15. 40306 pounds by 97.
16. 134062 quarts by 346.
17. 37012 × 1242 = ?
18. $460.76 × 804 = ?
19. 560036 × 2423 = ?
20. 764209 × 846 = ?
21. 135062 × 3405 = ?
22. 3027061 × 20674 = ?

78. For additional examples when the multiplier contains *two* or more figures, copy from Drill Table No. 2.

Commencing opposite **1**, copy multiplicands containing three, four, five, or more figures from columns A B C D, etc., and for the multipliers use the figures directly under the right-hand figures of the multiplicand. Thus,

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)
4926	1348	7684	3825	9367	5736
48	84	25	67	36	52

In like manner, copy examples from columns B C D E, C D E F, etc.

Then copy multiplicands with *five*, or more figures and multipliers with *three*, or more figures.

79. ***To multiply by the factors of a number.***

80. The **Factors** of a number are the numbers which multiplied together produce that number.

Thus, 18 is composed of three 6's, or six 3's; of two 9's, or nine 2's; hence 3 and 6, and 2 and 9 are the *factors* of 18.

81. The process of finding the product of several factors, by multiplying the first by the second, and the result by the third, and so on, is called *Continued Multiplication.*

Thus, $86 \times 24 = 2064$, or $86 \times 6 \times 4 = 2064$, or $86 \times 4 \times 3 \times 2 = 2064$. It is obvious that the product will be the same whether the multiplicand be multiplied by the multiplier or by any *set of factors* of which the multiplier may be composed (Prin. IV).

Multiply

	Factors.	
1. 3462 by 36,	or by 9×4.	4. \$643.26 by 96.
2. 8637 by 72,	or by $6 \times 4 \times 3$.	5. 30762 by 108.
3. \$406.16 by 64,	or by $8 \times 4 \times 2$.	6. 47623 by 125.

82. ***When there are ciphers at the right of one or both factors.***

1. Multiply 375 by 100.

EXPLANATION. Annexing one cipher to a number removes each order of units one place to the *left*, and thus increases its value ten times, or *multiplies* it by 10. For the same reason, annexing two ciphers multiplies by 100, etc. (**15**, III).

OPERATION.

$$\begin{array}{r} 375 \\ 100 \\ \hline 37500 \end{array}$$

2. Multiply 850 by 500.

EXPLANATION. The multiplicand is equal to 85×10, and the multiplier to 5×100; these taken in any order will give the same result. Hence, first multiply 85 by 5, and the product 425 by 10×100, or annex three ciphers, and the entire product is 425000.

OPERATION.

$$\begin{array}{r} 850 \\ 500 \\ \hline 425000 \end{array}$$

RULE. *To the product of the significant figures annex as many ciphers as there are on the right of both factors.*

3. Multiply 370 by 10; by 100; by 60; by 250.
4. Multiply 1360 by 140; by 700; by 9000; by 6700.
5. Multiply 2460 by 200; by 480; by 5000; by 2500.
6. Multiply 6304200 by 500; by 2700; by 3040.
7. What is the continued product of 310, 7060, and 600?
8. What is the value of 150 barrels of beef, each containing 220 pounds, at 15 cents a pound? At 20 cents?
9. At 64 cents a bushel, what are 46 acres of corn, averaging 36 bushels to the acre, worth?
10. Find the cost of 60 sacks of coffee, each containing 56 pounds, at 28 cents a pound; at 30 cents.
11. What is the value of 115 pieces of cloth, each piece containing 48 yards, at $3.75 a yard? If each piece contained 60 yards, at $4.45 a yard?
12. A barrel of salt weighs 280 pounds; what is the weight of 300 barrels? Of 450 barrels?
13. How many yards of muslin in 67 bales, each bale containing 35 pieces, and each piece 48 yards? What is its value, at 14 cents a yard?

Perform the multiplication of the following:

14. $5640 \times 80.$	17. $\$360.09 \times 72.$	20. $840 \times 200 \times 75.$
15. $3007 \times 600.$	18. $\$403.50 \times 60.$	21. $3214 \times 304 \times 90.$
16. $8621 \times 320.$	19. $\$25600 \times 700.$	22. $4006 \times 1200 \times 64.$

Complete the following equations:

23. $7 \times 12 + 16 - 20 = ?$	28. $17 + 13 - 5 \times ? = 10.$
24. $9 \times 8 - 12 = 5 \times ?$	29. $14 \times 0 \times 7 + 35 = 7 \times ?$
25. $7 \times 20 - 5 \times 25 = ?$	30. $56 - 40 + 8 = ? \times 8.$
26. $84 - (6 \times 11 + 10) = ?$	31. $10 \times 12 + 20 = 150 - ?$
27. $\overline{36 - 24} \times 8 = 100 - ?$	32. $(5 \times 20 - 75 + 5) \times 3 = ? \times 10.$

The operations of multiplication, indicated by signs, must be performed *before* those of addition and subtraction, unless otherwise indicated.

Written Exercises.

83. 1. The sum of three numbers is 1586; the least is 218, and the greatest 934; what is the third number, and the product of the three?

2. Multiply 6 times the difference between \$29045 and \$14000 by the sum of 462 and 208.

3. What is the sum of \$426.25 × 97 and \$84.05 × 146?

4. Multiply 1736 increased by 78 times 156 by 4756 diminished by 26 times 67.

5. A produce dealer bought 1488 barrels of potatoes at \$3.66 a barrel, and sold them at \$4.28 a barrel. What did he gain?

6. What is the difference in the cost of 174 head of cattle, at \$47.50 a head, and 1240 head of sheep at \$6 a head?

7. Paid \$15675 for a farm of 275 acres, and sold the same for \$63 an acre; what was the whole gain?

8. A drover paid \$42.50 a head for 216 head of cattle, \$9.40 a head to get them to market, and then sold them for \$55 a head; what was the gain?

9. A young man receiving a salary of \$1500 a year, pays \$425.75 for board, \$175 for clothing, \$84.75 for books, \$150 for donations, and \$163.87 for other expenses; how much can he save in 6 years? In 10 years?

10. A miller exchanged 175 barrels of flour at \$7.84 a barrel, for 860 bushels of wheat at \$2 a bushel, paying the balance in money; how much money did he pay?

11. A railroad company having built 286 miles of road, at \$12745 a mile, paid \$1500000; what did they still owe?

12. What was the weight and value of a crop of cotton which was put up in 394 bales, averaging 568 pounds each, and valued at 17 cents a pound?

13. What is the cost of the rails for a railroad 184 miles long, if 126 tons are required to the mile, at \$58.50 a ton?

14. What is the difference in the value of 920 bushels of wheat at \$1.84 a bushel, and 375 barrels of flour at \$7.50 a barrel?

15. A house is worth \$2475; the farm on which it stands is worth 6 times as much less \$1250, and the stock on the farm is worth \$750 more than 3 times the value of the house. What is the value of the whole, and of the farm and stock respectively?

16. A grocer bought 12 barrels of sugar at \$16 a barrel, and 17 barrels at \$13 a barrel; how much would he gain by selling the whole at \$18 a barrel?

17. Bought two farms worth \$4550 each, 20 shares of railroad stock at \$106 per share, and had \$250 left. How much had I at first?

18. A grocer bought 15 tubs of butter, each containing 48 pounds, at \$.28 a pound, and sold the same for \$.35 a pound. What was his gain?

19. A coal dealer bought 10 car loads of coal, each car containing 12 tons, at \$4.25 a ton. He sold 50 tons at \$5.40 a ton, 36 tons at \$6 a ton, and the remainder at \$6.20 a ton. How much did he make by the transaction?

20. Multiply $7520 + \overline{12500 - 1075} \times 7$ by 6 times the difference between 973 and $1000 - 368$.

21. Multiply $\overline{9760 + 1242} \times 12 - 32400$ by $1524 - 4 \times 350$.

22. Multiply $1425 + 84 \times 65 - 750$ by 12 times 16.

23. $(\overline{1462 + 684} \times \overline{5000 - 4 \times 1240}) \times 70 = ?$

24. $\$78672.75 - \$3650 + \$10600 \times \overline{286 - 156} = ?$

25. $\$.87 \times 32700 + \$10500 - (\$15376.40 - \$14700.50) \times 24 = ?$

26. $756400 - 3900 \times 70 + (\overline{167 + 328} \times 1260 - 40 \times 30) = ?$

DIVISION.

INDUCTIVE EXERCISES.

84. 1. How many feet are 12 feet less 6 feet, less 6 feet? 12 feet — 4 feet — 4 feet — 4 feet? 12 feet — 3 feet — 3 feet — 3 feet — 3 feet?

2. How many times can 6 feet be taken from 12 feet? 4 feet from 12 feet? 3 feet from 12 feet?

3. How many 6's in 12? How many 4's? 3's? 2's?

4. How many times can 5 pounds of tea be taken from a box containing 30 pounds? How many 5's in 30?

5. Into how many *equal parts* are 30 pounds separated? What is the size of each part? 5 pounds is contained in 30 pounds how many times?

6. Thirty is how many times 5? 6? 10? 3?

7. Distribute 30 pounds of tea equally among 6 families; how many pounds will each receive?

8. Can we say 6 *families* are contained in 30 *pounds* of tea 5 times? Why not?

9. Can we subtract 6 *families* from 30 *pounds* of tea 5 times? Why not?

10. How then shall we find *one* of 6 *equal parts* of 30 pounds of tea?

One of the 6 equal parts of 30 pounds of tea is 5 pounds, since *six* 5's or 6 times 5 make 30.

11. What is one of 6 equal parts of 30 pounds? Of 42 feet?

12. What is one of 3 equal parts of 30 pounds? Of 27 acres?

13. When we say, 7 cents is *contained* in 63 cents, is the result denominate or abstract?

14. When we say, one of 7 *equal parts* of 63 cents, is the result denominate or abstract?

85. The process of finding how many times the same number may be taken from a given number, or of finding one of the equal parts into which a number may be divided, is called *Division*, and the result obtained is called the *Quotient*.

Thus, if 24 marbles are divided equally among a number of boys, giving each boy 6 marbles, how many boys are there?

Here the whole number and one of the equal parts are given, to find the *number* of equal parts.

Again, if 24 marbles are divided equally among 4 boys, how many marbles will each boy receive?

Here the whole number and the number of equal parts are given, to find the *size* or *value* of *one* of the equal parts.

86. The number to be divided into equal parts of a known size, or which is to be separated into a given number of equal parts, is called the *Dividend*.

Thus, in the above examples, 24 marbles is the dividend.

87. *One* of the equal parts into which the dividend is to be divided, or the *number* of equal parts into which the dividend is to be divided, is called the *Divisor*.

Thus, in the first example, 6 marbles is the divisor; and in the second, 4 is the divisor.

88. Division may be regarded as the reverse of multiplication, since in multiplication *both factors* are given to find the *product*; while in division, *one factor* and the *product* (answering to the *divisor* and *dividend*) are given to find the other factor, called the *Quotient*.

Thus, $9 \times 4 = 36$, the factor 9 being taken 4 times gives the product 36; hence, there are *four* 9's in 36, or 9 is contained in 36, 4 times.

89. If anything remains after dividing the dividend, it is called the *Remainder*, and it must always be *less* than the divisor.

When there is no remainder, the division is said to be *exact*, and the dividend is the product of the divisor and quotient, and each expresses *one* of the *equal parts* into which the dividend may be separated.

Thus, 8 is contained in 32, 4 times; the divisor 8 expresses one of the *four* equal parts of 32, as also the quotient 4 expresses one of the *eight* equal parts of 32.

90. The **Sign of Division** is $\div$. It is read, *divided by*.

When placed between two numbers it shows that the one on the left is to be divided by the one on the right.

Thus, $63 \div 7 = 9$ is read, 63 *divided by* 7 *equals* 9, and signifies that there are *nine* 7's in 63, or 7 is contained in 63, 9 times, since 9 times 7 is 63.

91. Division is also indicated by writing the dividend *above* and the divisor *below* a short horizontal line.

Thus, $\frac{18}{6}$ is read, 18 *divided by* 6; $\frac{56}{8} = 7$ is read, 56 *divided by* 8 *equals* 7.

DRILL EXERCISES.

92. 1. First, to find *at sight* how many times any number from 2 to 12 inclusive, used as a factor, is contained in the several products taken from the Multiplication Table, the result being the other factor.

Thus, begin with the products which have 2 as one factor, taken in any order, and find how many 2's in 6, 8, 12, 18, etc.

Written, $2\,\underline{)\,6}$ $2\,\underline{)\,8}$ $2\,\underline{)\,12}$ $2\,\underline{)\,18}$ $2\,\underline{)\,20}$ $2\,\underline{)\,24}$, etc.
$\quad 3 \quad 4 \quad 6$

Write under the products the number of 2's each contains, which is the other factor, and say, 2's in 6, *three*; 2's in 8, *four*; 2's in 12, *six*, etc.

The number at the left of the curved line is the *divisor,* the one at the right the *dividend,* and the result obtained is the *quotient.*

Thus, 2) 6 may be expressed, $6 \div 2$.

2) 16 is the same as $16 \div 2 = 8$, read, 16 *divided by* 2 *equals* 8.
     8

Erase and rewrite the quotients, until they can be promptly given from memory.

2. Practice in the same manner upon the products which have 3, 4, 5, 6, to 12 inclusive, as factors taken from the table.

3. Divide by 2 orally, from 2 in 2, to 2 in 24.

Thus, 2 in 2, *once ;* 2 in 4, *twice ;* 2 in 6, 3 *times ;* 2 in 8. 4 *times,* etc

Also the following:

4. 3 in 3, to 3 in 36.
5. 4 in 4, to 4 in 48.
6. 5 in 5, to 5 in 60.
7. 6 in 6, to 6 in 72.
8. 7 in 7, to 7 in 84.
9. 8 in 8, to 8 in 96.
10. 9 in 9, to 9 in 108.
11. 10 in 10, to 10 in 120.
12. 11 in 11, to 11 in 132.
13. 12 in 12, to 12 in 144.

14. *Reverse* the above ; thus, 2 in 24, 12 times ; 2 in 22, 11 times ; 2 in 20, 10 times, etc.

15. Then *combine ;* thus, 3 in 3, *once ;* 3 in 6, *twice ;* 2 in 6. 3 times; 3 in 12, 4 times ; 4 in 12, 3 times, etc.

Express by the proper *signs* the following and their answers; thus, How many 3's in 12? $12 \div 3 = 4$, or $\frac{12}{3} = 4$.

16. How many 6's in 24? In 48? In 54? In 60? In 36?
17. How many 8's in 40? In 64? In 72? In 96? In 80?
18. How many 9's in 36? In 45? In 90? In 63? In 81?
19. How many 10's in 40? In 70? In 50? In 100?
20. How many 12's in 48? In 60? In 84? In 96?
21. How many times 6 men are 48 men? 5 feet are 45 feet? 7 days are 84 days? 8 cents are 72 cents?

For *dictation:*

$48 \div 8 = ?$ $54 \div 9 = ?$ $60 \div 5 = ?$ $72 \div 12 = ?$

$84 \div 7 = ?$ $96 \div 12 = ?$ $66 \div 11 = ?$ $108 \div 9 = ?$

$\frac{96}{8} = ?$ $\frac{84}{7} = ?$ $\frac{72}{9} = ?$ $\frac{88}{8} = ?$ $\frac{120}{10} = ?$ $\frac{144}{12} = ?$

93. This last form of indicating division is often used to simplify two or more operations to be performed in the same example. Thus,

Multiply 12 by 6 and divide the product by 8 may be expressed, $\frac{12 \times 6}{8} = 9$, since $12 \times 6 = 72$, and $72 \div 8 = 9$.

To 10 times 9 add 6 and divide the sum by 12.

Written, $\frac{10 \times 9 + 6}{12} = 8$, since $10 \times 9 = 90$, and plus 6 is 96, and $96 \div 12 = 8$.

Express by *signs* each of the following:

1. Divide 7 times 9 plus 3, by 11.
2. Divide the difference of 36 and 12, by 8.
3. Divide the product of 12 and 5, by 10.
4. Divide the sum of 4 times 8 and 5 times 8, by 6.
5. From the sum of 24 and 16 subtract 12, and divide the remainder by 7.

What is the value of the following expressions?

(6.) $\frac{45 - 15}{6}$. (7.) $\frac{11 \times 6 + 6}{9}$. (8.) $\frac{8 + 6 \times 12}{10}$. (9.) $\frac{7 \times 9 - 13}{5}$.

(10.) $\frac{8 \times 3 + 3 \times 10}{9}$. (11.) $\frac{12 \times 6 - 4 \times 8}{4}$. (12.) $\frac{42 + 38 - 20}{3 \times 4}$.

The operations of multiplication and division, when indicated by *signs*, must be performed before those of addition and subtraction, unless otherwise indicated.

MENTAL EXERCISES.

94. 1. How many times can 6 cents be taken from 42 cents? from 54 cents? from 60 cents? from 72 cents?

2. How many times 7 feet make 42 feet? 63 feet?

3. The product of two factors is 54 rods, and one of the factors is 9 rods; what is the other? Why?

4. If the product of two factors is 63 bushels, and one of the factors is 7, what is the other? If 7 bushels?

5. If the dividend is 72 and the divisor 8, what is the quotient? If the divisor is 9? 6? 12?

6. If the quotient is 9, and the divisor 8, what is the dividend? Why?

7. If the dividend is 72, and the quotient 8, what is the divisor? Why?

8. At $6 a ton, how much ice can be bought for $54?

ANALYSIS. As many tons, as $6 is contained times in $54, which is 9 times. Hence, 9 tons can be bought for $54.

9. At 7 cts. a quart, how many quarts of milk can be bought for 56 cts.? for 63 cts.? for 70 cts.? for 84 cts.?

10. How many days' labor, at $4 a day, will pay for 12 yards of cloth, at $3 a yard?

ANALYSIS. Twelve yards of cloth, at $3 a yard, will cost $36; and $4, the price of 1 day's labor, is contained in $36, 9 times. Hence, 9 days' labor will pay for 12 yards of cloth.

11. Six pounds of meat, at 12 cents a pound, are worth how many pounds of rice, at 8 cents a pound?

12. How many 5-dollar bills will pay for 9 cords of wood, at $4 a cord, and 4 barrels of flour, at $6 a barrel?

For *dictation:*

13. 3 times 8, divided by 6 = ?
14. 3 times 12, divided by 9 = ?
15. 12 times 6, divided by 8 = ?
16. 3 times 8, divided by 12 = ?
17. $(36-16)\div 5=$?
18. $(60-20)\div 8=$?
19. $(49-13)\div 9=$?
20. $(24+30)\div 6=$?

Find one of the

21. 2 equal parts of 14.
22. 3 equal parts of 27.
23. 4 equal parts of 40.
24. 5 equal parts of 60.
25. 6 equal parts of 54.
26. 7 equal parts of 77.
27. 8 equal parts of 64.
28. 9 equal parts of 108.

29. How is one of 2 equal parts of a number found? One of 3 equal parts? One of 4? Of 5? 6? 7? 8? 9? 10?

The *names* of the equal parts of a thing, or number, vary according to the *number* of the parts. Thus,

One of 2 equal parts is named **one half,** written $\frac{1}{2}$, which signifies 1 *divided by* 2. $\frac{1}{2}$ of 12 is $12 \div 2 = 6$.

One of 3 equal parts is named **one third,** written $\frac{1}{3}$, which signifies 1 *divided by* 3. $\frac{1}{3}$ of 24 is $24 \div 3 = 8$.

One of 4 equal parts is named **one fourth,** written $\frac{1}{4}$, which signifies 1 *divided by* 4. $\frac{1}{4}$ of 48 is $48 \div 4 = 12$.

In like manner we obtain the names of *fifths, sixths, sevenths, eighths, tenths, twelfths, twentieths,* etc.

The equal parts into which a unit is divided are called *Fractions.*

30. If an acre of land is divided into 2 equal parts, what is each part called? If into 3 equal parts? 4? 5? 6? 7? 8? 10? 12? 15? 20? 45?

31. What is *one third* ($\frac{1}{3}$) of 12 months? *One fourth* ($\frac{1}{4}$) of 28 days?

32. What is *one fifth* ($\frac{1}{5}$) of \$60? *One sixth* ($\frac{1}{6}$) of 60?

33. If 48 cts. are equally divided among 6 boys, what *part* of 48 cents will each boy receive? If among 8 boys?

34. If \$30 are paid to 5 men in equal parts, what part of \$30 is paid to 1 man? If paid to 3 men? To 6 men?

35. What is $\frac{1}{4}$ of 36 pounds? $\frac{1}{5}$ of 40 men? $\frac{1}{6}$ of 54 acres?

36. What is $\frac{1}{7}$ of 63 gallons? $\frac{1}{8}$ of 72 pints? $\frac{1}{9}$ of 81 men?

37. What is $\frac{1}{10}$ of 100 rods? $\frac{1}{11}$ of \$88? $\frac{1}{12}$ of 120 days?

38. How do we find $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, etc., of any number?

39. If 9 men pay $63 for the use of a room, what does each man pay?

ANALYSIS. Each man pays *one ninth* ($\frac{1}{9}$) of $63, which is $7.

40. If a man pays $96 for the rent of his house 8 months, what is his rent for 1 month?

41. What is $\frac{1}{4}$ of 48? $\frac{1}{2}$ of $\frac{1}{4}$ of 48? $\frac{1}{2}$ of 4 times 12? $\frac{1}{3}$ of $\frac{1}{5}$ of 60? $\frac{1}{6}$ of $\frac{1}{2}$ of 24? $\frac{1}{7}$ of 5 times 14? 4 times $\frac{1}{8}$ of 64?

42. How many are 8 times $\frac{1}{6}$ of 42 feet? 9 times $\frac{1}{7}$ of $63?

43. How many are 10 times $\frac{1}{3}$ of 36 days? 7 times $\frac{1}{12}$ of $96?

44. If 7 yards of cloth cost $35, what will 9 yards cost?

ANALYSIS. One yard will cost $\frac{1}{7}$ of $35, or $5; and 9 yards will cost 9 times $5, or $45. Hence, etc.

45. What will 12 pounds of beef cost, if 5 pounds cost 60 cents?

46. If a man earns $84 in 7 weeks, how much does he earn in 3 weeks? In 5 weeks? In 8 weeks? In 12 weeks?

47. If a man build 81 rods of fence in 9 days, how much does he build in 5 days? In 7 days?

48. If 7 yards of silk cost $27, for how much a yard should it be sold, to gain $8? To lose $6?

49. If a man pay 75 cents for 6 pounds of meat, and receive back 15 cents, what is the price of the meat a pound?

50. How many times is $\frac{1}{8}$ of $56 contained in $63? $\frac{1}{7}$ of 84 cents contained in 60 cents? $\frac{1}{9}$ of 81 yards contained in 108 yards?

51. If 6 coats can be cut from 30 yards of cloth, how many can be cut from 45 yards?

ANALYSIS. *One sixth* of 30 yards is 5 yards, and 5 yards is contained in 45 yards, 9 times. Hence, etc.

52. If 7 tons of coal can be bought for $42, how many tons can be bought for $54? For $60? For $72?

For *dictation:*

53. $\frac{1}{3}$ of 27, plus 42 = ?
54. $\frac{1}{5}$ of 60, plus 28 = ?
55. $\frac{1}{8}$ of 96, plus 64 = ?
56. $\frac{1}{9}$ of 108, minus 7 = ?
57. $\frac{1}{2}$ of 60, minus 15 = ?
58. $\frac{1}{3}$ of 30, plus 73 = ?
59. $\frac{1}{7}$ of $\overline{93 - 23} = ?$
60. $\frac{1}{9}$ of $\overline{19 + 44} = ?$

61. 80 + 4, ÷ 7, × 12, − 44, ÷ 10, + 25, − 15 = ?
Read, 80, 84, 12, 144, 100, 10, 35 = 20.

62. 12 × 3, ÷ 4, + 26, − 5, ÷ 10, × 12, + 4, − 20 = ?

63. 108 ÷ 12, + 9, + 10, ÷ 7, × 11, − 14, ÷ 6, + 50 = ?

64. 72 − 48, ÷ 6, ÷ 10, + 67, − 10, ÷ 12, × 9 = ?

65. 50 + 50, − 75, ÷ 5, × 11, + 15, − 4, ÷ 11 = ?

66. How do we obtain one third of a number? two thirds? one fourth? three fourths?

67. What part of 4 is 1? is 2? is 3?

ANALYSIS. *One* is $\frac{1}{4}$ of 4; 2 is 2 times $\frac{1}{4}$ or $\frac{2}{4}$ of 4; 3 is 3 times $\frac{1}{4}$ of 4, or $\frac{3}{4}$ of 4.

68. What part of 5 pounds is 1 pound? 2 pounds? Of 6 quarts is 1 quart? 2 quarts? 4 quarts? 5 quarts?

69. What part of 8 is 3? Of 7 is 2? Of 9 is 5?

70. Nine are how many times 4?

ANALYSIS. Four in 9, 2 times and 1 remainder, which is $\frac{1}{4}$ of 4. Hence, 9 is 2 times 4 and $\frac{1}{4}$ of 4.

71. 16 are how many times 5? *Ans.* 3 times 5 and $\frac{1}{5}$ of 5.

72. 43 are how many times 8? *Ans.* 5 times 8 and $\frac{3}{8}$ of 8.

73. 45 are how many times 4? 5? 6? 7? 8? 9? 10?

74. 60 are how many times 5? 6? 7? 8? 9? 10? 11? 12?

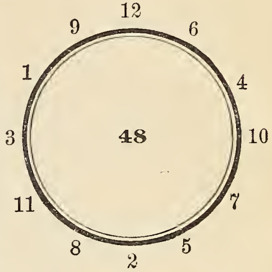
The preceding exercises and examples are intended simply as models, or forms, each to be largely increased, at the option of the teacher.

The pupil should be able to divide rapidly all the numbers from 12 to 100 by all the numbers less than 13.

This can be accomplished in a short time by spending a few minutes of each day's recitation in thorough drill.

The work may be prepared on the slate in the following form, $\frac{48}{4} = ?$ $\frac{48}{5}$, $\frac{48}{6}$, $\frac{48}{7}$, $\frac{48}{9}$, etc., and the answers given orally, *not* written.

The same device used to teach rapid multiplication **(67)** may be used here also to great advantage.



Place a small circle on the slate or blackboard, and write the numbers from 1 to 12 inclusive around the outside of it, as divisors.

Then use each of the numbers from 12 to 100 successively as a dividend, written in the centre of the circle, and divide *orally* the central number by each of the numbers outside the circle, pronouncing *results* only, as rapidly as possible. Commencing with any divisor, as 12, read to the right around the circle, thus, 48 is 4 times 12, 8 times 6, 12 times 4, 4 times 10 and $\frac{8}{10}$ of 10, 6 times 7 and $\frac{6}{7}$ of 7, etc. In the same manner, commence and read to the left. Then erase the central number, and insert some other number for the dividend, and treat in the same manner.

Principles of Division.

95. 1. When the object of the division is to find *how many times* one number contains another:

I. *The divisor and dividend are like numbers, and the quotient is an abstract number.*

2. When the object of the division is to find *one* of the *equal parts* of a number:

II. *The dividend and quotient are like numbers, and the divisor an abstract number.*

III. *The result of the continued division of the dividend by one number, and the resulting quotient by another, and so on, is the same in whatever order the divisors are taken.*

IV. *The product of the divisor and quotient, plus the remainder, if any, equals the dividend.*

DEFINITIONS.

96. A **Quotient** shows how many times one of two *like* numbers is contained in the other. Or,

It is *one* of the *equal parts* of a number.

97. **Division** is the process of finding the quotient.

98. The **Dividend** is the number to be divided.

99. The **Divisor** is the number by which we divide.

WRITTEN EXERCISES.

100. ***To divide by any divisor not greater than 12.***

1. Divide 924 by 7.

FIRST OPERATION.

Divisor. Dividend. Quotient.

```
7 ) 924 ( 132
    7
    --
    22        PROOF.
    21        132
    --          7
     14       ---
     14       924 (Prin. IV.)
     --
```

EXPLANATION. Write the divisor at the left of the dividend, separating them by a curved line.

Begin with the left-hand or highest order of the dividend, and divide the units of each order separately; thus, 7 is contained in 9 *once*. Writing 1 at the right of the dividend for the first figure of the quotient, multiply the divisor by this quotient figure and place the product under the part of the dividend used. Subtract, and to the remainder annex 2, the next lower order of the dividend. In the same manner, we find that 7 is contained in 22, 3 times and 1 remainder. Write 3 in the quotient, and to the remainder annex 4, the next lower order of the dividend. 7 is contained in 14, 2 times and no remainder. Hence, the entire quotient is 132.

The division is said to be *exact* when there is no remainder. The *divisor* and *quotient* then are *factors* of the dividend.

The solution of the preceding example may be abbreviated thus:

SECOND OPERATION.

$$7\ )\ \underline{924}$$
$$132$$

EXPLANATION. 7 is contained in 9, *once* and 2 remainder. Write 1, the quotient figure, under the part of the dividend used. Prefix the remainder to the next lower order of the dividend, 2, making 22. 7 is contained in 22, 3 times and 1 remainder. Prefix the remainder to the next lower order, 4, making 14. 7 is contained in 14, 2 times and no remainder. Hence, the entire quotient is 132.

Observe, since 9 of the dividend is hundreds, the 1 of the quotient is hundreds, and since 22 of the dividend is tens, the 3 of the quotient is tens; and lastly, as 14 of the dividend is units, the 2 of the quotient must be units. Hence,

The quotient figure will always be of the same order as the lowest order of the dividend used to obtain it.

The difference in the two preceding operations is apparent. In the first, the steps of the solution *are all written;* in the second, the quotient only is written, the other steps being performed *mentally.* The first is usually termed *Long Division,* the second, *Short Division;* the latter is generally used when the divisor does not exceed 12. The principles employed in both cases are the same.

In like manner, by Short Division, find the quotient, and prove the following:

(2.)	(3.)	(4.)	(5.)
6) 8124	8) 9416	5) 6840	7) 7805 feet.

(6.)	(7.)	(8.)	(9.)
5) 6245 cents.	6) 8024 days.	8) 9784 men.	7) 790 cts.

10. Divide 970656 by 4; by 6; by 8.
11. Divide 823718 by 7; 730524 by 6; 995320 by 8.
12. Divide 75610 by 5; 95326 by 7; 8135472 by 6.

13. Divide 21647 by 9.

OPERATION
$9 \overline{)\,21647}$
$2405\frac{2}{9}$

EXPLANATION. Since 9 is not contained in 2, the highest order of units, prefix it to the next lower, making 21, and say, 9 in 21, 2 times and 3 remainder; 9 in 36, 4 times and no remainder; 9 in 4, 0 times and 4 remainder; 9 in 47, 5 times and 2 remainder.

This last remainder 2 should be divided by 9, but we can only *indicate* the division by placing it over the divisor, thus, $\frac{2}{9}$, and write it as a part of the quotient. Hence, the entire quotient is $2405\frac{2}{9}$, read, two thousand four hundred five and a *remainder* of *two*, or *two divided by nine*, or *two ninths*.

PROOF. $2405 \times 9 + 2 = 21647$ (**95**, IV).

Divide the following numbers by each of the numbers from 2 to 12 inclusive:

14. 4632.	18. 16825.	22. 20731.	26. 187036.
15. 3047.	19. 40618.	23. 84640.	27. 300891.
16. 8659.	20. 81060.	24. 68902.	28. 716200.
17. 2806.	21. 73152.	25. 93007.	29. 209070.

30. Divide \$136.17 by \$.09.

OPERATION.
9 cts.) 13617 cts.
1513 *times.*

EXPLANATION. When the divisor and dividend are like numbers, they express units of the same kind, and the quotient is an abstract number (**95**, I). Hence, 9 cts. is contained in 13617 cts., 1513 times.

To change dollars to cents, annex two ciphers (**41**, 4), and omit the sign (\$); and to change dollars and cents to cents, omit the sign (\$) and the point (.), and write the word cents, or cts., at the right.

31. Divide \$136.17 by 9.

OPERATION.
9) \$136.17
\$15.13

EXPLANATION. When the divisor is an abstract number, the dividend and quotient are like numbers (**95**, II). Hence, 1 *ninth* ($\frac{1}{9}$) of \$136.17 is \$15.13.

Solve and prove the following:

(32.)	(33.)	(34.)	(35.)
\$7) \$8764	\$.08) \$268.00	6) \$37.80	8) \$4.00
1252 times.	3350 times.	\$6.30	\$.50

36. Divide \$34265 by \$7; \$98442 by \$6; \$765452 by \$8.

37. Divide \$5634.72 by 7; \$7766.55 by 9; \$2965.32 by 12.

38. Divide \$236.80 by 8 cts.; by 5 cts.; by 4 cts.

39. How many pounds of sugar, at 9 cts. a pound, can be bought for \$16.20? for \$27? for \$19.35? for \$175.68?

40. At 11 cts. a yard, how many yards of muslin can be bought for \$33? for \$12.87? for \$73.70? for \$3.63?

What is

41. 1 *fourth* of 37684 tons?
42. 1 *sixth* of 7304 bushels?
43. 1 *seventh* of 16753 miles?
44. 1 *eighth* of \$452.96?
45. $\frac{1}{5}$ of 30205 acres?
46. $\frac{1}{9}$ of 250863 pounds?
47. $\frac{1}{12}$ of 716052 votes?
48. $\frac{1}{11}$ of 806322 inches?

49. Find *one* of 9 equal parts of \$473803; of \$19706.40.

50. Find *one* of 7 equal parts of 1036041; of 81900632.

51. If 12 tons of coal cost \$61.20, what will be the cost of 5 tons? of 7 tons? of 16 tons? of 25 tons?

101. ***To divide any number by any divisor greater than 12.***

1. Divide 8657 by 42.

OPERATION.

Divisor. Dividend. Quotient.

$$\begin{array}{r} 42\ )\ 8657\ (\ 206\frac{5}{42} \\ \underline{84} \\ 257 \\ \underline{252} \\ 5\ \text{Rem.} \end{array}$$

Explanation. Since 42 is not contained in 8, the highest order of units, prefix it to the next lower order, making 86 *hundreds* for the *first partial* dividend. 42 is contained in 86 hundreds, 2 hundred times. Multiplying the divisor by this quotient figure, and subtracting the product from the part of the dividend used, there is a remainder of 2.

To this remainder annex 5, the next lower order of the dividend, making 25 *tens*, the *second partial* dividend. 42 is contained in 25 tens, no times. Place a cipher (0) in the quotient, and annex 7, the next lower, making 257 *units*, the *third partial* dividend. 42 is contained in 257, 6 times and a remainder of 5, which write over the divisor and annex to the quotient. Hence, the quotient is $206\frac{5}{42}$.

PROOF. $42 \times 206 + 5 = 8657$ (**95**, IV).

In like manner,

2. Divide 34762 by 18; by 26; by 38; by 46; by 54.
3. Divide 12605 by 75; by 104; by 116; by 92.
4. Divide 73062 by 241; 86327 by 162; 184321 by 109.

RULE. I. *Write the divisor at the left of the dividend.*

1. *Take for the first partial dividend the least number of orders on the left that will contain the divisor one or more times. Find how many times the divisor is contained in this partial dividend, and write the result as the first figure of the quotient, either at the right of the dividend, or under the lowest order of the partial dividend used.*

3. *Multiply the divisor by this quotient figure, and subtract the product from the partial dividend used, and to the remainder annex the next lower order of the dividend for a new partial dividend, and proceed as before; and thus continue until all the orders of the dividend have been divided.*

4. *If any partial dividend will not contain the divisor, write a cipher in the quotient and annex the next lower order of the dividend, and proceed as before.*

5. *If the division is not exact, write the last remainder with the divisor underneath, as a part of the quotient.*

To test or prove the work, see Prin. 4, **95**.

5. Divide 34706 by 28; by 33; by 48; by 54; by 66.
6. Divide 51699 by 116; by 203; by 314; by 109; by 94.
7. Divide 4477705 by 204; 103467 by 156; 340061 by 231.

Find the result in the following:

8. Of \$1081.92 ÷ 28.
9. Of \$183.12 ÷ 44.
10. Of \$663.60 ÷ 105.
11. Of \$1531.60 ÷ 56 cts.
12. Of \$346.92 ÷ \$.42.
13. Of \$15625 ÷ \$125.
14. Of \$54978.00 ÷ \$1.75.
15. Of 721445 feet ÷ 72 feet.
16. Of 184752 men ÷ 144.
17. Of $236 \times 20 \div 29$.
18. Of $(4720 + 5778) \div 38$.
19. Of $3670054 \div \overline{6 \times 63}$.
20. Of $(809671 - 76850) \div 126$.
21. Of $\overline{26314 \times 7} \div \overline{4763 - 4500}$.
22. Of $(\$25000 \times 23) \div \overline{\$46 \times 9} - 200$.
23. Of $(3761 \times 24 - 4000) \div \overline{640 - 506}$.
24. Of (7000 acres — 2500 acres) × 18 ÷ $\overline{21 \times 8}$.
25. Of (9052 pounds × 105 — 40 pounds) ÷ $\overline{100 \times 126}$.
26. Paid \$17125 for 137 acres of land. What was paid an acre?
27. If an army consume 312805 pounds of beef in 365 days, what is the average amount for 1 day?
28. How many cows, at \$43 a head, will \$19608 buy?
29. How many bales, each weighing 475 pounds, can be made of 93450 pounds of cotton?
30. How many United States bonds, at \$109 each, can be bought for \$163500? for \$107256? for \$214512?
31. How many loads of bricks, each load containing 1248 bricks, are there in a pile of 91856 bricks?
32. The property of a town, containing 4674 inhabitants, is valued at \$5870544; what is the average to each person?
33. A merchant bought 375 barrels of flour, at \$7.50 a barrel, and sold it for \$3600. What was his whole gain, and his gain on each barrel?
34. Divide 1653817 by 412; by 508; by 784.
35. Divide 3065284 by 1326; by 2465; by 3108.
36. Divide 87630046 by 4036; by 5174; by 16325.
37. Divide 206180439 by 53021; by 163042.

102. Drill Table No. 2, page 52, will furnish all the additional examples for the slate and blackboard required.

Commencing with the second line, opposite **2**, copy dividends with *three, four, five,* or more figures, as may be desired, from the columns A B C D E, etc., in the same manner and order as multiplicands were copied in multiplication, and for divisors use the figure or figures directly above the right-hand figure or figures of the dividend. Thus,

When the dividend contains *four* figures and the divisor *one,* copy from the columns A B C D:

(1.)	(2.)	(3.)	(4.)	(5.)
6) 1348	8) 7684	4) 3825	5) 9367	7) 5736

In like manner, copy dividends from columns B C D E, C D E F, D E F G, etc.

When the dividend contains *five* figures and the divisor *two,* copy from the columns A B C D E:

(6.)	(7.)	(8.)	(9.)
65) 13482 (	82) 76844 (	44) 38257 (	57) 93673 (

In like manner, copy from columns B C D E F, C D E F G, D E F G H, etc.

When the dividend contains *five* figures and the divisor *three,* copy as before; thus, from columns B C D E F:

(10.)	(11.)	(12.)	(13.)
657) 34825 (	825) 68443 (	443) 82579 (	579) 36735 (

In like manner, copy from columns C D E F G, D E F G H, etc.

In the same manner, copy dividends containing more than *five* figures, and divisors containing *three, four,* or more figures.

103. ***To divide when the divisor can be separated into factors.***

The process of dividing any number by a factor of the divisor, and the resulting quotient by another, and so on, is called *Continued Division.*

1. Divide 25380 by 108.

$$108 = 9 \times 4 \times 3, \quad \text{or} \quad 6 \times 6 \times 3, \quad \text{or} \quad 9 \times 6 \times 2.$$

108) 25380 (235	9) 25380	6) 25380	9) 25380
216	4) 2820	6) 4230	6) 2820
378	3) 705	3) 705	2) 470
324	235	235	235
540			
540			

It is obvious that the quotient will be the same, whether the dividend be divided by the divisor, or by any set of factors of which the divisor may be composed (**95**, III).

2. Divide 5664 by 96, or by the factors 3, 4, and 8.
3. Divide 60165 by 105, or by the factors 7, 5, and 3.
4. Divide 170604 by 126, or by the factors 9, 7, and 2.

5. Divide 893 by 30, using the factors 2, 3, and 5.

EXPLANATION. Dividing by 2, we obtain 446 *twos* and 1 remainder; dividing 446 *twos* by 3, we have 148 *sixes* and 2 *twos*, or 4, remainder; again, dividing 148 *sixes* by 5, we have 29 *thirties* and 3 *sixes*, or 18, remainder; hence, $18+4+1$, or 23, is the true remainder.

OPERATION.

2) 893	
3) 446	1
5) 148, 2 *twos* =	4
29, 3 *sixes* =	18
True remainder,	23

RULE. *Find the product of each remainder by all the divisors preceding the one that produced it, and the sum of the products, with the first remainder, if any, will be the true remainder.*

Find the *true remainder* in the following:

6. 178584 ÷ 72, or by 9 and 8.
7. 4619 ÷ 125, or by 5, 5, and 5.
8. 63825 ÷ 135, or by 3, 5, and 9.
9. 735479 ÷ 168, or by 7, 6, and 4.

104. ***When the divisor has ciphers at the right.***

1. Divide 37621 by 100.

OPERATION.

$$1|00\ )\ 376|21$$
$$376 \ldots\ldots 21 \text{ Rem.}$$
$$\text{Quotient, } 376\tfrac{21}{100}.$$

EXPLANATION. Removing any order of units one place to the *right*, by cutting off or taking the right-hand figure of a number, *divides* it by 10 (**15**, IV). For similar reasons, cutting off *two* figures *divides* it by 100; *three* figures *divides* by 1000, etc. The remaining figures are the *quotient*, and those cut off are the *remainder*.

2. Divide 76340 by 500.

OPERATION.

$$5|00\ )\ 763|40$$
$$152 \ldots\ldots 340 \text{ Rem.}$$
$$\text{Quotient, } 152\tfrac{340}{500}.$$

EXPLANATION. The divisor is composed of the factors 5 and 100. First divide by 100, by cutting off the two right-hand figures of the dividend, the quotient is 763. Dividing this quotient by 5 gives a quotient of 152, and a remainder of 3 hundreds, which prefixed to the first remainder 40 gives the *true* remainder, 340.

3. Divide 134750 by 10; by 100; by 1000; by 10000.
4. Divide 42067200 by 600; by 1200; by 9000; by 24000.
5. Divide 31403700 by 9100; by 12500; by 3710; by 26000.

RULE. I. *Cut off the ciphers from the right of the divisor, and as many figures from the right of the dividend, and divide the remaining part of the dividend, by the remaining part of the divisor, for the quotient.*

II. *Annex the figures cut off, to the remainder, if any, and the result will be the true remainder.*

6. At a cost of \$1200 each, how many cottages can be built for \$76800? For \$138000?

7. How many lots, at \$700 each, can be bought for \$58100?

8. If \$745600 be distributed equally among 960 soldiers, what will each man receive?

Find the value of the following expressions:

9. $75460 \div 3600$.	14. $\$130760 \div 1280$.
10. $137214 \div 1320$.	15. $(\overline{840 + 524} \times 5) \div 200$.
11. $\$846000 \div \$2500$.	16. $(\overline{12680 - 3265} \times 10) \div 320$.
12. 203040 cts. $\div$ 600 cts.	17. $(864000 \div 400 + 3740) \times 9$.
13. $4307561 \div 3170$.	18. $(63070 \times 10) \div (28 \times 500)$.

General Principles of Division.

105. The value of the quotient depends upon the *relative* values of the dividend and divisor.

There are six cases of changing the terms by multiplication and division, and which may be illustrated by comparing the equation $\frac{64}{8} = 8$ with the following, thus:

I. $\frac{64 \times 4}{8}$ or $\frac{64}{8 \div 4} = 32.$ *Multiplying the dividend, or dividing the divisor,* MULTIPLIES *the quotient by the same number.*

II. $\frac{64 \div 4}{8}$ or $\frac{64}{8 \times 4} = 2.$ *Dividing the dividend, or multiplying the divisor,* DIVIDES *the quotient by the same number.*

III. $\frac{64 \times 4}{8 \times 4}$ or $\frac{64 \div 4}{8 \div 4} = 8.$ *Multiplying or dividing both the dividend and divisor by the same number,* DOES NOT CHANGE *the quotient.*

Hence, the following *General Principle* may be deduced:

106. *A change in the* DIVIDEND *by multiplication or division produces a* LIKE *change in the quotient; but such a change in the* DIVISOR *produces an* OPPOSITE *change in the quotient.*

MENTAL EXERCISES.

107. 1. If \$120 are equally distributed among 6 poor families, and each family consists of 4 persons, what will each receive?

ANALYSIS. Each *family* will receive $\frac{1}{6}$ of \$120, or \$20; and each *person* will receive $\frac{1}{4}$ of \$20, or \$5.

2. What is $\frac{1}{3}$ of $\frac{1}{8}$ of 96? $\frac{1}{2}$ of $\frac{1}{5}$ of 100? $\frac{1}{4}$ of $\frac{1}{8}$ of 64? $\frac{1}{5}$ of $\frac{1}{9}$ of 90? $\frac{1}{2}$ of $\frac{1}{10}$ of 200 feet?

3. If $\frac{1}{12}$ of 144 pineapples are bad, how many are good?

4. How many days' work, at \$4 a day, will pay for 5 weeks' board, at \$7 a week, and \$13 borrowed money?

5. Paid \$1.05 for 5 boxes of soap, each box containing 3 cakes; what was the price of each cake? Of 6 cakes?

6. If 8 yards of cloth are worth \$40, how many yards must be given for 15 cords of wood, at \$3 a cord?

7. A grocer bought two kinds of sugar, one for 9 cts. and the other for 11 cts. a pound. What was the *average* price per pound?

The *average* of *two* numbers is *one half* their sum; of *three* numbers, *one third* their sum, etc.

8. Find the average of 3, 5, and 7; of 5, 8, and 14; of 4, 6, 7, and 11; of 3, 5, 7, 9, and 6.

9. What is the average price per pound of three kinds of coffee, at 20 cts., 25 cts., and 30 cts. a pound, respectively?

10. A grocer bought 4 barrels of apples of different qualities, at \$2.50, \$3, \$4, and \$4.50, and sold them for \$20. How much above the average price did he receive a barrel?

Find the second member of the following equations:

11. $36 - 9 + 13 \times 3 = ?$
12. $71 + 14 - 20 \times \overline{44 \div 11} = ?$
13. $(13 \times 3 + 12 \times 5) \div 9 = ?$
14. $\overline{24 - 12} \times \overline{64 \div 8} = ?$
15. $\frac{11 \times 6}{3} - \frac{6 \times 10}{5} + 14 = ?$
16. $\frac{24 + 12}{4} \times \frac{48 - 24}{8} = ?$

CANCELLATION

108. **Cancellation** is a short method of obtaining results by rejecting or cancelling equal factors both from the dividend and the divisor.

109. The **Sign of Cancellation** is an oblique mark (/), drawn through the number from which the factor is cancelled.

110. Principles. I. *Rejecting a factor from a number divides the number by that factor.*

Thus, $36 = 9 \times 4$. Rejecting 4 from 36, we have 9, the quotient of 36 divided by 4.

II. *Rejecting equal factors from both the dividend and the divisor does not change the quotient.*

Thus, $72 \div 24$, $\frac{72}{24} = 3$. Or, $\frac{72}{24} = \frac{12 \times \cancel{6}}{4 \times \cancel{6}} = \frac{12}{4} = 3$. Because, dividing both the dividend and the divisor by 6, or rejecting from both the factor 6, does not change the quotient (**105**, III).

Written Exercises.

111. 1. Divide 48 by 24.

OPERATION.

$$48 \div 24, \quad \text{or} \quad \frac{48}{24} = \frac{\cancel{2} \times \cancel{3} \times 8}{\cancel{2} \times \cancel{3} \times 4} = \frac{8}{4} = 2.$$

Explanation. By inspection, we find that the dividend contains the factors 2, 3, and 8, and that the divisor contains the factors 2, 3, and 4. Rejecting the factors that are common both to the dividend and divisor, which is the same as dividing by those factors (**110**, II), there remain the factor 8 in the dividend and the factor 4 in the divisor. Hence, the quotient is $\frac{8}{4} = 2$.

2. Divide 150 by 25.

Explanation. Separate the dividend into the factors 5, 5, and 6, and the divisor into the factors 5 and 5. Rejecting equal factors from both, there remains the factor 6 in the dividend. Hence, the quotient is 6.

OPERATION.

$$\frac{150}{25} = \frac{\cancel{5} \times \cancel{5} \times 6}{\cancel{5} \times \cancel{5}} = \frac{6}{1} = 6$$

3. Divide $28 \times 49 \times 75$ by $7 \times 15 \times 84$.

OPERATION.

$$\frac{\overset{4}{\cancel{28}} \times \overset{7}{\cancel{49}} \times \overset{5}{\cancel{75}}}{\cancel{7} \times \cancel{15} \times \underset{\cancel{12}\ 3}{\cancel{84}}} = \frac{7 \times 5}{3} = \frac{35}{3} = 11\tfrac{2}{3}. \quad \text{Or,} \quad \begin{array}{r|l} \cancel{7} & \cancel{28} \\ \cancel{15} & \cancel{49}\ ^{7} \\ ^{3}\ \cancel{84} & \cancel{75}\ ^{5} \\ \hline 3 & 35 \\ \hline & 11\frac{2}{3} \end{array}$$

Explanation. Since 7 is a factor common to 7 of the divisor and 28 of the dividend, reject it from both, retaining the factor 4 in the dividend. For a like reason, reject the factor 15 from the divisor and from 75 in the dividend, retaining the factor 5. Then cancel the common factor 7 from 84 in the divisor and from 49 in the dividend, retaining the factors 12 and 7 respectively. Next cancel the common factor 4 from 12 in the divisor and from 4 in the dividend, and there remains the uncancelled factors 7 and 5 in the dividend and 3 in the divisor. Hence, the quotient is $7 \times 5 \div 3 = 11\frac{2}{3}$.

It is thought by many to be more convenient to write the *dividend*, or its factors on the *right* of a *vertical* line, and the *divisor*, or its factors on the *left*, and then, after cancelling equal factors, perform the multiplication and division.

4. Divide $5 \times 9 \times 7 \times 11$ by $7 \times 5 \times 3 \times 11$.
5. Divide $80 \times 56 \times 18$ by $28 \times 20 \times 9$.
6. Divide $3 \times 6 \times 8 \times 12$ by $2 \times 3 \times 4 \times 6$.
7. Divide $70 \times 39 \times 13$ by $26 \times 21 \times 7$.

Rule. *Cancel the factors common to the divisor and dividend; divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor.*

After the cancellation, if the unit 1 alone remains in the dividend, it must be retained; if in the divisor, it may be omitted.

8. Find the quotient of $\frac{183 \times 70 \times 9 \times 24 \times 5}{126 \times 60 \times 30}$.

9. $(100 \times 64 \times 18 \times 7) \div (49 \times 25 \times 16) = ?$

10. Multiply 18 times 66 by 27 times 25, and divide the product by 7 times 84 multiplied by 30 times 45.

11. Gave 20 pounds of butter, at 27 cents a pound, for 15 pounds of coffee. What was the coffee worth a pound?

EXPLANATION. 20 pounds of butter will cost 27 cents $\times$ 20; and 1 pound of coffee will be worth *one fifteenth* of 27 cents $\times$ 20, or $\frac{27 \times 20}{15} = 36$ cts.

OPERATION.

	$\cancel{27}$ 9
$\cancel{15}$	$\cancel{20}$ 4
	36

12. Gave 36 bushels of wheat for 9 barrels of flour, at \$10 a barrel. What was the wheat worth a bushel?

13. If 5 yards of cloth cost \$8.20, what will 18 yards cost?

EXPLANATION. One yard will cost *one fifth* of \$8.20, or \$8.20 $\div$ 5; and 18 yards will cost 18 times \$8.20 $\div$ 5, or $\frac{\$8.20 \times 18}{5} = \$29.52$.

OPERATION.

	$\cancel{\$8.20}$ 1.64
$\cancel{5}$	18
	\$29.52

14. If 20 tons of coal cost \$147, what will 12 tons cost?

15. What will 27 pounds of tea cost, if 9 pounds cost \$4.50?

16. How many bushels of corn, at \$.60 a bushel, will pay for 12 tons of coal, at \$7.20 a ton?

EXPLANATION. 12 tons of coal will cost \$7.20 $\times$ 12; and it will take as many bushels of corn, at \$.60 a bushel, to pay for it, as \$.60 is contained times in \$7.20 $\times$ 12, or $\frac{\$7.20 \times 12}{\$.60} = 144$.

OPERATION.

	$\cancel{\$7.20}$ 144
.05 $\cancel{\$.60}$	$\cancel{12}$
	144

17. How many bushels of potatoes, worth 40 cts. a bushel, must a farmer give for 25 pounds of coffee, worth 30 cts. a pound?

18. How many chickens, at \$.70 each, must be given for 2 barrels of flour, at \$8.40 a barrel?

19. Sold 8 tubs of butter, each containing 54 pounds, at 28 cts. a pound; and received in payment 6 barrels of sugar, each containing 216 pounds. What was the sugar worth a pound?

EXPLANATION. The value of the butter is equal to 28 cts. × 54 × 8; the number of pounds of sugar is 216 × 6, and the price of 1 pound of sugar is equal to

$$\frac{28 \text{ cts.} \times 54 \times 8}{216 \times 6} = 9\tfrac{1}{3} \text{ cts.}$$

OPERATION.

$$\begin{array}{r|l} & 28\text{ cts.} \\ {}^{3}\,\cancel{216} & \cancel{54} \\ \cancel{6} & \cancel{8} \\ \hline 3 & 28 \\ \cline{2-2} & 9\tfrac{1}{3}\text{ cts.} \end{array}$$

20. A farmer exchanged 29 bushels of barley, worth \$.75 a bushel, with his neighbor, for corn, worth \$.87 a bushel. How many bushels of corn did he receive?

21. A grocer sold 16 boxes of soap, each containing 66 pounds, at 9 cts. a pound, and received in payment 88 barrels of potatoes, each containing 3 bushels. What were the potatoes worth a bushel?

22. If 35 yards of cloth cost \$140, what will 28 yards cost?

23. Bought 180 yards of cotton cloth, at 12 cts. a yard. How many bushels of oats, at \$.45, will pay for the cloth?

24. How much tea, at \$.96 a pound, will pay for 6 tubs of butter, each containing 36 pounds, worth 27 cts. a pound?

25. If 24 pounds of rice are worth 18 pounds of sugar, how many pounds of rice will pay for 217 pounds of sugar?

26. Sold 15 barrels of apples, each containing 3 bushels, at \$.84 a bushel. How many pieces of cotton cloth, each containing 45 yards, at 12 cents a yard, will pay for the apples?

27. A farmer sold 14 bushels of potatoes, at \$.56 a bushel, and took his pay in sugar, at 8 cents a pound; how many pounds did he receive?

DEFINITIONS

112. Quantity, in business transactions, is the amount of anything considered, or of any commodity bought, or sold.

113. Price is the value in money of *one*, or of a *given unit* of any commodity.

114. Cost is the value in money of the *entire quantity* bought, or sold.

115. An **Aliquot Part** of a number is an exact divisor of that number; or, it is *one* of the *equal parts* of the number.

Thus, $7\frac{1}{2}$, 5, 3, and $2\frac{1}{2}$ are *aliquot* parts of 15, since $7\frac{1}{2}$ is one of *two* equal parts of 15, 5 is one of *three* equal parts, 3 is one of *five* equal parts, etc.

Practical Applications.

116. ***The price and quantity given, to find the cost.***

1. Find the cost of 63 pounds of coffee, at 26 cts. a pound?

Explanation. 63 pounds will cost 63 times the price of 1 pound, or $\$.26 \times 63 = \$16.38$. Hence,

Price × Quantity = Cost.

OPERATION.

Price,	26 cts.
Quantity,	63
Cost,	$16.38

2. At 12 cts. a pound, what is the cost of 96 pounds of sugar?
3. At $.75 each, what is the cost of 48 books? Of 125?
4. What cost 67 yards of cloth, at $2.36 a yard? At $4.12?
5. What is the cost of 146 barrels of flour, at $7.56 a barrel?
6. What is the cost of 108 days' work, at $.98 a day?
7. Find the cost of 248 bushels of wheat, at $12 for 8 bushels; at $7.80 for 5 bushels; at $11.48 for 7 bushels.

117. ***The quantity and cost given, to find the price.***

1. If 75 baskets of peaches cost $112.50, what is the price?

EXPLANATION. Since 75 baskets cost $112.50, the price of 1 basket is *one seventy-fifth* of $112.50, or $1.50. Hence,

Cost ÷ Quantity = Price.

OPERATION.

Quantity.	Cost.
75)	$112.50
Price,	$1.50

2. If 45 pounds of tea cost $33.75, what is the price per pound?

3. If $448 is paid for 28 tons of hay, what is the price per ton? If $228 is paid for 16 tons? $440 for 25 tons?

4. What is the price per pound, when 160 pounds of mutton cost $14.40? When 87 pounds cost $10.44?

5. If 33 tons of coal cost $206.25, what is the price per ton? What is the cost of 14 tons? Of 37 tons?

6. If a man earns $11.22 in 6 days, what does he earn in 9 days? In 16 days? In 29 days? In 54 days?

118. ***The cost and price given, to find the quantity.***

1. How many pounds of butter, at 25 cts. a pound, can be bought for $42?

EXPLANATION. At 25 cents a pound, $42 will pay for as many pounds of butter as 25 cents is contained times in $42, which is 168 times. Hence,

Cost ÷ Price = Quantity.

OPERATION.

$42 = 4200 cts. (**41**, 4).

Price.	Cost.
25 cts.)	4200 cts.
Quantity,	168 pounds.

2. How many pounds of sugar can be bought for $72, at 8 cts. a pound? At 10 cts.? At 11 cts.? At 12 cts.?

3. At $.30 a box, how many boxes of grapes can be bought for $5.40? For $12? For $17.10? For $3.60?

4. At $136 each, how many horses can be bought for $2040? For $2720? For $5032?

119. Aliquot Parts of One Dollar.

50 cents $= \frac{1}{2}$ of \$1.	$12\frac{1}{2}$ cents $= \frac{1}{8}$ of \$1.
$33\frac{1}{3}$ cents $= \frac{1}{3}$ of \$1.	10 cents $= \frac{1}{10}$ of \$1.
25 cents $= \frac{1}{4}$ of \$1.	$8\frac{1}{3}$ cents $= \frac{1}{12}$ of \$1.
20 cents $= \frac{1}{5}$ of \$1.	$6\frac{1}{4}$ cents $= \frac{1}{16}$ of \$1.
$16\frac{2}{3}$ cents $= \frac{1}{6}$ of \$1.	5 cents $= \frac{1}{20}$ of \$1.

120. ***To find the cost of any number of things, or of a quantity, when the price is an aliquot part of \$1.***

1. What cost 96 yards of cambric, at 25 cts. a yard?

OPERATION.

$\$.25 \times 96 = \$24$. Or,

$\frac{1}{4}$ of $\$1 \times 96$, or $\$96 \div 4 = \$24$.

Explanation. If the price was \$1 a yard, the cost would be \$96; but since the price is $\$\frac{1}{4}$, the cost is $\frac{1}{4}$ of \$96, or \$24. Hence,

Take such a part of the given number, or quantity, as the price is of \$1; the result will represent the cost.

What is the cost of

2. 108 yards of calico, at $8\frac{1}{3}$ cts. a yard? At 10 cts.?
3. 120 pounds of sugar, at $12\frac{1}{2}$ cts. a pound? At $8\frac{1}{3}$ cts.?
4. 75 pounds of coffee, at $33\frac{1}{3}$ cts. a pound? At 25 cts.?
5. 18 dozen of eggs, at 20 cents a dozen? At $16\frac{2}{3}$ cts.?
6. 37 baskets of peaches, at \$.50 a basket?
7. How many bushels of oats, at \$.45 a bushel, will pay for 4 pieces of sheeting, each piece containing 36 yards, at $16\frac{2}{3}$ cts. a yard, and 2 sacks of coffee, each containing 35 pounds, at 20 cts. a pound?
8. What is the cost of 84 yards of carpet, at $\$1.33\frac{1}{3}$ a yard?

OPERATION.

$$\$1\frac{1}{3} \times 84 = \$84 + \frac{1}{3} \text{ of } \$84, \text{ or } \$112.$$

Explanation. Since the price is $\$1\frac{1}{3}$, the cost of 84 yards is \$84 plus $\frac{1}{3}$ of \$84, or $\$84 + \$28 = \$112$.

9. What is tho cost of 144 bushels of wheat, at $1.16⅔ a bushel? At $1.25? At $1.33⅓? At $1.50? At $2.25?

10. What cost 120 hats, at $2.20 each? At $3.12½?

121. ***To find the quantity, when the cost is given, and the price is an aliquot part of $1.***

1. How many slates, at 25 cts. each, can be bought for $42?

EXPLANATION. Since the price is $¼, $1 will buy 4 slates, and $42 will buy 42 times 4 slates, or 168 slates. Hence,

OPEEATION.

$\$1.00 \div \$.25 = 4;$

$4 \times 42 = 168.$

Multiply the number of dollars in the cost by the number of times the price is contained in one dollar.

2. At 33⅓ cents a box, how many boxes of grapes can be bought for $1.33⅓? For $12? For $7.66⅔?

3. How many pounds of sugar can be bought for $36, at 6¼ cts. a pound? At 8⅓ cts.? At 10 cts.? At 12½ cts.?

4. How many melons can be bought for $63, at $.25 each? At 20 cts.? At 16⅔ cts.? At 12½ cts.?

5. How many baskets of pears can be bought for $1.50, at 25 cts. a basket? At $.50? At $.75?

REVIEW.

WRITTEN EXERCISES.

122. 1. Find the quotient, by short division, of 76538959 divided by 28; by 64; by 96; by 108; by 132.

2. If the divisor is 3857, the quotient 489, and the remainder 1305, what is the dividend?

3. How many times in succession can 3589 be subtracted from 241462, and what will be the final remainder?

4. What must be subtracted from 57385, so that it can be exactly divided by 387? or what number added?

5. What number divided by 36 will give a quotient of 327 and a remainder of 26 ?

6. How many times in succession must 1739 be added to 83487 to make the final sum 200000 ?

7. If 4, 7, and 9 are three factors of 3276, find the fourth.

8. The quotient is 404, the divisor 365, and the remainder 215; what is 1 *fifth* of the dividend?

9. How many pounds of sugar, at 14 cts. a pound, can be bought for 84 bushels of apples, worth 36 cts. a bushel?

10. What number multiplied by 216 will produce 1554768?

11. The sum of two numbers is 4560, and the less 1970 ; what is the product of the two numbers?

12. How many tubs of butter, each containing 58 pounds, at 28 cts. a pound, will pay for 12 barrels of flour, at $8.12 a barrel?

13. What number added to 800924 makes 3000001 ?

14. The divisor is 48, the quotient 76920 ; if the divisor is 1 *fourth* as great, what is the quotient? If the divisor is *increased* 4 times?

15. A builder bought four city lots for $1260, $2083, $2500, and $2637; what was the average cost per lot?

16. Bought a farm for $5000. Paid at one time $1236, at another, $908.75, and at another, enough to reduce the debt to $1075.50 ; what was the third payment?

17. How many bushels of potatoes, at $.75 a bushel, will pay for 5 pieces of sheeting, each piece containing 36 yards, at 15 cents a yard; and 3 sacks of coffee, each containing 30 pounds, at 25 cts. a pound?

18. There are 36 men employed on a job, each receiving the same wages; at the end of 21 days, they receive $1701. What are their daily wages?

19. If it require 250 bushels of wheat to make 50 barrels of flour, how many bushels will be required to make 19 barrels?

20. If 26 acres of land cost \$2236, what will 127 acres cost, at the same rate? 236 acres? $\frac{1}{8}$ of 640 acres?

21. In what time will a boy, at \$.75 a day, earn as much as a man earns in 70 days, at \$2.25 a day?

22. A farmer sold 150 bushels of wheat, at \$1.60 a bushel, 125 barrels of apples, at \$2 a barrel, and 250 bushels of potatoes, at \$.48 a bushel, and with the money bought sheep, at \$5 a head; how many sheep did he buy?

23. The cost of a piece of cloth was \$112.70, and the price \$2.45 a yard; how many yards in the piece?

24. A coal dealer bought 1428 tons of coal for \$5712, and sold 975 tons of it, at \$5.60 a ton, and the remainder, at cost; what did he gain?

25. If 24 pounds of cheese cost \$2.88, what is the price per pound?

26. A farm containing 157 acres sold for \$4474.50; what was the price per acre?

27. The sum of two numbers is 372, and their difference is 64; what are the numbers?

EXPLANATION. Since 372 equals the sum of the numbers, and since the greater diminished by the difference equals the less, if the difference be subtracted from the *sum*, the remainder will be twice the less number. Hence, $\overline{372-64} \div 2 = 154$, the *less* number; and $154 + 64 = 218$, the *greater* number.

Or, if the difference be added to the sum, the amount will be twice the greater number. Hence, $\overline{372+64} \div 2 = 218$; and $218 - 64 = 154$.

PROOF. $218 + 154 = 372$, the sum.

28. There are two numbers whose difference is 45, and whose sum is 455; find their product.

29. A house and lot are valued at \$12500; the house is valued at \$2640 more than the lot; what is the value of each?

30. A merchant made \$7387 profits in two years. He gained the second year \$1053 more than in the first; what was his gain each year?

31. In an election, the whole number of votes cast for two candidates was 3789, and the majority for the successful candidate was 227. How many votes did each receive?

32. A man having \$6850, paid out all but \$1572 in 7 weeks; what was the average amount paid out each week?

33. A man took 3 loads of potatoes to market, each load containing 20 bags, and each bag 2 bushels. He sold them at \$.50 a bushel, and received in payment 4 chests of tea, each containing 25 pounds; what was the tea worth a pound?

34. A number of cattle, that cost \$4896, sold for \$6048, by which a profit of \$12 a head was made. How many head of cattle were there, and what was the price per head?

Find the second members of the following equations:

35. $\$475 \times \overline{120 - 74} \div 437 + \$87 \times 42 = ?$

36. $(3775 \div 151 + 14 \times 376 - 3000) \div 109 = ?$

37. $(67893 - 8637) \div 823 + 754 \times (235 - 94) - 987 = ?$

38. $\left(8085 \div 35 + \frac{817}{19} - \frac{328 \times 5}{40}\right) \times 64 = ?$

39. $\$1728 \times 8 \div \frac{\$180 \times 24}{\$108 \times 10} - (\$164 \times \overline{96 - 81}) = ?$

40. $\$7608 + \frac{(\$560 - \$374) \times 10}{1080 \div 36} = \$10000 - ?$

123. All operations in Arithmetic are classed under one or more of the problems belonging to the four fundamental rules.

Let the pupil be required to illustrate each of the following problems by several original examples.

I. ADDITION.

Given: 1. The parts, to find their sum or amount.

2. The less of two numbers and their difference, to find the greater.

II. SUBTRACTION.

Given: 1. The sum of two numbers, and one of them, to find the other number.

2. Two numbers, to find their difference.

III. MULTIPLICATION.

Given: 1. Two numbers, to find their product.

2. Any number of factors, to find their continued product.

3. The divisor and quotient, to find the dividend.

IV. DIVISION.

Given: 1. The dividend and divisor, to find the quotient.

2. The dividend and quotient, to find the divisor.

3. The product and one of two factors, to find the other.

4. The continued product of several factors, and the product of all but one factor, to find that one.

V. MISCELLANEOUS.

Given: 1. Several different quantities, or prices, to find the *average* quantity or price.

2. The sum and difference of two numbers, to find the numbers.

3. The price and quantity, to find the cost.

4. The cost and quantity, to find the price.

5. The cost and price, to find the quantity.

6. The quantity and the price, when the price is an *aliquot part* of $1, to find the cost.

7. The cost and the price, when the price is an *aliquot part* of $1, to find the quantity.

PROPERTIES OF NUMBERS

DEFINITIONS

124. An **Integer** is a number that represents *whole* things.

Thus, 6, 8, 12 men, 46 days are integers.

Every number is an *integer*, a *fraction*, or a *mixed number*.

125. Every integer is

1. Even, or Odd.
2. Prime, or Composite.
3. Abstract, or Denominate (**4, 5**).
4. Simple, or Compound (**280**).

126. A **Factor** of a number is one of the integers whose continued product is that number.

Thus, 6 and 9 are factors of 54; 2, 4, and 6, of 48.

127. An **Exact Divisor** of a number is an integer that will divide that number without a remainder.

Thus, 8 is an exact divisor of 56; and 12, of 60.

1. A *factor* is also an *exact divisor* of a number; and conversely, an *exact divisor* of a number is always a *factor* of the same number.

2. Every exact divisor or factor of a number is also called a *measure* of that number.

3. When a number is a factor or divisor of *two* or *more* numbers, it is called a *common* factor, or *common* measure of those numbers.

4. The use of the terms *number, factor,* and *divisor,* is here restricted to *integral* numbers.

128. A **Prime Number** is a number exactly divisible only by itself and by unity.

Thus, 2, 3, 5, 7, 11, etc., are prime numbers.

129. A **Prime Factor** is any prime number used as a *factor.*

Thus, 2, 3, and 5 are prime factors of 30.

130. Two numbers are *prime to each other* when they have no factor or divisor, except 1, common to both.

Thus, 8 and 15 are prime to each other.

131. A **Composite Number** is a number that has other factors or divisors than itself and 1.

Thus, 24 is a composite number, since it is the product of, and can be divided by 4 and 6, or by 3 and 8, or by 2, 3, and 4.

132. The **Greatest Common Divisor** (G. C. D.) of two or more numbers is the greatest number that divides each of them exactly, and is the product of *all* the common *prime factors.*

Thus, 9 is the G. C. D. of 27 and 36, since it is the product of 3 and 3, the only prime factors common to 27 and 36.

133. A **Multiple** of a number is any dividend exactly divisible by that number; or, it is any product obtained by using the given number as a factor.

Thus, 8, 12, and 16 are multiples of 4.

134. The **Least Common Multiple** (L. C. M.) of two or more numbers is the least number that is exactly divisible by each of them.

Thus, 15 is the L. C. M. of 3 and 5, since it is the least number that is exactly divisible by 3 and 5.

There can be but *one* L. C. M. of two or more numbers.

135. One number is a *measure,* or a *sub-multiple,* or an *aliquot part* of another number, when it exactly divides it.

Thus, 6 is a measure, or a sub-multiple, or an aliquot part of 24.

136. An **Even Number** is a number exactly divisible by 2.

All *even* numbers end with 0, 2, 4, 6, or 8.

137. An **Odd Number** is a number not exactly divisible by 2.

All *odd* numbers end with 1, 3, 5, 7, or 9.

138. Any number is exactly divisible

1. By 2, if it ends with a cipher or a digit divisible by 2.
2. By 3, if the sum of its digits is divisible by 3.
3. By 4, if the number expressed by the two figures on the right is exactly divisible by 4.
4. By 5, if it ends with a cipher or 5.
5. By 6, if it is exactly divisible by 2 and 3.
6. By 8, if the three figures on the right are ciphers, or if the number expressed by them is exactly divisible by 8.
7. By 9, if the sum of its digits is exactly divisible by 9.
8. By 10, if it ends with a cipher.
9. By 12, if it is exactly divisible by 3 and 4.

Mental Exercises.

139. 1. Name the factors of 25; 32; 45; 60.

2. Name the prime numbers from 1 to 30.
3. Name the composite numbers from 2 to 40.
4. Of what integers is 60 the continued product?
5. Of what prime numbers is 9 the product? 15? 21?
6. Name four numbers of which 7 is a common factor.
7. Name the smallest exact divisor of 30 and 42.
8. Name the prime factors of 35; 36; 50; 54.
9. What are the prime divisors of 25? 30? 55? 63?
10. Name two numbers that will exactly divide or measure 28 acres; 32 hours; 45 rods; 60 pounds.
11. Name two common divisors or measures of 36, 48, and 60; the greatest common measure; the least.
12. Name three measures of 24 yards; of 36 miles.
13. What factors are common to 12, 24, and 36? To 9. 27, and 45?

14. What prime divisor is common to 14 and 3 times 14?

15. What is the least number exactly divisible by 3, 2, and 7?

16. Name three *odd* numbers of which 9 is a common factor or divisor; three *even* numbers.

17. What is the greatest number that will exactly divide or measure 36 and 45? 45 and 60? 24, 36, and 48?

18. What two numbers will exactly measure 20 and 60? Their *sum?* Their *difference?* Their *product?*

19. What is the G. C. D. of 32, 40, and 64?

20. Name four numbers of which 7 is the G. C. D.

21. Name the L. C. M. of 9 pints and 6 pints.

22. What number is the L. C. M. of $5, $6, and $10?

140. PRINCIPLES. 1st. Of Factors and Divisors.

I. *Every prime factor of a number is an exact divisor of that number.*

II. *Every composite number is the product of all its prime factors.*

III. *A common divisor of two numbers is a divisor of their sum and of their difference; also, of any multiple of either of them.*

IV. *The* G. C. D. *of two or more numbers is the product of all their common prime factors.*

2d. Of Multiples and Dividends.

V. *Every multiple of a number contains every prime factor of that number.*

VI. *The* L. C. M. *of two or more numbers contains every prime factor of each number, and no other factors; and any prime factor occurs in the* L. C. M. *as often as it occurs in that number which contains it the greatest number of times.*

VII. *If two or more numbers are prime to each other, their* L. C. M. *is their product.*

FACTORS AND DIVISORS.

WRITTEN EXERCISES.

141. *To resolve a number into its prime factors.*

1. What are the prime factors of 390?

EXPLANATION. Since 390 is an even number, divide it by its least prime divisor 2, and the quotient by the prime divisor 3, and the resulting quotient by 5, and as the quotient 13 is prime, the division can be carried no further. Hence, $2 \times 3 \times 5 \times 13 = 390$ (Prin. II), and 2, 3, 5, and 13 are the prime factors of 390.

OPERATION.

$$\begin{array}{r|l} 2 & 390 \\ \hline 3 & 195 \\ \hline 5 & 65 \\ \hline & 13 \end{array}$$

In like manner, find the prime factors or divisors of

2. 495. | 3. 968. | 4. 756. | 5. 1089. | 6. 8064. | 7. 1728.

142. *To find the prime factors common to two or more numbers.*

1. What are the prime factors common to 42, 63, and 105?

EXPLANATION. The prime number 3 is an exact divisor of all the given numbers, and 7 of all the quotients obtained; and since no number greater than 1 will divide the last set of quotients, 3 and 7 are the only prime factors common to 42, 63, and 105.

OPERATION.

$$\begin{array}{r|rrr} 3 & 42, & 63, & 105 \\ \hline 7 & 14, & 21, & 35 \\ \hline & 2, & 3, & 5 \end{array}$$

RULES FOR FINDING THE PRIME FACTORS.

1st. Of a Composite Number:

Divide the given number by any prime divisor of it, and the quotient in the same manner, and so continue the division until the quotient is a prime number. The several divisors and the last quotient are the prime factors required.

2d. Common to two or more numbers:

Divide by any prime number that is an exact divisor of each of them, and the resulting quotients in the same manner, until they contain no common factor. The divisors will be the prime factors required.

Find the prime factors common to

2. 144 and 180.	5. 36, 48, and 72.	8. 96, 144, and 130.
3. 336 and 420.	6. 42, 63, and 84.	9. 168, 256, and 320.
4. 462 and 460.	7. 48, 72, and 96.	10. 325, 540, and 625.

143. ***To find the G. C. D. of two or more numbers.***

1. What is the G. C. D. of 126, 168, and 210?

EXPLANATION. By factoring, we find 2, 3, and 7 to be the prime factors common to all the given numbers. The product of these prime divisors is the G. C. D. of 126, 168, and 210 (Prin. IV). $2 \times 3 \times 7 = 42$, G. C. D.

OPERATION.

2)	126,	168,	210
3)	63,	84,	105
7)	21,	28,	35
	3,	4,	5

In the same manner, find the G. C. D. of

2. 128 and 324.	5. 28, 42, and 56.	8. 16, 24, 48, and 72.
3. 394 and 672.	6. 84, 126, and 210.	9. 60, 84, 108, and 132.
4. 272 and 425.	7. 52, 78, and 416.	10. 28, 84, 168, and 336.

11. Find the G. C. D. of 814 and 962.

FIRST METHOD.

```
814 ) 962 ( 1
      814
      148 ) 814 ( 5
            740
             74 ) 148 ( 2
                  148
```

SECOND METHOD.

5	814	962	1
	740	814	
	74	148	2
		148	

Explanation. The operation is the same, whether the ordinary form of division is used, or the vertical lines. The latter is the shorter and more convenient.

Dividing 962 by 814 gives the remainder 148; next, dividing 814 by 148 gives the remainder 74; lastly, dividing 148 by 74 gives no remainder. The last divisor, 74, is the G. C. D.

This may be shown as follows:

1. Since 74 is a divisor of 148, it is a divisor of 148×5 or 740 (Prin. III); therefore 74 is a divisor of $740 + 74$ or 814 (Prin. III). Again, since 74 is a divisor of 814 and 148, it is a divisor of $814 + 148$, or 962. Hence, 74 is a *common* divisor of 814 and 962.

2. *Every* common divisor of 814 and 962 is a divisor of $962 - 814$, or 148 (Prin. III); hence, every common divisor of 814 and 962 is a common divisor of 814 and 148. Again, every common divisor of 814 and 148 is a divisor of $814 - 148 \times 5$, or 74.

3. It has been proved (1) that 74 is a *common* divisor of 814 and 962. and (2) that *every* common divisor of 814 and 964 is a divisor of 74. Therefore, since no number greater than 74 is a divisor of 74, it follows that 74 is the G. C. D. required.

Find the G. C. D. of the following sets of numbers:

12. 825 and 960.	15. 961 and 1178.	18. 5355 and 6545.
13. 689 and 1573.	16. 1379 and 2401.	19. 4155 and 24720.
14. 304 and 1072.	17. 2121 and 1313.	20. 715 and 264429.

Rules for Finding the G. C. D.

1st. By Factoring:

Resolve the given numbers into their prime factors; the product of all the prime factors that are common is the G. C. D. (Prin. IV.)

2d. By Continued Division:

Divide the greater number by the less, and the divisor by the remainder, if any, and so continue to divide the last divisor by the last remainder, till nothing remains. The last divisor is the G. C. D.

Find the G. C. D. of the following numbers:

21. 1233 and 19180.
22. 388, 399, and 580.
23. 174, 580, and 448.
24. 72, 84, 66, and 176.
25. 84, 336, 420, and 504.
26. 132, 396, 528, and 660.

MULTIPLES AND DIVIDENDS.

WRITTEN EXERCISES.

144. ***To find the L. C. M. of two or more numbers.***

1. Find the L. C. M. of 18, 28, and 42.

FIRST METHOD.

$$18 = 3 \times 3 \times 2$$
$$28 = 2 \times 2 \times 7$$
$$42 = 2 \times 3 \times 7$$

EXPLANATION. Resolving the given numbers into their prime factors, we find the L. C. M. must contain the factor 3, *twice*, 2, *twice*, and 7, *once*, and no other factors, (Prin. VI). Hence, $3 \times 3 \times 2 \times 2 \times 7 = 252$, the L. C. M.

SECOND METHOD.

$$\begin{array}{r|rrr} 2 & 18, & 28, & 42 \\ \hline 3 & 9, & 14, & 21 \\ \hline 7 & 3, & 14, & 7 \\ \hline & 3, & 2, & 1 \end{array}$$

Hence,

$$2 \times 3 \times 7 \times 3 \times 2 = 252.$$

EXPLANATION. Divide the given numbers by the prime factor 2, and write the quotients in the second line. Divide 9 and 21 by 3, and write the quotients with 14, which is not exactly divisible by 3, in the third line. Again, divide 14 and 7 by 7, and write the quotients with 3 in the fourth line. The numbers in this line being prime to each other, the process of division terminates here. Now find the continued product of the divisors 2, 3, 7, and the numbers 3, 2, and 1 in the fourth line. The result, 252, is the L. C. M., since it contains every prime factor of the given numbers, and no other prime factors (Prin. VI).

Find the L. C. M. of the following:

2. 15 and 16. | 3. 16, 48, 54, and 72. | 4. 14, 18, 24, and 120.

RULE. 1. *Resolve the numbers into their prime factors; the product of these factors, each used the greatest number of times it occurs in any one of the given numbers is the* L. C. M. Or,

2. *Write the numbers in a horizontal line, omitting such of the smaller as are factors of the larger, and divide by any prime factor that is contained in two or more of the given numbers, writing the quotients and undivided numbers underneath.*

3. *In the same manner, divide the quotients and undivided numbers, and so continue to divide till all the results are prime to each other; then the continued product of all the divisors and the numbers in the last line will be the* L. C. M.

Find the L. C. M. of the following sets of numbers:

5. 14, 21, 45.	8. 14, 18, 24, 120.	11. 40, 80, 200, 320.
6. 28, 14, 35.	9. 13, 52, 78, 130.	12. 28, 154, 84, 198.
7. 26, 189, 56.	10. 39, 52, 78, 117.	13. 20, 126, 150, 490.

REVIEW.

MENTAL EXERCISES.

145. 1. How do prime numbers differ from composite numbers?

2. What is the smallest, and what the largest prime factor of 66? Name the composite factors of 66.

3. Name all the different values of an exact number of articles that can be bought for 36 cents; for 48 cents.

4. Show that a factor of any number is also a factor of any multiple of that number.

5. What is the least number of which 3, 4, and 5 are factors? Why? (Prin. VII.)

6. Show that a common divisor of two numbers is also a divisor of their *sum*; of their *difference*.

7. Show that any number which will exactly divide one of two numbers, will divide their product.

8. Name three common multiples of 6 feet and 9 feet.

9. Name three common measures of $12 and $30.

10. Name three exact measures of 36 pints; of 48 inches.

11. Why must the L. C. M. of 6 and 20 be greater than 20?

12. Why cannot the L. C. M. of 18 and 36 be greater than 36?

WRITTEN EXERCISES.

146. 1. What are the prime divisors common to 1428 and 1092? To 1050 and 3150? To 105, 315, and 525?

2. Find the G. C. D. of 336 feet and 280 feet; of 825 tons and 1372 tons; of $764 and $822; of $48.33 and $62.37.

3. What is the length of the longest board that will make an exact number of lengths of fence, enclosing a garden 210 feet long and 154 feet wide?

4. Divide the L. C. M. of 312, 260, and 390, by their G. C. D.

5. What is the smallest quantity of grain that will fill an exact number of bins, whether they hold 45, 54, 72, or 81 bushels?

6. Find the least number which, divided by 30, 55, and 105, will give a remainder of 27.

7. Find the greatest number that will divide 748 and 927, and give the remainders 13 and 17 respectively.

8. At what price per head must cattle be purchased by parties who have respectively $740, $999, and $1147, so that each man may purchase an exact number? How many head can each purchase?

9. What is the smallest sum of money for which a person can purchase an exact number of horses, at $170 each, or of oxen, at $70 each?

10. The L. C. M. of two numbers is 765072, and the G. C. D. 42. One of the numbers is 3024; find the other.

FRACTIONS.

INDUCTIVE EXERCISES.

147. 1. If a melon is cut into two *equal parts*, what is each part called? (**94**, p. 65.) If into 3 *equal parts?* If into 4 *equal parts?*

2. Into how many *halves* can a unit or thing be divided? Into how many *thirds? Fourths? Fifths? Sixths? Sevenths? Eighths? Ninths? Twelfths? Twentieths?* etc.

3. What are the largest *equal* parts into which a unit or whole thing can be divided? The next largest? etc.

4. If a pie is cut into 6 equal pieces, what part of the whole pie is 1 piece? 2 pieces? 3 pieces? 4 pieces? 5 pieces?

5. If a cheese is cut into 2 equal pieces, what part of the whole cheese is 1 piece? If cut into 6 equal pieces? If into 8 equal pieces?

6. What is 1 of 4 equal parts of a unit, or whole thing, called? 3 of 4 equal parts? 2 of 5 equal parts? 5 of 8 equal parts? etc.

7. Numbers representing *equal parts* of a unit or thing are written thus:

One half	is written	$\frac{1}{2}$.	Two thirds	is written	$\frac{2}{3}$.
One third	"	$\frac{1}{3}$.	Five sixths	"	$\frac{5}{6}$.
One fourth	"	$\frac{1}{4}$.	Nine tenths	"	$\frac{9}{10}$.
One fifth	"	$\frac{1}{5}$.	Eleven sixteenths	"	$\frac{11}{16}$.
One sixth	"	$\frac{1}{6}$.	Seven fiftieths	"	$\frac{7}{50}$.

Forty-three two-hundredths is written $\frac{43}{200}$.

8. What is meant by 1 *half* of an apple? 1 *third* of a pound? 1 *fourth* of a mile? 1 *sixth* of $12?

9. If 10 quarts of chestnuts are divided among 5 boys, into how many parts is the whole divided? What part of the whole does 1 boy receive? 2 boys? 3 boys? 4 boys?

10. Which is greater, thirds, or fourths? Sixths, or fifths? Fourths, or fifths? Sevenths, or ninths?

148. PRINCIPLES. I. *The* VALUE *of one of the* EQUAL PARTS *into which a unit or whole thing is divided, depends upon their* NUMBER.

II. *The* LESS *the* NUMBER *of parts, the* GREATER *their* VALUE; *and the* GREATER *the* NUMBER *of parts, the* LESS *their* VALUE.

DEFINITIONS.

149. A **Fraction** is one or more of the *equal parts* of a *unit*, or of anything regarded as a *whole*.

150. The **Unit** of a **Fraction** is the unit, or whole, which is divided into equal parts.

151. A **Fractional Unit** is *one* of the equal parts into which the unit of the fraction is divided, and takes its *name* and *value* from the number of parts into which the unit or the whole thing is divided.

Thus, if 1 pound is divided into 4 equal parts, the *unit of the fraction* is 1 pound, and the *fractional unit*, 1 *fourth* of a pound.

152. A fraction is usually expressed by a sign of division (**91**), being composed of two numbers, called the *Numerator* and *Denominator*, one written above the other, with a line between them. Hence,

153. A fraction may be regarded as an expression of *unexecuted* division, the *numerator* corresponding to the *dividend*, the *denominator* to the *divisor*, and the fraction itself to the *quotient*.

154. The **Denominator** of a fraction is the number written *below* the line, and shows the *number* of equal parts into which the integer is divided.

155. The **Numerator** of a fraction is the number written *above* the line, and shows *how many* of the equal parts are used.

Thus, in the fraction $\frac{5}{8}$, 8, the *denominator*, shows that the integer is divided into 8 equal parts, named *eighths,* and 5, the *numerator*, shows that 5 of the 8 equal parts are used.

156. The **Terms** of a fraction are the *numerator* and the *denominator.*

Thus, 7 and 8 are the *terms* of the fraction $\frac{7}{8}$.

Fractions are named *Proper*, or *Improper*, according to their *value.*

157. A **Proper Fraction** is a fraction whose numerator is less than its denominator; hence, its *value* is less than 1.

Thus, $\frac{3}{4}$, $\frac{7}{10}$, and $\frac{55}{91}$ are *proper* fractions.

158. An **Improper Fraction** is a fraction whose numerator equals or exceeds its denominator; hence, its *value* is equal to, or greater than 1.

Thus, $\frac{4}{4}$, $\frac{3}{2}$, $\frac{25}{6}$, and $\frac{108}{24}$ are *improper* fractions.

159. A **Mixed Number** is a number composed of an integer and a fraction united.

Thus, $9\frac{4}{5}$ is a *mixed* number, and is equivalent to $9 + \frac{4}{5}$.

160. The **Value** of a fraction is the quotient of its numerator divided by its denominator.

Thus, the value of $\frac{9}{3}$ is 3; of $\frac{10}{5}$ is 2; of $\frac{12}{7}$ is $1\frac{5}{7}$.

161. The **Reciprocal** of a number is 1 divided by that number.

Thus, the *reciprocal* of 7 is $1 \div 7$ or $\frac{1}{7}$; of 13, it is $1 \div 13$, or $\frac{1}{13}$; the reciprocal of $\frac{6}{7}$ is $1 \div \frac{6}{7} = 1 \times \frac{7}{6} = \frac{7}{6}$, or the *reciprocal of a fraction* is the fraction *inverted.*

162. An integer is changed to a fractional form by writing 1 under it for a denominator. Thus, $7 = \frac{7}{1}$; $15 = \frac{15}{1}$, etc.

163. Since a fraction may be regarded as an expression of unexecuted division, all changes in the *terms* of a fraction will affect the *value* of the fraction according to the laws of division (**105**), and is equivalent to substituting the words *numerator, denominator*, and *value of the fraction*, for the words *dividend, divisor*, and *quotient*, respectively, as shown in the following illustrations:

$$\text{I.} \left\{ \begin{array}{l} \dfrac{6 \times 3}{9} = \dfrac{18}{9} \\[2ex] \dfrac{6}{9 \div 3} = \dfrac{6}{3} \end{array} \right.$$

1st. The *value* of the fractional unit remains the same, but the *number* of fractional units is 3 times as great.

2d. The *value* of the fractional unit is increased 3 times, but the *number* of fractional units remains the same.

Hence, *the value of the fraction, in both cases, is multiplied.* (**105**, I.)

$$\text{II.} \left\{ \begin{array}{l} \dfrac{6 \div 3}{9} = \dfrac{2}{9} \\[2ex] \dfrac{6}{9 \times 3} = \dfrac{6}{27} \end{array} \right.$$

1st. The *value* of the fractional unit remains the same, but the *number* of fractional units is 3 times as small.

2d. The *value* of the fractional unit is diminished 3 times, but the *number* of fractional units remains the same.

Hence, *the value of the fraction, in both cases, is divided.* (**105**, II.)

$$\text{III.} \left\{ \begin{array}{l} \dfrac{6 \times 3}{9 \times 3} = \dfrac{18}{27} \\[2ex] \dfrac{6 \div 3}{9 \div 3} = \dfrac{2}{3} \end{array} \right.$$

1st. The *value* of the fractional unit is 3 times as small, but the *number* of fractional units is 3 times as great.

2d. The *value* of the fractional unit is 3 times as great, but the *number* of fractional units is 3 times as small.

Hence, *the value of the fraction, in either case, is not changed.* (**105**, III.)

The foregoing illustrations render obvious the following:

164. GENERAL PRINCIPLES.

I.	*Multiplying the numerator*, or *Dividing the denominator*,	MULTIPLIES *the value of the fraction.*
II.	*Dividing the numerator*, or *Multiplying the denominator*,	DIVIDES *the value of the fraction.*
III.	*Multiplying or dividing both terms, by the same number*	DOES NOT CHANGE *the value of the fraction.*

REDUCTION OF FRACTIONS.

165. The **Reduction** of a **Fraction** consists in changing its terms without altering its value.

166. ***To reduce fractions to higher or lower terms.***

MENTAL EXERCISES.

1. In 3 *fourths* how many *eighths?*

ANALYSIS.—Since 1 equals 8 eighths, 1 fourth equals $\frac{1}{4}$ of 8 eighths, or $\frac{2}{8}$; and 3 fourths equal 3 times $\frac{2}{8}$, or $\frac{6}{8}$. Hence, $\frac{3}{4}=\frac{6}{8}$.

2. One half is how many *eighths? Tenths? Twelfths?*
3. One third of a year is how many *sixths* of a year?
4. Two thirds of a mile are how many *ninths* of a mile?
5. Express the value of $\frac{4}{5}$ in terms 3 times as large.
6. Name some equivalent fractions for *thirds. Fourths.*
7. Show that multiplying both terms of $\frac{2}{3}$ by 4 does not change the value of the fraction.
8. The number of twelfths in a unit is how many times the number of *fourths? Thirds? Sixths?*
9. Express $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{6}$ each as 12ths; as 24ths; as 36ths.
10. How are 6ths changed to 30ths? 7ths to 28ths?
11. Express $\frac{5}{9}$ in 27ths. In 36ths. In 63ds. In 45ths.

12. Of what numbers are the denominators of $\frac{3}{8}$, $\frac{11}{12}$, and $\frac{7}{16}$ multiples?

Change

13. $\frac{4}{5}$ to *six* equivalent fractions having higher terms.
14. $\frac{3}{4}$ and $\frac{6}{7}$ to fractions having terms 4 times as great.
15. How are fractions changed to higher terms?

Ans. By Multiplication. **(163,** III, 1st.)

16. How many *thirds* are 6 *ninths?*

Analysis.—Since 1 third equals 3 ninths, 6 ninths are as many thirds as 3 ninths are contained times in 6 ninths, which is 2 times. Hence, 6 ninths equal 2 thirds. $\frac{6}{9} = \frac{2}{3}$.

17. How many *fourths* in $\frac{9}{12}$? In $\frac{8}{16}$? *Sixths* in $\frac{12}{18}$?
18. Express $\frac{16}{24}$ in terms 1 eighth as large.
19. Show that dividing the terms of $\frac{15}{20}$ by 5 does not change the value of the fraction.
20. Change $\frac{15}{25}$ to a fraction having lower terms; $\frac{21}{28}$; $\frac{32}{40}$.
21. What common divisor have the terms of $\frac{12}{18}$? $\frac{20}{25}$? $\frac{27}{36}$?
22. Express the value of $\frac{12}{24}$ in terms $\frac{1}{2}$ as great; $\frac{1}{3}$; $\frac{1}{4}$.
23. Express $\frac{12}{15}$ in parts 3 times as great in *value.*
24. Is $\frac{16}{24}$ in its lowest terms? Why not? Name two common factors of its terms. Why is $\frac{2}{3}$ equal to $\frac{16}{24}$?
25. Express $\frac{15}{20}$ in its lowest terms; $\frac{18}{27}$; $\frac{30}{40}$; $\frac{24}{36}$; $\frac{16}{32}$.
26. How is a fraction reduced to lower terms?

Ans. By Division. **(163,** III, 2d.)

27. How is a fraction changed to its *lowest* terms?

All higher terms of a fraction are *multiples* of its lowest terms. A fraction is in its *lowest terms* when its numerator and denominator are prime to each other.

Written Exercises.

167. 1. Change $\frac{7}{9}$ to *thirty sixths.*

Explanation.—Since the denominator of the required fraction is 4 times that of the given fraction, we multiply the terms of the fraction by 4. **(166.)**

OPERATION.

$$\frac{7 \times 4}{9 \times 4} = \frac{28}{36}$$

2. Reduce $\frac{3}{8}$ to 24ths; to 48ths; to 64ths; to 96ths.
3. Change $\frac{4}{5}$, $\frac{7}{8}$, $\frac{9}{10}$, and $\frac{3}{4}$ each to 40ths.
4. Change $\frac{14}{21}$, $\frac{7}{12}$, $\frac{23}{28}$, and $\frac{6}{42}$ each to 84ths.
5. Change $\frac{11}{14}$ and $\frac{19}{21}$ each to 126ths; to 168ths.
6. Reduce $\frac{72}{126}$ to its lowest terms.

OPERATION.

$$\frac{72 \div 6}{126 \div 6} = \frac{12}{21}; \quad \frac{12 \div 3}{21 \div 3} = \frac{4}{7}.$$

$$\text{Or, } \frac{72 \div 18}{126 \div 18} = \frac{4}{7}.$$

EXPLANATION.—First divide the terms of the given fraction by 6; then the terms of the resulting fraction by 3, and the result is $\frac{4}{7}$; and since the terms of $\frac{4}{7}$ are *prime to each other*, the lowest terms of $\frac{72}{126}$ is $\frac{4}{7}$. Or, divide the terms of the given fraction by 18, the G. C. D.*

In the same manner reduce to their lowest terms:

7. $\frac{24}{36}$.	11. $\frac{46}{104}$.	15. $\frac{81}{567}$.	19. $\frac{304}{1072}$.	23. $\frac{1428}{1596}$.
8. $\frac{28}{40}$.	12. $\frac{119}{168}$.	16. $\frac{432}{576}$.	20. $\frac{660}{1158}$.	24. $\frac{4272}{9256}$.
9. $\frac{34}{85}$.	13. $\frac{288}{480}$.	17. $\frac{512}{676}$.	21. $\frac{320}{1080}$.	25. $\frac{684}{11172}$.
10. $\frac{36}{81}$.	14. $\frac{156}{195}$.	18. $\frac{91}{119}$.	22. $\frac{805}{875}$.	26. $\frac{5184}{6912}$.

168. ***To change an integer, or a mixed number, to the form of an improper fraction.***

MENTAL EXERCISES.

1. How many *halves* in 1? *Thirds? Fourths? Sixths?* How many *sixths* in 5?

ANALYSIS.—Since in 1 there are 6 sixths, in 5, there are 5 times 6 sixths, or 30 sixths. Hence, $5 = \frac{30}{6}$.

2. How many *fourths* of a yard in 3 yards? 6 yards?
3. How many *eighths* of a dollar in \$4? \$7? \$9?
4. In 6 bushels how many half-bushels? How many *fourths? Sixths? Eighths? Twelfths?*

---

* *Rules* are omitted, where the mental exercises, analyses, and explanations are so full and explicit as to make the rule obvious. The pupil, however, should be required to frame a rule for all such cases.

5. How is an integer changed to fourths? To sevenths?

6. How many *fourths* in $6\frac{3}{4}$?

ANALYSIS.—Since in 1 there are 4 fourths, in 6 there are 6 times 4 fourths, or $\frac{24}{4}$. Hence, $6\frac{3}{4} = \frac{24}{4} + \frac{3}{4} = \frac{27}{4}$.

7. How many *sevenths* of a week in 3 weeks? In $4\frac{5}{7}$ weeks? In $2\frac{6}{7}$ weeks?

8. How many *twelfths* in a year? In $3\frac{5}{12}$ years?

9. How is an integer or mixed number changed to the form of an improper fraction?

WRITTEN EXERCISES.

169. 1. Change 57 to a fraction whose denominator is 32.

OPERATION.—$57 \times 32 = 1824$. Hence, $57 = \frac{1824}{32}$.

2. Change $36\frac{17}{45}$ to an improper fraction.

OPERATION.—$36\frac{17}{45} = 36 + \frac{17}{45}$; $36 \times 45 + 17 = 1637$. Hence, etc. For explanation, see "Analysis" in Mental Exercises.

3. Change 125 to a fraction whose denominator is 25.
4. In 49 bushels of wheat how many *sixtieths* of a bushel?
5. How many 16ths of a pound in 25 pounds?
6. In \$316 how many 8ths of a dollar? 25ths? 12ths?
7. Change 14 and 29 each to 36ths.
8. Change 42 and 75 each to 50ths.
9. Change 128 and 206 each to 63ds.
10. Change $31\frac{11}{12}$ and $57\frac{5}{12}$ each to 12ths.
11. Change $87\frac{5}{27}$ and $105\frac{23}{27}$ each to 27ths.
12. Change $245\frac{3}{40}$ and $92\frac{19}{40}$ each to 40ths.
13. Change $27\frac{29}{30}$ months to thirtieths of a month.
14. Change 45 to a fraction having 18 for its denominator.
15. Change to improper fractions $19\frac{15}{17}$; $65\frac{7}{41}$; $204\frac{13}{56}$.
16. What improper fraction is equivalent to $421\frac{11}{63}$?
17. How many poor families can be supplied with $\frac{1}{8}$ of a ton of coal each from $7\frac{5}{8}$ tons?

170. ***To change an improper fraction to the form of an integer or mixed number.***

MENTAL EXERCISES.

1. Change $\frac{25}{6}$ to the form of a mixed number.

ANALYSIS.—Since 6 sixths equal 1, 25 sixths equal as many times 1 as 6 sixths is contained times in 25 sixths, which is $4\frac{1}{6}$ times. Hence, $\frac{25}{6} = 25 \div 6 = 4\frac{1}{6}$.

2. How many times 1 is $\frac{14}{4}$? $\frac{21}{9}$? $\frac{47}{8}$? $\frac{75}{12}$? $\frac{84}{9}$?

3. How many yards in $\frac{50}{4}$ of a yard? How many pecks in $\frac{45}{8}$ of a peck? How many dollars in $\$\frac{75}{10}$?

4. How many pounds in 42 quarter-pound packages of tea? In 36 half-pound packages? In 65 eighth of a pound packages?

5. How is any number of sixths changed to *ones*, or to an integer?

6. How is any number of eighths changed to *ones?* Tenths? Twelfths? Why?

7. How is an improper fraction changed to the form of an integer or mixed number?

WRITTEN EXERCISES.

171. 1. Change $\frac{245}{14}$ to the form of a mixed number.

OPERATION.— $\frac{245}{14} = 245 \div 14 = 17\frac{1}{2}$. Hence, $\frac{245}{14} = 17\frac{1}{2}$.

2. In $\frac{347}{8}$ of a dollar, how many dollars?

3. In $\frac{1025}{32}$ of a bushel, how many bushels?

Change to integers or mixed numbers:

4. $\frac{81}{16}$.	8. $\frac{1025}{9}$.	12. $\frac{256}{64}$.	16. $\frac{6084}{72}$.
5. $\frac{33}{13}$.	9. $\frac{2464}{16}$.	13. $\frac{2380}{35}$.	17. $\frac{2500}{65}$.
6. $\frac{100}{27}$.	10. $\frac{976}{21}$.	14. $\frac{3742}{12}$.	18. $\frac{1261}{356}$.
7. $\frac{275}{15}$.	11. $\frac{664}{29}$.	15. $\frac{5064}{28}$.	19. $\frac{513}{133}$.

20. What is the value of $\frac{775}{31}$? Of $\frac{810}{24}$? Of $\frac{1465}{76}$? Of $\frac{8635}{140}$? $\frac{16956}{234}$? $\frac{37000}{820}$?

172. ***To change fractions to equivalent fractions having a common denominator.***

MENTAL EXERCISES.

1. How many 6ths in 1? In $\frac{1}{2}$? $\frac{2}{3}$? 8ths in $\frac{1}{4}$? $\frac{3}{4}$?

2. Change $\frac{1}{2}$ and $\frac{3}{4}$ to 4ths. To 8ths. To 12ths.

3. How can $\frac{2}{3}$ and $\frac{5}{6}$ be changed to 12ths? To 24ths? To 36ths? (**166,** 1 Ex.)

4. If the denominator be multiplied by any number, how is the value of the fraction preserved? Why?

5. Change $\frac{4}{5}$ and $\frac{7}{10}$ to 20ths. To 30ths. To 40ths.

6. How can $\frac{8}{10}$ and $\frac{12}{20}$ be changed to 5ths? (**166,** 16 Ex.)

7. If the denominator be divided by any number, how is the value of the fraction preserved? Why?

8. Change $\frac{8}{12}$ and $\frac{6}{18}$ to thirds. To 6ths.

9. Change $\frac{1}{3}$, $\frac{2}{3}$, and $\frac{5}{9}$ to 9ths. To 18ths.

10. Change $\frac{5}{8}$ and $\frac{7}{12}$ to 24ths. To 48ths.

11. What is a multiple of a number? A common multiple of two or more numbers? (**133.**)

12. Name a multiple of 3. Of 4. Of 6. Of 8.

13. Name a common multiple of 3 and 4. Of 6 and 9.

14. Name the least common multiple of 4 and 5.

15. What is the least common multiple of the denominators of $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$? Of $\frac{1}{2}$, $\frac{3}{8}$, and $\frac{5}{6}$? Of $\frac{4}{5}$, $\frac{3}{4}$ and $\frac{9}{10}$?

16. Name two common multiples of 4 and 6. Their least common multiple.

17. Name four fractions that can be changed to 24ths.

18. Reduce $\frac{3}{4}$ and $\frac{5}{7}$ to equivalent fractions having 28 for their common denominator.

19. What is the least number of which the denominators of $\frac{1}{6}$, $\frac{3}{8}$, and $\frac{7}{12}$ are common factors?

20. Change $\frac{5}{6}$, $\frac{3}{4}$, and $\frac{1}{2}$ to fractions of equal value having the least common denominator.

173. A **Common Denominator** is a denominator common to two or more fractions.

A common denominator of two or more fractions is a *common multiple* of their denominators.

174. The **Least Common Denominator** of two or more fractions is the *least common multiple* of their denominators.

Written Exercises.

175. 1. Change $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ to equivalent fractions having a common denominator.

OPERATION.

$$\frac{1\times3\times4}{2\times3\times4}=\frac{12}{24}$$
$$\frac{2\times2\times4}{3\times2\times4}=\frac{16}{24}$$
$$\frac{3\times2\times3}{4\times2\times3}=\frac{18}{24}$$

Explanation.—Multiply both terms of each of these fractions by the product of all the denominators except its own. The values of the fractions will not be changed (**164,** III), and the denominators of the new fractions will be equal, because each will be equal to the product of all the given denominators, 2, 3, and 4.

Change to fractions having a common denominator:

2. $\frac{2}{3}$ and $\frac{4}{5}$.	4. $\frac{9}{14}$ and $\frac{6}{7}$.	6. $\frac{3}{5}$, $\frac{1}{4}$, and $\frac{3}{10}$.	8. $\frac{7}{15}$, $\frac{2}{3}$, and $\frac{5}{6}$.
3. $\frac{1}{6}$ and $\frac{5}{7}$.	5. $\frac{7}{15}$ and $\frac{5}{9}$.	7. $\frac{5}{6}$, $\frac{4}{5}$, and $\frac{2}{3}$.	9. $\frac{7}{12}$, $\frac{4}{9}$, $\frac{2}{3}$, and $\frac{1}{4}$.

10. Reduce $\frac{5}{6}$, $\frac{8}{15}$, and $\frac{7}{12}$ to equivalent fractions having the least common denominator.

OPERATION.

3	6	15	12
2	2	5	4
	1	5	2

$3\times2\times5\times2=60$ L. C. M.

$$\frac{5\times10}{6\times10}=\frac{50}{60}$$
$$\frac{8\times4}{15\times4}=\frac{32}{60}$$
$$\frac{7\times5}{12\times5}=\frac{35}{60}$$

Explanation.—The L. C. M. of the denominators is 60. Dividing this by each of the denominators, the quotients are 10, 4, and 5. Multiplying both terms of $\frac{5}{6}$ by 10, both terms of $\frac{8}{15}$ by 4, and both terms of $\frac{7}{12}$ by 5, the results are $\frac{50}{60}$, $\frac{32}{60}$, and $\frac{35}{60}$. These have the least common denominator, since 60 is the L. C. M. of the given denominators. (**174.**)

Fractions should first be reduced to their lowest terms, and integral and mixed numbers to the form of improper fractions.

Change to equivalent fractions having the least common denominator :

11. $\frac{5}{9}$, $\frac{7}{8}$, and $\frac{11}{24}$.
12. $\frac{3}{7}$, $\frac{2}{3}$, and $\frac{13}{21}$.
13. $\frac{3}{4}$, $\frac{7}{12}$, and $\frac{9}{20}$.
14. $\frac{6}{7}$, $\frac{5}{14}$, and $\frac{24}{49}$.
15. $1\frac{6}{7}$, $\frac{9}{14}$, $\frac{5}{12}$, and $\frac{15}{28}$.
16. $\frac{2}{3}$, $\frac{15}{12}$, $\frac{4}{9}$, and $\frac{6}{15}$.
17. $\frac{3}{4}$, $3\frac{1}{8}$, $\frac{7}{12}$, and 6.
18. $2\frac{5}{6}$, $1\frac{2}{3}$, 4 and $\frac{4}{5}$.
19. $\frac{7}{11}$, $4\frac{1}{2}$, $\frac{2}{33}$, and $7\frac{3}{8}$.
20. $\frac{2}{7}$, $\frac{1}{3}$, $\frac{3}{14}$, and $3\frac{1}{4}$.
21. $\frac{8}{9}$, $\frac{5}{18}$, $\frac{4}{27}$, $\frac{2}{3}$, and $\frac{7}{12}$.
22. $\frac{3}{8}$, $2\frac{3}{4}$, $\frac{5}{16}$, $\frac{21}{64}$, and $\frac{7}{256}$.

ADDITION AND SUBTRACTION OF FRACTIONS.

MENTAL EXERCISES.

176. 1. What is the fractional unit of $\frac{3}{5}$? Of $\frac{4}{5}$? How many *fifths* are $\frac{3}{5}$ and $\frac{4}{5}$? How many times 1?

2. What is the *sum* of $\frac{5}{9}$ and $\frac{7}{9}$? The *difference?*

3. What is the *numerator* of the sum of $\frac{3}{8}$ and $\frac{7}{8}$? Of the difference?

4. What is the *denominator* of the sum, and of the difference, of $\frac{3}{8}$ and $\frac{7}{8}$?

5. How are fractions having a common denominator added or subtracted?

6. What is the fractional unit of $\frac{2}{3}$? Of $\frac{5}{6}$? Can $\frac{2}{3}$ and $\frac{5}{6}$ in their present form be added? Why not? Can one be subtracted from the other? Why not?

7. How many 6ths is $\frac{2}{3}$? What is the sum of $\frac{4}{6}$ and $\frac{5}{6}$? The difference?

8. What change must be made in $\frac{3}{4}$ and $\frac{2}{3}$ before their sum or their difference can be found? What is their sum? How many times 1? What is their difference?

9. How are fractions having different denominators added or subtracted?

10. Find the sum of $\frac{1}{2}$ and $\frac{2}{3}$. Of $\frac{3}{4}$ and $\frac{1}{3}$. Of $\frac{1}{2}$ and $\frac{5}{8}$.

11. Find the sum of $\frac{4}{5}$ and $\frac{3}{4}$. Of $\frac{3}{5}$ and $\frac{2}{3}$. Of $\frac{4}{9}$ and $\frac{5}{6}$.

12. Find the difference between $\frac{2}{3}$ and $\frac{4}{5}$.

13. Find the difference between $\frac{5}{8}$ and $\frac{5}{6}$; $\frac{3}{4}$ of a ton and $\frac{3}{10}$ of a ton.

14. Belle having \$1, gave \$$\frac{3}{8}$ for a book and \$$\frac{1}{4}$ for a slate. What did she pay for both? What part of \$1 had she left?

15. James having \$$\frac{5}{6}$ gave \$$\frac{1}{4}$ for a neck-tie; what had he left?

16. If the sum of two fractions is $\frac{7}{12}$ and the less is $\frac{1}{6}$, what is the greater?

17. A man bought $3\frac{1}{2}$ tons of coal at one time, and $4\frac{7}{8}$ at another; how much did he buy in all?

ANALYSIS.—He bought the sum of $3\frac{1}{2}$ and $4\frac{7}{8}$ tons. 3 and 4 are 7, and the sum of $\frac{1}{2}$ and $\frac{7}{8}$ is $\frac{11}{8}$, or $1\frac{3}{8}$, which added to 7 makes $8\frac{3}{8}$ tons. Hence, etc.

18. Walter had \$3, he earned \$$2\frac{3}{4}$ more, and his father gave him \$$\frac{7}{8}$. How much had he then?

19. From a piece of silk containing $9\frac{1}{2}$ yards, $3\frac{3}{4}$ yards were cut. How many yards remained?

ANALYSIS.—The difference between $9\frac{1}{2}$ and $3\frac{3}{4}$. $\frac{1}{2}$ equals $\frac{2}{4}$; and as $\frac{3}{4}$ cannot be taken from $\frac{2}{4}$, take 1 or $\frac{4}{4}$ from 9, leaving 8, and add it to $\frac{2}{4}$, making $\frac{6}{4}$; $\frac{3}{4}$ from $\frac{6}{4}$ leaves $\frac{3}{4}$, and 3 from 8 leaves 5. Hence, $5\frac{3}{4}$ yards remained. $9\frac{1}{2} - 3\frac{3}{4} = 5\frac{3}{4}$.

20. From a piece of land containing $12\frac{3}{8}$ acres, $5\frac{2}{3}$ acres were sold. How much land was left?

21. How much less than 3 is $\frac{3}{7} + \frac{1}{2} + \frac{3}{8}$?

22. Find the value of

$\frac{7}{8}+\frac{1}{3}$	$\frac{2}{3}+\frac{5}{8}+\frac{1}{6}$	$\frac{6}{7}+\frac{1}{3}+\frac{4}{21}$	$5\frac{1}{8}+6\frac{1}{2}$
$\frac{1}{7}+\frac{3}{4}$	$\frac{7}{8}+\frac{3}{4}+\frac{1}{3}$	$\frac{5}{12}+\frac{2}{3}+\frac{5}{6}$	$12\frac{4}{9}+4\frac{2}{3}$
$\frac{5}{6}-\frac{1}{3}$	$\frac{4}{5}+\frac{3}{4}+\frac{1}{2}$	$\frac{17}{20}+\frac{2}{5}-\frac{7}{10}$	$14\frac{7}{8}-7\frac{3}{4}$
$\frac{1}{2}-\frac{3}{7}$	$\frac{7}{10}+\frac{4}{5}-\frac{3}{4}$	$\frac{1}{4}+\frac{13}{24}-\frac{3}{8}$	$9\frac{3}{5}-2\frac{7}{10}$

177. PRINCIPLE. *In order that fractions may be added or subtracted, they must express parts of like units, and be reduced to equivalent fractions having a common denominator, if their denominators are different.*

WRITTEN EXERCISES.

178. 1. Find the sum of $\frac{5}{9}$, $1\frac{4}{5}$, and $\frac{11}{15}$.

OPERATION.— $\frac{5}{9} + 1\frac{4}{5} + \frac{11}{15} = \frac{25}{45} + \frac{81}{45} + \frac{33}{45} = 3\frac{4}{45}$.

EXPLANATION.—Change the given numbers to equivalent fractions having the least common denominator, which is 45. (**174.**) Write the sum of the numerators over the common denominator, and $\frac{139}{45} = 3\frac{4}{45}$ is the required sum.

2. Find the sum of $\frac{5}{12}$, $\frac{7}{10}$, $\frac{3}{4}$, and $\frac{7}{30}$. Of $\frac{13}{27}$, $\frac{7}{9}$, $\frac{1}{6}$, and $\frac{5}{18}$.

3. Find the sum of $16\frac{5}{9}$, $23\frac{2}{3}$, and $41\frac{3}{5}$.

OPERATION.— $16\frac{5}{9} + 23\frac{2}{3} + 41\frac{3}{5} = 80 + \frac{82}{45} = 81\frac{37}{45}$.

EXPLANATION.—The sum of the integers is 80, and of the fractions $\frac{82}{45} = 1\frac{37}{45}$, which added to 80 makes $81\frac{37}{45}$, the required sum.

4. Find the sum of 27, $36\frac{9}{14}$, and $57\frac{2}{3}$. Of $84\frac{7}{8}$, $102\frac{6}{7}$.

5. Find the difference between $\frac{7}{9}$ and $\frac{13}{24}$.

OPERATION.— $\frac{7}{9} - \frac{13}{24} = \frac{56}{72} - \frac{39}{72} = \frac{56-39}{72} = \frac{17}{72}$.

EXPLANATION.—Change the given fractions to equivalent fractions having the least common denominator, which is 72. Write the difference of the numerators over the common denominator, and $\frac{17}{72}$ is the required difference. $\frac{7}{9} - \frac{13}{72} = \frac{17}{72}$.

6. From $\frac{13}{15}$ subtract $\frac{3}{4}$. From $1\frac{7}{8}$ subtract $\frac{9}{14}$.

7. From $156\frac{7}{21}$ subtract $85\frac{5}{12}$.

EXPLANATION.—Reduce $\frac{7}{21}$ and $\frac{5}{12}$ to equivalent fractions having the least common denominator. Then, since $\frac{35}{84}$ cannot be taken from $\frac{28}{84}$, take 1 or $\frac{84}{84}$ from 156, leaving 155, and add to $\frac{28}{84}$, making $\frac{112}{84}$, and $\frac{35}{84}$ from $\frac{112}{84}$ leaves $\frac{77}{84}$ or $\frac{11}{12}$, and 85 from 155 leaves 70. Hence, etc.

OPERATION.

$$156\frac{7}{21} = 156\frac{28}{84}$$
$$85\frac{5}{12} = 85\frac{35}{84}$$
$$70\frac{77}{84}$$

8. From $73\frac{5}{8}$ subtract $27\frac{3}{11}$. From $106\frac{8}{13}$ subtract $38\frac{4}{5}$.

1. When the integers or mixed numbers are small, they may be reduced to improper fractions.

2. All fractional results should be reduced to their lowest terms, and improper fractions to whole or mixed numbers.

3. From the preceding exercises and explanations the rules are obvious.

Find the value of

9. $\frac{5}{12}+\frac{13}{8}$.
10. $\frac{3}{24}+\frac{21}{32}$.
11. $1\frac{7}{12}-\frac{7}{15}$.
12. $\frac{17}{18}-\frac{5}{8}$.
13. $\frac{2}{3}+\frac{7}{9}+\frac{5}{21}$.
14. $\frac{5}{7}-\frac{2}{9}+1\frac{1}{3}$.
15. $6\frac{4}{5}-1\frac{1}{6}+\frac{7}{10}$.
16. $28\frac{1}{2}+4\frac{9}{16}-\frac{11}{12}$.
17. $40-16\frac{7}{9}$.
18. $73\frac{9}{10}-2\frac{11}{15}$.
19. $91\frac{1}{9}+67\frac{3}{24}$.
20. $86-49\frac{7}{36}$.
21. $\frac{17}{28}+5\frac{6}{7}-1\frac{1}{8}$.
22. $26-3\frac{9}{22}+7\frac{1}{4}$.
23. $\frac{37}{48}+10\frac{21}{32}-1\frac{7}{8}$.

24. From the sum of $28\frac{1}{2}$ and $16\frac{5}{8}$ subtract the difference between $47\frac{2}{3}$ and $32\frac{4}{5}$.

25. From a cask of sugar containing $178\frac{1}{2}$ pounds, $92\frac{5}{8}$ pounds were taken. How much remained?

26. A farmer having $246\frac{7}{10}$ acres of land, bought $57\frac{1}{4}$ acres more, and then sold $120\frac{2}{3}$ acres. How much had he left?

27. A young man received a salary of $\$60\frac{5}{8}$ a month, and paid for his board $\$30\frac{1}{2}$, for washing $\$6\frac{1}{4}$, and for other expenses $\$12\frac{9}{10}$. What had he left?

28. What number added to $45\frac{5}{14}-7\frac{8}{21}$ makes $250\frac{6}{7}$?

29. Two horses cost $\$525\frac{7}{8}$, and one of them cost $\$284\frac{1}{3}$; what was the cost of the other?

30. If I pay $\$3475\frac{3}{4}$ for a painting, and sell it at a loss of $\$535\frac{1}{2}$, what do I receive for it? If at a gain of $\$225\frac{1}{3}$?

31. A merchant owned $\frac{11}{14}$ of a vessel, and sold $\frac{3}{7}$ of it. What part did he still own?

32. From 350 bushels take the difference between $156\frac{1}{8}$ bushels and $208\frac{7}{32}$ bushels.

33. A boat has on board $387\frac{1}{4}$ tons of coal, of which $108\frac{2}{3}$ tons are cannel, and the remainder anthracite. How much is anthracite?

Find the second member of the following equations:

34. $\frac{5}{11}-\frac{1}{3}+\frac{2}{3}-\frac{3}{5}=?$

35. $4\frac{1}{2}+16\frac{5}{18}-\overline{7-3\frac{7}{9}}=?$

36. $16\frac{17}{24}+11\frac{3}{8}-\overline{21\frac{7}{16}-9\frac{1}{4}}=?$

37. $18\frac{4}{15}+38-11\frac{7}{10}-17\frac{23}{30}=?$

MULTIPLICATION OF FRACTIONS.

MENTAL EXERCISES.*

179. 1. How many sevenths are 3 times 4 sevenths?

2. What part of a mile is 2 times $\frac{3}{8}$ of a mile?

3. At $\$\frac{7}{8}$ a yard, what will 4 yards of ribbon cost?

ANALYSIS.—4 times $\$\frac{7}{8}$, or $\$\frac{7}{8}\times 4 = \$\frac{28}{8} = \$3\frac{1}{2}$. Or, $\$\frac{7}{8\div 4} = \$\frac{7}{2} = \$3\frac{1}{2}$.

4. If a boy earn $\$\frac{1}{12}$ a day, what will he earn in 3 days? In 4 days? In 5 days? In 6 days?

Show that multiplying the numerator of $\frac{6}{10}$ by 5, or dividing the denominator by 5, multiplies the fraction by 5. (**163**, I.)

5. Multiply $\frac{7}{16}$ by 4; $\frac{5}{9}$ by 7; $\frac{9}{20}$ by 5; $\frac{11}{27}$ by 9; $\frac{3}{11}$ by 8.

A fraction is *multiplied* by a number equal to its denominator by removing the denominator. Thus, $\frac{6}{7}\times 7 = 6$.

6. Multiply $\frac{7}{8}$ by 8; $\frac{9}{10}$ by 10; $\frac{9}{17}$ by 17; $\frac{8}{13}$ by 13.

Cancelling a *factor* of the denominator *multiplies* the fraction by that factor. Thus, $\frac{7}{10}\times 5 = \frac{7}{2} = 3\frac{1}{2}$; $\frac{11}{18}\times 6 = \frac{11}{3} = 3\frac{2}{3}$.

7. Multiply $\frac{14}{20}$ by 5; $\frac{7}{24}$ by 8; $\frac{8}{15}$ by 3; $\frac{13}{21}$ by 7.

8. At $\$\frac{9}{16}$ a pound, what will 4 pounds of tea cost?

9. If a horse eats $\frac{3}{8}$ of a bushel of oats in a day, how much will 2 horses eat? 4 horses? 8 horses?

10. At $\$5\frac{3}{8}$ each, what will 4 hats cost?

ANALYSIS.—$\$5\frac{3}{8} = \$\frac{43}{8}$, and 4 times $\$\frac{43}{8} = \$\frac{43}{2} = \$21\frac{1}{2}$. Or, 4 times $\$\frac{3}{8} = \$\frac{12}{8} = 1\frac{1}{2}$, and 4 times $\$5 = \$20$; $\$20 + \$1\frac{1}{2} = \$21\frac{1}{2}$.

11. Multiply $4\frac{5}{6}$ by 3; $7\frac{5}{6}$ by 4; $8\frac{6}{7}$ by 6; $7\frac{9}{10}$ by 5.

12. At $\$4\frac{3}{4}$ a ton, what will be the cost of 2 tons of coal?

13. What is $\frac{1}{3}$ of 15 yards? $\frac{1}{5}$ of $\$25$?

14. Multiplying by $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, etc., is the same as dividing by what integers?

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* Whenever practicable, every recitation in arithmetic should be commenced with a brief *mental* exercise or *drill*, followed by a corresponding *written* one.

15. $\frac{5}{8}$ is how many times $\frac{1}{8}$? What is $\frac{5}{8}$ of 16 ounces?

16. Multiplying $\frac{1}{6}$ of 24 by 5 is the same as multiplying 24 by what fraction?

17. Which is greater, $\frac{4}{5}$ of 20, or $20 \times \frac{4}{5}$? $24 \times \frac{3}{4}$, or $\frac{3}{4}$ of 24?

18. When the multiplier is a fraction, the word "*of*" should be used, and not "*times*."

19. At $9 a barrel, what will $\frac{2}{3}$ of a barrel of flour cost?

ANALYSIS. Since 1 barrel costs $9, $\frac{2}{3}$ of a barrel costs 2 times $\frac{1}{3}$ of $9, or $\frac{2}{3}$ of $9. $\frac{1}{3}$ of $9 is $3, and $\frac{2}{3}$ of $9 is 2 times $3, or $6.

20. What is $\frac{3}{4}$ of 12? $\frac{5}{9}$ of 45? $\frac{2}{5}$ of 31? $\frac{3}{8}$ of 42?

21. If a man can do a job of work in 30 days, in what time can he do $\frac{1}{2}$ of it? $\frac{2}{3}$ of it? $\frac{3}{4}$ of it?

22. At the rate of $16 for a ton of hay, what is the value of $\frac{3}{8}$ of a ton? Of $\frac{3}{5}$ of a ton?

Show that multiplying an integer by a fraction is the same as taking the fractional part of the integer.

23. What is $\frac{3}{4}$ of 16? $16 \times \frac{3}{4}$? $\frac{3}{5}$ of 21? $21 \times \frac{3}{5}$?

Multiplying a fraction by an integer, or an integer by a fraction is the same, since either factor may be regarded as the multiplier.

24. At $8 a yard, what will $6\frac{2}{3}$ yards of cloth cost?

ANALYSIS. $6\frac{2}{3}$ times $8. 6 times $8 are $48, and $\frac{2}{3}$ of $8 are $$5\frac{1}{3}$; $$48 + \$5\frac{1}{3} = \$53\frac{1}{3}$. Hence, etc.

25. Multiply 7 by $5\frac{1}{4}$; 12 by $7\frac{5}{6}$; 6 by $9\frac{2}{3}$.

26. At 12 cents a pound, what will $4\frac{1}{4}$ pounds of sugar cost? $7\frac{3}{8}$ pounds? $9\frac{2}{3}$ pounds?

27. What is $\frac{1}{2}$ of 4 yards? $\frac{1}{3}$ of 9 days? $\frac{1}{2}$ of 6 tenths?

28. What part of 1 is $\frac{1}{3}$ of $\frac{1}{2}$? $\frac{1}{4}$ of $\frac{1}{3}$? $\frac{1}{2}$ of $\frac{1}{6}$?

29. Taking $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ of a fraction is the same as dividing by what integers?

30. What is $\frac{1}{3}$ of $12? $\frac{2}{3}$ of $12? $\frac{1}{4}$ of $\frac{8}{9}$? $\frac{3}{4}$ of $\frac{8}{9}$?

31. What is $\frac{1}{4}$ of $$\frac{4}{25}$? $\frac{2}{3}$ of $\frac{9}{10}$ of a mile?

The word "*of*" between two fractions is equivalent to the sign ($\times$) of multiplication. Thus, $\frac{3}{4}$ of $\frac{4}{5} = \frac{3}{4} \times \frac{4}{5}$; $\frac{6}{7}$ of $10 = \frac{6}{7} \times 10$.

32. At $\$\frac{7}{8}$ a yard, what is $\frac{2}{3}$ of a yard of cloth worth?

ANALYSIS. *Two* times $\frac{1}{3}$, or $\frac{2}{3}$ of $\$\frac{7}{8}$. $\frac{1}{3}$ of $\$\frac{7}{8} = \$\frac{7}{24}$, and 2 times $\$\frac{7}{24} = \$\frac{7}{12}$.

33. Multiply $\frac{6}{7}$ by $\frac{1}{4}$; by $\frac{3}{4}$; by $\frac{2}{5}$; by $\frac{1}{6}$; by $\frac{5}{6}$; by $\frac{3}{5}$.

34. What is $\frac{2}{3}$ of $\frac{3}{6}$? $\frac{3}{4}$ of $\frac{8}{9}$? $\frac{4}{5}$ of $\frac{3}{8}$? $\frac{5}{6} \times \frac{3}{4}$?

35. At $\$\frac{9}{10}$ a gallon, what will $\frac{3}{8}$ of a gallon of syrup cost?

36. At $\$6\frac{2}{9}$ a ton, what will $\frac{3}{4}$ of a ton of coal cost?

ANALYSIS. It will cost $\frac{3}{4}$ of $\$6\frac{2}{9}$, or 3 times $\frac{1}{4}$ of $\$6\frac{2}{9}$; $\$6\frac{2}{9} = \$\frac{56}{9}$, and $\frac{3}{4}$ of $\$\frac{56}{9} = \$\frac{56}{9} \times \frac{3}{4} = \$\frac{14}{3} = \$4\frac{2}{3}$. Hence, etc.

By cancelling the factors common to the numerators and denominators before multiplying, the operation is *shortened*, and the answer obtained in the *lowest terms.*

37. What is $\frac{4}{5}$ of $3\frac{1}{3}$ miles? $\frac{5}{6}$ of $2\frac{2}{5}$ acres? $\frac{3}{4}$ of $\$5\frac{1}{3}$?

38. What is $\frac{3}{7} \times \frac{2}{9}$? $\frac{7}{10} \times \frac{5}{14}$? $2\frac{1}{3} \times \frac{3}{4}$? $\frac{4}{9} \times 3\frac{2}{3}$?

How does cancelling equal factors in the numerator and denominator affect the value of the fraction? Why? (**110**, II).

39. What will $4\frac{2}{3}$ cords of wood cost, at $\$3\frac{3}{4}$ a cord?

ANALYSIS. $4\frac{2}{3} = \frac{14}{3}$, and $\$3\frac{3}{4} = \$\frac{15}{4}$; $\$\frac{15}{4} \times \frac{14}{3} = \$\frac{35}{2} = \$17\frac{1}{2}$.

40. What is $2\frac{2}{3}$ times $\$\frac{6}{7}$? $\frac{3}{5}$ of $4\frac{1}{6}$ pounds? $3\frac{1}{2}$ times $\$2\frac{1}{3}$?

41. If a man owning $\frac{7}{8}$ of a vessel, sells $\frac{2}{3}$ of his share, what part of the whole vessel does he retain?

42. If a horse trots $6\frac{3}{4}$ miles an hour, how far will he trot in $\frac{4}{5}$ of an hour? In $\frac{5}{9}$ of an hour? In $1\frac{1}{3}$ hours?

Multiply

43. $\frac{3}{4}$ by 8.	47. 12 by $\frac{3}{4}$.	51. $4\frac{1}{2}$ by 5.	55. 9 by $3\frac{3}{4}$.
44. $\frac{4}{5}$ by 7.	48. 16 by $\frac{2}{3}$.	52. $6\frac{1}{5}$ by 8.	56. 12 by $4\frac{2}{3}$.
45. $\frac{5}{6}$ by 6.	49. 24 by $\frac{5}{8}$.	53. $12\frac{2}{5}$ by 6.	57. 11 by $7\frac{1}{5}$.
46. $\frac{6}{17}$ by 14.	50. 40 by $\frac{7}{10}$.	54. $10\frac{7}{9}$ by 10.	58. 8 by $10\frac{3}{8}$.

What is the value of

59. $\frac{3}{5} \times \frac{5}{6}$?	62. $4\frac{1}{3} \times \frac{5}{6}$?	65. $3\frac{1}{2} \times 4\frac{1}{3}$?	68. $\frac{2}{5}$ of $\frac{1}{2} \times 4$?
60. $\frac{6}{7} \times \frac{2}{3}$?	63. $\frac{7}{8} \times 2\frac{1}{2}$?	66. $1\frac{5}{8} \times 2\frac{3}{4}$?	69. $1\frac{1}{4} \times \frac{3}{4}$ of $\frac{1}{2}$?
61. $\frac{9}{10} \times \frac{4}{3}$?	64. $\frac{7}{12} \times 3\frac{3}{7}$?	67. $4\frac{9}{10} \times 1\frac{3}{7}$?	70. $\frac{4}{5}$ of $\frac{10}{7}$ of $\frac{7}{9}$?

71. If a peach-basket hold $\frac{5}{8}$ of a bushel, how much will 3 baskets hold? 5 baskets? 6 baskets?

72. At \$$5\frac{2}{3}$ a yard, what will 4 yards of cloth cost? 6 yards? 7 yards? 8 yards?

73. When hay is \$20 a ton, what must be paid for $\frac{1}{2}$ of a ton? $\frac{1}{3}$ of a ton? $\frac{2}{3}$? $\frac{3}{4}$? $\frac{4}{5}$? $\frac{3}{8}$? $\frac{5}{6}$? $\frac{7}{10}$?

74. At $6\frac{1}{4}$ cents a pound, what will $\frac{4}{5}$ of a pound of rice cost? $1\frac{2}{3}$ pounds? 8 pounds?

75. John had \$$\frac{9}{10}$, and gave $\frac{1}{3}$ of it to his sister Nellie. What part of a dollar did he give her? What part had he left?

76. At \$$\frac{8}{9}$ a bushel, what will $\frac{3}{8}$ of a bushel of pears cost? $\frac{3}{4}$ of a bushel? $\frac{3}{10}$ of a bushel?

77. If I give $\frac{2}{3}$ of \$$\frac{9}{10}$ to a beggar, what part of a dollar do I give him? What part have I left?

78. How far is it from New York to Philadelphia, if $\frac{3}{5}$ of 50 miles is $\frac{1}{3}$ of the whole distance?

79. What will $2\frac{1}{7}$ bushels of peaches cost, at \$$2\frac{4}{5}$ a bushel?

Find the value of

80. $\frac{5}{8}$ of 12.
81. $14 \times 1\frac{5}{7}$.
82. $5\frac{1}{8} \times \frac{8}{9}$.
83. $\frac{11}{12} \times 6$.
84. $\frac{3}{4}$ of $\frac{1}{2}$ of $\frac{1}{6}$.
85. $10 \times \frac{3}{4}$ of $5\frac{1}{3}$.
86. $\frac{7}{12} - \frac{1}{6}$ of $2\frac{1}{2}$.
87. $\frac{8}{9} + 4\frac{2}{3} \times 1\frac{1}{4}$.
88. $4\frac{1}{2} + \frac{3}{4} - \frac{5}{6}$ of $3\frac{1}{2}$.
89. $\frac{7}{8} \times \overline{7\frac{1}{2} - 4\frac{3}{4}} + 1\frac{1}{3}$.
90. $\frac{5}{12}$ of $4\frac{1}{6} + \frac{3}{8}$ of 6.
91. $15 \times \frac{7}{10} - \frac{1}{2}$ of $7\frac{4}{5}$.

180. PRINCIPLES. I. *The product of two or more factors, whether integral or fractional, is the same in whatever order they are used.*

II. *The value of a fraction is multiplied, by multiplying its numerator, or dividing its denominator by any integer.* **(164, I).**

III. *The product of an integral or fractional number by a fraction is equal to such a part of the multiplicand as the multiplier is of the unit 1.*

WRITTEN EXERCISES.

181. In the multiplication of fractions, either

1st. *One* of the factors is a fractional number, and the *other* integral. Or,

2d. *Both* factors are fractional.

1. Multiply $\frac{5}{24}$ by 8.

OPERATION. $\frac{5}{24}\times 8 = \frac{5\times 8}{24} = \frac{5}{3} = 1\frac{2}{3}$. Or, $\frac{5}{24}\times 8 = \frac{5}{24\div 8} = \frac{5}{3} = 1\frac{2}{3}$.

2. Multiply 8 by $\frac{5}{24}$.

OPERATION. $8\times\frac{5}{24} = \frac{8\times 5}{24} = \frac{5}{3} = 1\frac{2}{3}$. Or, $\overline{8\div 24}\times 5 = \frac{5}{3} = 1\frac{2}{3}$. (**180**, Prin. I).

Or, both factors in the above examples may be put in fractional form and written thus: $\frac{5}{24}\times\frac{8}{1}$, or $\frac{8}{1}\times\frac{5}{24} = 1\frac{2}{3}$.

It is obvious, that *multiplying a fraction by an integer, or an integer by a fraction, is essentially the same.* (Prin. I).

By the use of a *vertical line* and *cancellation,* we have *one uniform* process, by which *all* the operations in multiplication of fractions are much abbreviated and simplified.

When either factor is an integer, place it on the *right* of the line; when both factors are fractional, place the numerators on the *right,* and the denominators on the *left.* The product of the numbers on the *right,* divided by the product of those on the *left,* will give the required product.

3. Multiply $16\frac{5}{9}$ by 12.

$$\begin{array}{rcl} & & 16\frac{5}{9} \\ & & 12 \\ \hline 192 & = & 16 \times 12 \\ 6\frac{2}{3} & = & \frac{5}{9}\times 12 \\ \hline 198\frac{2}{3} & = & 16\frac{5}{9}\times 12 \end{array}$$

Or, $16\frac{5}{9} = \frac{149}{9}$

$$\begin{array}{r|l} \overset{3}{\cancel{9}} & 149 \\ & \overset{4}{\cancel{12}} \\ \hline 3 & 596 \\ \hline & 198\frac{2}{3} \end{array}$$

4. Multiply 18 by $6\frac{4}{9}$.

$$\begin{array}{rcl} & & 18 \\ & & 6\frac{4}{9} \\ \hline 108 & = & 18\times 6 \\ 8 & = & \frac{4}{9} \text{ of } 18 \\ \hline 116 & = & 18\times 6\frac{4}{9} \end{array}$$

Or, $6\frac{4}{9} = \frac{58}{9}$

$$\begin{array}{r|l} & \overset{2}{\cancel{18}} \\ \cancel{9} & 58 \\ \hline & 116 \end{array}$$

We may multiply the integral and fractional parts separately, and unite the results; or, reduce the mixed numbers to improper fractions, and multiply as in Examples 1 and 2.

5. Multiply $\frac{9}{14}$ by $\frac{8}{15}$.

$$\frac{\overset{3}{\cancel{9}}}{\underset{7}{\cancel{14}}} \times \frac{\overset{4}{\cancel{8}}}{\underset{5}{\cancel{15}}} = \frac{12}{35}.$$

Or,
$$\begin{array}{r|l} {}^{7}\cancel{14} & \cancel{9}\,{}^{3} \\ {}^{5}\cancel{15} & \cancel{8}\,{}^{4} \\ \hline 35 & 12 = \frac{12}{35} \end{array}$$

6. Multiply $\frac{5}{8}$ of $2\frac{2}{3}$ by $\frac{3}{4}$ of 12.

$$\frac{5}{\cancel{8}} \times \frac{\cancel{8}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{4}} \times \frac{\overset{3}{\cancel{12}}}{1} = 15.$$

Or,
$$\begin{array}{r|l} \cancel{8} & 5 \\ \cancel{3} & \cancel{8} \\ \cancel{4} & \cancel{3} \\ 1 & \cancel{12}\,{}^{3} \\ \hline 1 & 15 = 15 \end{array}$$

Multiply

7. 56 by $\frac{7}{8}$.
8. 120 by $\frac{8}{15}$.
9. $\frac{14}{25}$ by 70.
10. $\frac{9}{28}$ by $\frac{7}{15}$.
11. $8\frac{1}{2}$ by $\frac{4}{5}$.
12. $\frac{16}{21}$ by $3\frac{1}{9}$.
13. $7\frac{2}{9}$ by $5\frac{3}{10}$.
14. $\frac{5}{6}$ of $\frac{2}{3}$ by $\frac{4}{5}$ of $4\frac{1}{2}$.
15. 15 by $\frac{7}{9}$ of $\frac{11}{21}$ of 6.
16. Multiply $142\frac{7}{15}$ by 30; 84 by $32\frac{9}{16}$; $47\frac{5}{8}$ by $18\frac{2}{3}$.

Rules for the Multiplication of Fractions.

I. When *either* factor is a fractional number:

Multiply together the integer and the numerator of the fraction, and write the product over the denominator.

II. When *both* factors are fractional numbers:

Multiply together the numerators for the numerator of the product, and the denominators for the denominator of the product.

All factors common to the numerators and the denominators should be cancelled before multiplying.

Find the cost of

17. 125 bushels of potatoes, at $\$\frac{4}{5}$ a bushel.
18. $28\frac{5}{16}$ yards of cloth, at \$4 a yard.
19. 56 pounds of coffee, at $\$\frac{3}{8}$ a pound.
20. $\frac{7}{8}$ of a yard of silk, at $\$\frac{9}{10}$ a yard.
21. 120 pounds of wool, at $37\frac{1}{2}$ cents a pound.
22. $\frac{3}{4}$ of $15\frac{1}{3}$ yards of satin, at $\$4\frac{2}{5}$ a yard.

23. $214\frac{3}{4}$ pounds of beef, at $9\frac{1}{2}$ cents a pound.
24. $32\frac{5}{8}$ tons of coal, at $\$6\frac{4}{7}$ a ton.
25. $15\frac{1}{4}$ quarts of milk, at $6\frac{1}{4}$ cents a quart.
26. $\frac{4}{5}$ of 20 cords of wood, at $\$4\frac{9}{16}$ a cord.
27. $26\frac{3}{8}$ pounds of fish, at $4\frac{3}{4}$ cents a pound.
28. $\frac{5}{12}$ of $87\frac{1}{2}$ acres of land, at $\frac{4}{5}$ of $\$60\frac{3}{10}$ an acre.
29. $75\frac{5}{6}$ bushels of wheat, at $\$1\frac{7}{10}$ a bushel.
30. $112\frac{1}{2}$ barrels of flour, at $\$7\frac{1}{5}$ a barrel.
31. $106\frac{1}{2}$ baskets of peaches, at $\$\frac{9}{10}$ a basket.
32. $\frac{1}{3}$ of $\frac{5}{6}$ of 9 tons of hay, at $\$15\frac{1}{5}$ a ton.

33. A machinist's wages were $\$4\frac{1}{2}$ a day, and by overwork, he made $\$\frac{3}{5}$ a day more. What did he earn in $10\frac{2}{3}$ days?

34. If I purchase a carriage for $\$245\frac{1}{8}$, and sell it for $\frac{5}{8}$ of the cost, what do I lose? If for $1\frac{1}{8}$ times the cost, what do I gain?

35. If a family burn $1\frac{3}{8}$ tons of coal a month, how much will 4 families burn in $5\frac{7}{11}$ months?

36. The sum of three factors is 25; the least is $4\frac{1}{2}$, the greatest is $12\frac{3}{4}$. What is the product of the three?

Find the product of

37. $\frac{11}{12}$ and $\frac{15}{18}$.
38. $\frac{32}{72}$ and $\frac{27}{29}$.
39. $6\frac{3}{4}$ and $2\frac{14}{27}$.
40. $15\frac{4}{5}$ and $3\frac{8}{9}$.
41. $7\frac{2}{9}$ and $\frac{2}{5}$ of $6\frac{2}{3}$.
42. $\frac{3}{16}$, $\frac{21}{24}$, and $\frac{48}{49}$.
43. $8\frac{8}{23}$, $4\frac{3}{8}$, and $2\frac{13}{28}$.
44. $19\frac{5}{7}$ and $\frac{3}{5}$ of $\frac{10}{23}$.
45. $\frac{5}{6}$, $2\frac{3}{8}$, and $3\frac{7}{15}$.
46. $4\frac{1}{8}$, $2\frac{1}{11}$, and $1\frac{1}{6}$.
47. $7\frac{1}{9}$, $1\frac{5}{16}$, and $11\frac{3}{7}$.
48. $\frac{5}{9}$ of 8 and $\frac{3}{4}$ of $3\frac{1}{2}$.

Find the value of

49. $\frac{6}{7} \times \frac{4}{5} + \frac{2}{3}$ of $\frac{4}{5} \times 2$.
50. $6\frac{2}{3} \times 1\frac{3}{4} \times \overline{1\frac{1}{11} - \frac{1}{2}}$.
51. $(\frac{6}{7} - \frac{2}{3}) \times (\frac{3}{5} + \frac{2}{3})$.
52. $3\frac{6}{7} \times \frac{5}{9} - \overline{2\frac{2}{3} \times \frac{3}{10}}$.
53. $3\frac{2}{5} \times 5\frac{1}{2} \times \frac{7}{9} - \frac{1}{5}$ of $2\frac{13}{18}$.
54. $(4\frac{1}{2} \times \frac{7}{8} + 1\frac{3}{5}) \times \overline{3\frac{1}{3} - \frac{9}{10}}$.
55. $(19\frac{4}{5} - 3\frac{3}{4}) \times (3\frac{4}{5} - 2\frac{3}{7})$.
56. $19\frac{4}{5} - \overline{3\frac{3}{4} \times 3}\frac{4}{5} - 2\frac{3}{7}$.
57. $19\frac{4}{5} - (3\frac{3}{4} \times 3\frac{4}{5} - 2\frac{3}{7})$.
58. $3\frac{5}{9} + 2\frac{7}{8} \times 10\frac{2}{7} + 12\frac{6}{7} - 8\frac{9}{63}$.
59. $6\frac{2}{3} \times 5\frac{3}{4} - 4\frac{4}{5} \times 2\frac{5}{6} + 1\frac{17}{30}$.
60. $6\frac{2}{3} \times \overline{5\frac{3}{4} - 4\frac{4}{5}} \times 2\frac{5}{6} + \frac{19}{20}$.

DIVISION OF FRACTIONS.

MENTAL EXERCISES.

182. 1. What is the quotient of 6 sevenths divided by 3?

2. What is the quotient of 8 ninths divided by 2? by 4?

3. What is $\frac{1}{4}$ of $\frac{8}{9}$? $\frac{1}{3}$ of $\frac{9}{10}$? $\frac{1}{2}$ of $\frac{6}{11}$? $\frac{1}{6}$ of $\frac{12}{17}$?

4. Dividing by 2, 3, 4, 5, etc., is the same as multiplying by what fractions?

5. If 3 pounds of coffee cost $\$\frac{6}{7}$, what will 1 pound cost?

ANALYSIS.—$\frac{1}{3}$ of $\$\frac{6}{7}$, or $\$\frac{6\div 3}{7}=\$\frac{2}{7}$; or, $\$\frac{6}{7\times 3}=\$\frac{2}{7}$. Hence, etc.

6. If 4 dozen eggs cost $\$\frac{5}{6}$, what is the cost of 1 dozen?

How may a fraction be divided by an integer?

7. Find the value of $\frac{1}{4}$ of $\frac{8}{11}$; of $\frac{10}{11}\div 5$; of $\frac{1}{7}$ of $\frac{14}{15}$.

Show that dividing the numerator of $\frac{9}{10}$ by 3, or multiplying the denominator by 3, divides the fraction by 3. (**163**, II).

8. Divide $\frac{4}{5}$ by 2; $\frac{7}{9}$ by 3; $\frac{9}{10}$ by 4; $\frac{14}{25}$ by 7.

9. At \$4 a yard, how many yards will $\$18\frac{2}{3}$ buy?

ANALYSIS.—As many yards as \$4 is contained times in $\$18\frac{2}{3}$, or $\frac{1}{4}$ of $18\frac{2}{3}$. $18\frac{2}{3}=\frac{56}{3}$, and $\frac{1}{4}$ of $\frac{56}{3}=\frac{56\div 4}{3}=\frac{14}{3}=4\frac{2}{3}$. Or, $\frac{1}{4}$ of $18\frac{2}{3}$ is 4, and $2\frac{2}{3}$ remainder; and $\frac{1}{4}$ of $2\frac{2}{3}$ or of $\frac{8}{3}$ is $\frac{2}{3}$, and $4+\frac{2}{3}=4\frac{2}{3}$ yards.

10. Divide $7\frac{1}{2}$ by 5; $9\frac{1}{4}$ by 7; $10\frac{4}{5}$ by 9; $12\frac{2}{3}$ by 4.

11. How many times will $16\frac{3}{4}$ gallons of vinegar fill a vessel that holds 2 gallons? 3 gallons?

12. If a family burn $\frac{3}{5}$ of a ton of coal in a week, how many weeks will $6\frac{1}{2}$ tons last? $8\frac{3}{4}$ tons?

13. Divide $\frac{3}{4}$ of 18 by $\frac{2}{5}$ of 15; $\frac{3}{8}$ of 28 by $\frac{3}{10}$ of 30.

14. Divide $16\frac{2}{3}$ by 5; $12\frac{3}{5}$ by 9; $25\frac{1}{2}$ by 12; $3\frac{1}{9}$ by 7.

15. How many *thirds* in 1? in 2? in 3? in 4?

16. In 1 how many times $\frac{1}{2}$? $\frac{1}{3}$? $\frac{1}{4}$? $\frac{1}{5}$? $\frac{1}{6}$? $\frac{1}{7}$? $\frac{1}{8}$? $\frac{1}{10}$?

Show that dividing by $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc., is the same as multiplying the dividend by 2, 3, 4, etc.

17. How many times is 1 contained in 3? $\frac{1}{2}$ in 3? $\frac{1}{3}$ in 3?

18. In 2 how many times $\frac{1}{4}$? $\frac{3}{4}$? In 3 how many times $\frac{1}{5}$? $\frac{2}{5}$? $\frac{3}{5}$?

19. Dividing 5 by $\frac{3}{4}$ is the same as multiplying 5 by what fraction?

20. At $\$\frac{5}{6}$ each, how many shovels can be bought for \$9?

ANALYSIS.—As many as $\$\frac{5}{6}$ is contained times in \$9; $\$9=\$\frac{54}{6}$, and 5 sixths is contained in 54 sixths $10\frac{4}{5}$ times. Or, $\$\frac{5}{6}$ is contained in \$1 $\frac{6}{5}$ times, and in \$9, 9 times $\frac{6}{5}$, or $\frac{54}{5}=10\frac{4}{5}$ times. Hence, etc.

21. 6 is how many times $\frac{2}{3}$? $\frac{3}{4}$? $\frac{6}{7}$? $\frac{3}{8}$? $\frac{5}{12}$? $\frac{7}{9}$? $\frac{9}{20}$?

How may an integer be divided by a fraction?

22. How many books can be bought for \$8, at $\$\frac{4}{5}$ each? At $\$\frac{5}{8}$ each? At $\$\frac{6}{7}$? At $\$\frac{3}{10}$? At $\$\frac{7}{12}$?

23. If $\frac{3}{4}$ of a yard of cloth make a vest, how many vests will 9 yards make? 12 yards? 21 yards?

24. At $\$1\frac{2}{3}$ each, how many diaries will \$10 purchase?

ANALYSIS.—As many as $\$1\frac{2}{3}=\$\frac{5}{3}$ is contained times in \$10, or $\$\frac{30}{3}\div\$\frac{5}{3}=6$ times; hence, 6 diaries can be bought for \$10.

25. Divide 14 feet by $\frac{5}{6}$ of a foot; by $\frac{7}{8}$; by $1\frac{2}{3}$; $2\frac{3}{4}$; $3\frac{1}{2}$; $4\frac{1}{2}$.

26. How many barrels, each holding $2\frac{1}{2}$ bushels, will 9 bushels of apples fill? 15 bushels? 20 bushels? $27\frac{1}{2}$ bushels?

What is the *reciprocal* of a fraction? (**161**).
Show by an example that multiplying an integer by the reciprocal of a fraction *divides* the integer by that fraction.

27. How many times can $\frac{3}{16}$ of a yard of ribbon be cut from $\frac{9}{16}$ of a yard? From $\frac{12}{16}$? $\frac{8}{16}$? $\frac{15}{16}$?

When the divisor and dividend have a common denominator, how is the quotient found?

28. How many boxes of figs at $\$\frac{2}{5}$ a box will $\$\frac{2}{3}$ purchase?

ANALYSIS.—As many as $\$\frac{2}{5}$ is contained times in $\$\frac{2}{3}$; $\$\frac{2}{5}=\$\frac{6}{15}$, and $\$\frac{2}{3}=\$\frac{10}{15}$; $\$\frac{6}{15}$ is contained in $\$\frac{10}{15}$ $1\frac{2}{3}$ times. Hence, etc.

29. Divide $\frac{3}{4}$ by $\frac{2}{3}$; $\frac{4}{5}$ by $\frac{1}{6}$; $\frac{5}{8}$ by $\frac{1}{3}$; $\frac{7}{10}$ by $\frac{2}{5}$.

30. How many pounds of rice, at $\$\frac{1}{8}$ a pound, can be bought for $\$\frac{3}{4}$? For $\$\frac{9}{16}$? For $\$1\frac{1}{12}$?

When the divisor and dividend have not a common denominator, how is the quotient found?

31. At $\$\frac{7}{8}$ a day, in what time will a boy earn $\$5\frac{1}{4}$? $\$10\frac{1}{2}$?

Reduce mixed numbers to improper fractions; then divide as you divide one fraction by another.

32. How many times can a measure holding $\frac{3}{4}$ of a bushel be filled from $6\frac{1}{2}$ bushels? $7\frac{2}{3}$ bushels?

33. How many days work, at $\$1\frac{1}{2}$ a day, will $\$6\frac{3}{4}$ obtain?

ANALYSIS.—As many as $\$1\frac{1}{2}$ is contained times in $\$6\frac{3}{4}$. $\$1\frac{1}{2}=\$\frac{3}{2}=\$\frac{6}{4}$; $\$6\frac{3}{4}=\frac{27}{4}$; $\$\frac{27}{4}\div\$\frac{6}{4}=\$\frac{27}{4}\times\frac{4}{6}=\frac{27}{6}=4\frac{1}{2}$. Hence, etc. Or, multiply the dividend by the *reciprocal* of the divisor, $\frac{2}{3}$.

34. Divide $4\frac{1}{2}$ by $1\frac{2}{3}$; $5\frac{1}{4}$ by $2\frac{3}{4}$; $6\frac{1}{3}$ by $2\frac{1}{6}$.

35. At $\$1\frac{1}{8}$ a bushel, how many bushels of peaches can be bought for $\$6\frac{3}{4}$? For $\$11\frac{1}{4}$? For $\$9\frac{1}{2}$?

36. If $\frac{3}{4}$ of a yard of silk cost $\$\frac{7}{8}$, what will 1 yard cost?

ANALYSIS.—1 yard will cost 4 times $\frac{1}{3}$, or $\frac{4}{3}$ of $\$\frac{7}{8}=\$\frac{28}{24}=\$1\frac{1}{6}$.

37. If $\frac{3}{8}$ of a box of figs is worth $\$\frac{4}{5}$, what is 1 box worth?

38. How old is Jessie, if $7\frac{1}{2}$ years is $\frac{5}{12}$ of her age?

39. At $\$\frac{4}{5}$ a pound, how much tea can be bought for $\$\frac{7}{8}$?

Divide

40. $\frac{9}{10}$ of a rod by 6.
41. $\frac{16}{25}$ of an acre by 8.
42. $\frac{1}{2}$ of $\$2\frac{1}{4}$ by $\$\frac{4}{5}$.
43. 14 quarts by $3\frac{1}{2}$.

Divide

44. $3\frac{1}{2}$ gallons by 12.
45. $7\frac{1}{5}$ miles by 9.
46. $3\frac{2}{3}$ weeks by $\frac{6}{7}$.
47. $10\frac{1}{4}$ acres by $2\frac{1}{2}$ acres.

Find the value of

48. $\frac{5}{6}$ of $2\frac{1}{2}\div\frac{3}{4}$.
49. $3\frac{1}{8}\div\frac{1}{4}$ of $1\frac{1}{2}$.
50. $8\frac{1}{3}+6\frac{1}{4}\div 5$.
51. $12\frac{1}{4}\div 3\frac{1}{2}-\frac{15}{16}$.
52. $\frac{3}{8}\times 6\div\frac{5}{6}$ of $1\frac{2}{3}$.
53. $(\frac{1}{7}\times\frac{5}{4}+1\frac{1}{2})\div\frac{3}{7}$.
54. $8\div(\frac{2}{3}$ of $\frac{3}{4}\times 6+\frac{1}{4})$.
55. $4\frac{1}{8}\div\frac{11}{24}+2\frac{1}{8}\times\frac{2}{3}$.
56. $\overline{16\frac{1}{2}-2\frac{5}{8}}\times\overline{1\frac{1}{2}\div\frac{3}{8}}$.
57. $(\frac{5\times 2}{6}+\frac{6-4}{3})\div 8$.

183. Principles.—I. *A fraction is divided by dividing its numerator, or by multiplying its denominator by any integer.* (**163**, II.)

II. *Any integral or fractional number is divided by multiplying it by the* RECIPROCAL *of the divisor.*

Written Exercises.

184. In the division of fractions, the divisor and dividend are either—

1st. The *one* an integral and the *other* a fractional number. Or, 2d. *Both* are fractional.

By the use of the *vertical line* and *cancellation*, all the operations in division of fractions may be reduced to *one uniform* process, simple, and often much abbreviated, only differing from multiplication by inverting the terms of the divisor (Prin. II).

1. Divide $\frac{15}{16}$ by 5.

OPERATION.

$$\begin{array}{r|l} 16 & \cancel{15}\ ^{3} \\ \cancel{5} & \\ \hline 16 & 3 = \frac{3}{16}. \end{array} \qquad (\mathbf{163}, \text{II.})$$

2. Divide 5 by $\frac{15}{16}$.

OPERATION.

$$\begin{array}{r|l} & \cancel{5} \\ ^{3}\ \cancel{15} & 16 \\ \hline 3 & 16 \\ \hline & 5\frac{1}{3} \end{array} \qquad (\mathbf{183}, \text{II.})$$

3. Divide $\frac{5}{6}$ of $3\frac{7}{8}$ by $2\frac{3}{4} \times \frac{5}{12}$.

OPERATION.

$$\frac{5}{6} \text{ of } 3\frac{7}{8} = \frac{155}{48};\ 2\frac{3}{4} \times \frac{5}{12} = \frac{55}{48}.$$

$$\frac{155}{48} \div \frac{55}{48} = \frac{155}{48} \times \frac{48}{55} = \frac{31}{11} = 2\frac{9}{11} \text{ Ans.}$$

Or,

$$\begin{array}{r|l} \cancel{6} & \cancel{5} \\ \cancel{8} & 31 \\ 11 & \cancel{4} \\ \cancel{5} & \cancel{12} \\ \hline 11 & 31 \\ \hline & 2\frac{9}{11} \text{ Ans.} \end{array}$$

The *references*, with the *analyses* given in the *Mental Exercises*, are quite sufficient to enable the pupil to understand and to explain every operation under the head of Written Exercises.

Divide

4. $\frac{16}{17}$ by 12.
5. $\frac{35}{36}$ by 15.
6. 42 by $\frac{7}{9}$.
7. $\frac{9}{13}$ by $\frac{12}{39}$.
8. $7\frac{1}{9}$ by $2\frac{2}{7}$.
9. $\frac{12}{17}$ by $5\frac{1}{7}$.
10. $15\frac{1}{2}$ by 14.
11. $\frac{9}{14}$ by $2\frac{5}{7}$.
12. $6\frac{9}{10}$ by $2\frac{3}{5}$.
13. $6\frac{7}{8}$ by $\frac{4}{11}$ of 5.
14. $\frac{3}{11} \times \frac{4}{9} \times 2\frac{3}{4}$ by $1\frac{5}{8}$.
15. $\frac{3}{4}$ of $\frac{12}{27}$ by $3\frac{5}{9}$.

Rules for the Division of Fractions.

I. When the divisor is an integral number—

1. *Divide the numerator, or multiply the denominator of the fraction by the integer.* **(163,** II.) Or,

2. *If a mixed number, and more convenient, divide the integer and fraction separately and unite the results.*

II. When the divisor is a fractional number—

3. *Change integers and mixed numbers to the form of fractions; then multiply the dividend by the reciprocal of the divisor.* **(183,** II.)

When the pupil is familiar with the various processes and the principles involved, as taught in the preceding Mental and Written Exercises, all operations in Multiplication and Division of Fractions may be performed by this one

General Rule.

Change all integers and mixed numbers to the form of improper fractions. Inverting the terms of the divisors, place all numerators on the RIGHT, *and all denominators on the* LEFT *of a vertical line. Cancel equal factors, if any, and divide the product of the remaining factors on the right by the product of the remaining factors on the left.*

Find the value of

16. $45 \div \frac{4}{15}$.
17. $112 \div \frac{9}{28}$.
18. $15\frac{3}{8} \div \frac{7}{16}$.
19. $\frac{51}{64} \div 3\frac{1}{6}$.
20. $119 \div \frac{7}{9}$.
21. $\frac{56}{165} \div \frac{4}{7}$.
22. $15\frac{5}{8} \div 1\frac{7}{8}$.
23. $73\frac{1}{2} \div 7\frac{1}{2}$.
24. $\frac{14}{23} \div 2\frac{1}{3}$.

25. $43\frac{1}{5} \div 9$.
26. $\frac{18}{19} \div 7\frac{1}{2}$.
27. $56 \div 1\frac{5}{9}$.
28. $\frac{69}{80} \div \frac{5}{8}$ of $3\frac{2}{7}$.
29. $3\frac{23}{35} \div 2\frac{2}{3} \times 3\frac{1}{5}$.
30. $\frac{4}{5}$ of $8\frac{8}{9} \div 7\frac{8}{15}$.

31. What number multiplied by $\frac{9}{14}$ will produce $64\frac{2}{7}$?

32. What number divided by $12\frac{3}{8}$ will give $9\frac{7}{9}$?

33. The dividend is 4 times $3\frac{3}{8}$, the divisor $\frac{3}{7}$ of $70\frac{7}{10}$; what is the quotient?

34. If 175 bushels of potatoes are raised on $2\frac{1}{3}$ acres, what is the average yield an acre?

35. At $\$\frac{3}{4}$ a pound, how many pounds of tea can be bought for $\$37\frac{1}{2}$? For $\$28\frac{5}{8}$? For $\$40\frac{4}{5}$?

36. At $\$4\frac{2}{3}$ a bushel, how many bushels of clover-seed can be bought for $\$17\frac{1}{2}$? For $\$37\frac{1}{3}$? For $\$75\frac{5}{6}$?

37. If 1 man consume $\frac{4}{5}$ of a pound of meat, how many men will consume $\frac{3}{4}$ of $9\frac{3}{5}$ pounds? $\frac{5}{8}$ of 32 pounds?

38. At $\$1\frac{4}{5}$ a bushel, how many bushels of wheat will pay for $\frac{2}{3}$ of a barrel of flour, at \$10 a barrel?

39. If $9\frac{1}{3}$ bushels of corn weigh $528\frac{1}{2}$ pounds, what is the average weight of a bushel?

40. How many pounds of coffee can be bought for $\$45\frac{1}{3}$, at $\$\frac{3}{8}$ a pound? At $\$\frac{2}{5}$? At $\$\frac{8}{10}$? At $\$\frac{5}{16}$?

41. If $\frac{4}{7}$ of a lot is worth \$1200, what is the whole worth?

42. How many tons of hay, at $\$16\frac{1}{4}$ a ton, will $\$203\frac{1}{3}$ buy?

43. How many acres of land will $\$2187\frac{1}{2}$ purchase, at $\$131\frac{1}{4}$ an acre? At $\$84\frac{3}{8}$? At $\$75\frac{7}{10}$?

44. If a man spend $\$\frac{3}{8}$ a day for cigars, in what time will he spend $\$25\frac{1}{2}$? \$50? $\$125\frac{9}{10}$? \$75?

45. Allowing $1\frac{7}{8}$ bushels to an acre, how many acres can be sown with $156\frac{1}{4}$ bushels of wheat? $112\frac{2}{3}$ bushels?

46. If $\frac{3}{10}$ of a farm cost \$3155, what is the cost of the whole?

Find the value of

47. $3\frac{3}{4}$ of $2\frac{3}{16} \div 5\frac{5}{8}$ of $8\frac{8}{9}$; of $\overline{2\frac{2}{5} \times 2\frac{5}{8}} \div \overline{2\frac{3}{5} - 2\frac{1}{4}}$.

48. $\overline{3\frac{1}{2} \times \frac{4}{5}} \div \overline{2\frac{1}{10} \times \frac{4}{13}}$; of $(\frac{4}{5} - \frac{6}{25} + \frac{24}{125}) \div (\frac{2}{3} - \frac{2}{9} + \frac{52}{81})$.

49. Find the value of $\dfrac{15\frac{5}{8}}{8\frac{1}{3}}$.

OPERATION.

$$\frac{15\frac{5}{8}}{8\frac{1}{3}} = 15\tfrac{5}{8} \div 8\tfrac{1}{3} = \tfrac{125}{8} \div \tfrac{25}{3} = \tfrac{125}{8} \times \tfrac{3}{25} = 1\tfrac{7}{8}.$$

EXPLANATION. This example simply means that $15\frac{5}{8}$ is to be divided by $8\frac{1}{3}$. Similar expressions are sometimes called *Complex Fractions*, and performing the division is called *reducing a complex fraction to a simple one.*

When the terms of the dividend or divisor are connected by + or —, these operations must be performed before that of division.

Find the value of

50. $\dfrac{8\frac{1}{4}}{6\frac{3}{5}}$, or $8\frac{1}{4} \div 6\frac{3}{5}$.

51. $\dfrac{7\frac{2}{3}}{9}$, or $7\frac{2}{3} \div 9$.

52. $\dfrac{12\frac{9}{10}}{\frac{1}{3} \text{ of } 6\frac{3}{4}}$.

53. $\dfrac{\frac{8}{9}}{\frac{13}{18}}$.

54. $\dfrac{\frac{5}{2} - \frac{2}{5}}{\frac{4}{3} - \frac{3}{4}}$.

55. $\dfrac{\frac{2}{3} + \frac{5}{6}}{8\frac{7}{15}}$.

56. $\dfrac{11\frac{5}{11} - \frac{6}{11}}{3\frac{1}{2} + 5\frac{3}{22}}$.

57. $\dfrac{2\frac{2}{3} \times 3\frac{3}{4}}{2\frac{2}{3} + 3\frac{3}{4}}$.

The following sets of numbers will give all the exercise upon fractional numbers that may be desired.

Thus, take the first set, $\frac{4}{5}$ and $\frac{2}{3}$; find in their simplest form their *sum, difference, product,* and lastly, the *quotient* of the first divided by the other. Treat each set in the double columns in like manner.

58. $\frac{4}{5}$, $\frac{2}{3}$.
59. $\frac{7}{8}$, $\frac{7}{9}$.
60. $\frac{5}{12}$, $\frac{2}{7}$.
61. $\frac{9}{10}$, $\frac{5}{8}$.
62. $\frac{5}{9}$, $\frac{3}{7}$.
63. $\frac{7}{18}$, $\frac{2}{5}$.
64. $\frac{15}{24}$, $\frac{5}{8}$.
65. $\frac{6}{13}$, $\frac{8}{9}$.
66. $\frac{6}{20}$, $\frac{5}{12}$.
67. $\frac{17}{30}$, $\frac{1}{6}$.
68. $\frac{4}{5}$, $\frac{20}{21}$.
69. $\frac{8}{14}$, $\frac{3}{5}$.
70. $\frac{8}{12}$, $\frac{13}{20}$.
71. $\frac{28}{35}$, $\frac{9}{12}$.
72. $\frac{25}{75}$, $\frac{30}{10}$.
73. $\frac{7}{8}$, $\frac{16}{20}$.
74. $\frac{15}{25}$, $\frac{5}{9}$.
75. $\frac{21}{51}$, $\frac{13}{17}$.
76. $\frac{96}{144}$, $\frac{36}{60}$.
77. $\frac{32}{56}$, $\frac{28}{77}$.
78. $\frac{7}{9}$, $\frac{15}{25}$.
79. $\frac{51}{84}$, $\frac{9}{28}$.
80. $\frac{63}{90}$, $\frac{11}{121}$.
81. $\frac{112}{333}$, $\frac{16}{111}$.
82. $4\frac{5}{8}$, $12\frac{1}{2}$.
83. $9\frac{1}{5}$, $\frac{14}{20}$.
84. 56, $3\frac{7}{8}$.
85. $16\frac{4}{9}$, $11\frac{2}{3}$.
86. $7\frac{3}{4}$, $3\frac{5}{12}$.
87. $\frac{18}{42}$, $6\frac{1}{6}$.
88. $9\frac{9}{10}$, 15.
89. $1\frac{17}{20}$, $14\frac{1}{4}$.
90. $28\frac{1}{3}$, $13\frac{2}{3}$.
91. $\frac{16}{17}$, $17\frac{3}{8}$.
92. 125, $8\frac{1}{8}$.
93. $\frac{75}{90}$, 80.
94. $33\frac{4}{25}$, $7\frac{3}{4}$.
95. $130\frac{3}{4}$, $5\frac{1}{8}$.
96. $6\frac{4}{21}$, $15\frac{3}{8}$.
97. $90\frac{6}{11}$, $4\frac{8}{9}$.
98. $100\frac{5}{7}$, $\frac{9}{14}$.
99. $191\frac{1}{5}$, $159\frac{1}{3}$.
100. $\dfrac{\frac{7}{8}}{16}$, $\dfrac{\frac{5}{6}}{\frac{15}{16}}$.
101. $\dfrac{\frac{7}{8}}{4}$, $\dfrac{3}{7\frac{1}{2}}$.
102. $\dfrac{5}{2\frac{2}{3}}$, $\dfrac{14\frac{1}{4}}{3\frac{9}{16}}$.

RELATION OF NUMBERS.

185. The relation of one number to another is expressed by the quotient obtained by dividing the number compared by the number with which it is compared.

186. PRINCIPLE. *Only like numbers, whether integral or fractional, can be compared with each other.*

MENTAL EXERCISES.

187. ***To find what part one integer is of another.***

1. What part of 5 is 4?

ANALYSIS. 1 is $\frac{1}{5}$ of 5, and 4 is 4 times $\frac{1}{5}$ of 5, or $\frac{4}{5}$ of 5. Hence, etc.

2. What part of 7 is 3? Of 8 is 7? Of 9 is 2?
3. What part of 6 cts. is 1 ct.? Of \$11 is \$6?
4. What part of 60 is 40? Of 36 is 4? Of 42 is 7?
5. If a watch cost \$60, and a chain \$15, what part of the cost of the watch equals the cost of the chain?
6. If a chest of tea is worth \$36, what part of a chest can be bought for \$9? For \$6? \$12? \$4? \$18? \$27?

188. ***To find what part a fraction is of an integer.***

1. What part of 7 is $\frac{2}{3}$?

ANALYSIS. 1 is $\frac{1}{7}$ of 7, and $\frac{2}{3}$ of 1 is $\frac{2}{3}$ of $\frac{1}{7}$, or $\frac{2}{21}$ of 7; or, $\frac{2}{3} \div 7 = \frac{2}{21}$.

2. What part of 8 is $\frac{2}{5}$? is $\frac{3}{4}$? is $\frac{2}{7}$? $\frac{5}{6}$? $\frac{7}{8}$? $\frac{8}{9}$? $\frac{7}{12}$?
3. What part of \$12 is \$$\frac{4}{5}$? Of 10 weeks is $\frac{6}{7}$ of a week?
4. What part of 15 acres is $1\frac{1}{2}$ or $\frac{3}{2}$ acres? Of 8 is $2\frac{1}{3}$?
5. What part of \$20 is \$$\frac{4}{5}$? \$$\frac{7}{10}$? \$$\frac{3}{4}$? \$$\frac{2}{3}$? \$$\frac{7}{8}$?
6. If flour bought at \$7 a barrel is sold at \$$8\frac{1}{4}$, what part of the cost equals the gain?

189. ***To find what part a fraction is of a fraction.***

1. What part of $\frac{7}{8}$ is $\frac{3}{4}$?

ANALYSIS. 1 is $\frac{8}{7}$ of $\frac{7}{8}$, and $\frac{3}{4}$ of 1 is $\frac{3}{4}$ of $\frac{8}{7}$, or $\frac{24}{28} = \frac{6}{7}$. Or, $\frac{3}{4} \div \frac{7}{8} = \frac{3}{4} \times \frac{8}{7} = \frac{6}{7}$. Hence, etc.

2. What part of $\frac{4}{5}$ is $\frac{1}{2}$? Of $\frac{5}{7}$ is $\frac{1}{6}$? Of $\frac{3}{8}$ is $\frac{1}{7}$? Of $\frac{2}{3}$ is $\frac{3}{5}$?
3. What part of $\frac{9}{10}$ is $\frac{7}{20}$? Of $1\frac{1}{4}$ is $\frac{7}{8}$? Of $4\frac{1}{2}$ is $1\frac{5}{8}$?
4. What part of \$$8\frac{3}{4}$ is \$$5\frac{1}{2}$? Of $5\frac{5}{6}$ miles is $3\frac{1}{3}$ miles?
5. Ella had \$$\frac{9}{10}$, and spent \$$\frac{3}{8}$. What part of \$$\frac{9}{10}$ had she left?
6. At \$$1\frac{3}{8}$ a pound, what part of a pound will \$$\frac{3}{8}$ buy?
7. A man bought $\frac{7}{8}$ of an acre of land, and sold $\frac{1}{2}$ of an acre; what part of $\frac{7}{8}$ of an acre had he left?

190. ***To find a number when a fractional part is given.***

1. 24 is $\frac{4}{5}$ of what number?

ANALYSIS. If $\frac{4}{5}$ of a number is 24, $\frac{1}{5}$ of it is $\frac{1}{4}$ of 24, or 6; hence the number is 5 times 6, or 30.

2. 60 is $\frac{5}{3}$ of what number? $\frac{6}{11}$ of what? $\frac{4}{5}$? $\frac{12}{7}$? $\frac{3}{4}$?
3. $9\frac{3}{5}$ is $\frac{1}{4}$ of what number? $\frac{3}{4}$ of what? $\frac{3}{2}$? $\frac{4}{5}$? $\frac{6}{7}$?
4. $\frac{8}{9}$ is $\frac{1}{2}$ of what number? $\frac{2}{3}$ of what? $\frac{4}{7}$? $\frac{5}{6}$? $\frac{7}{8}$?
5. Frank gave 60 cents for a knife, which was $\frac{5}{12}$ of what he paid for a sled; what did the sled cost him?
6. If $\frac{7}{8}$ of a ton of hay is worth \$$9\frac{5}{8}$, what is 1 ton worth?
7. 24 is $\frac{4}{5}$ of how many times 6?

ANALYSIS. If $\frac{4}{5}$ of a number is 24, $\frac{1}{5}$ of it is 6; hence, the number is 5 times 6, or 30; and 6 in 30, 5 times.

8. 42 is $\frac{7}{8}$ of how many times 12? 8? 6? 11? 9?
9. $14\frac{2}{3}$ is $\frac{4}{9}$ of how many times 3? 11? 10? 8? 12?
10. $12\frac{6}{7}$ is $\frac{9}{14}$ of how many times $\frac{1}{8}$ of 40? $\frac{3}{5}$ of 10?
11. $\frac{7}{8}$ is $\frac{3}{4}$ of how many times $\frac{1}{3}$ of $\frac{1}{2}$? $\frac{1}{2}$ of $\frac{2}{3}$? $\frac{1}{4}$ of $\frac{2}{3}$?
12. A lady paid \$50, which was $\frac{5}{3}$ of what she paid for 10 yards of silk; what was the cost of the silk a yard?

13. $\frac{4}{7}$ of 56 is $\frac{4}{9}$ of how many times 20? 9? 8? 10? 12?

14. $\frac{4}{5}$ of $\frac{3}{4}$ of 40 is $\frac{2}{3}$ of $\frac{1}{2}$ of how many times 8? 6? 7?

15. $\frac{3}{4}$ of 32 is $\frac{6}{7}$ of 4 times what number?

ANALYSIS. $\frac{3}{4}$ of 32 is 24, and 24 is $\frac{6}{7}$ of 28; 28 is 4 times $\frac{1}{4}$ of 28, which is 7. Hence, etc.

16. $\frac{5}{8}$ of 72 is $\frac{9}{10}$ of 5 times what number?

17. A merchant sold $\frac{2}{7}$ of a piece of cloth for \$45, at \$3 a yard; how many yards in the piece?

18. Paid \$10 for a pair of pantaloons, which was $\frac{4}{5}$ of 3 times what I paid for a coat; what did the coat cost?

19. A grocer, after selling $\frac{5}{8}$ of a barrel of sugar, had 60 pounds left; how many pounds did the barrel contain?

Review

MENTAL EXERCISES.

191. 1. The sum of two fractions is $\frac{11}{18}$, and one of them is $\frac{1}{3}$; what is the other? What their difference?

2. From what number must $4\frac{1}{8}$ be taken to leave $7\frac{5}{6}$?

3. The greater of two numbers is $9\frac{4}{7}$, and their difference $2\frac{1}{2}$; what is 3 times the less number?

4. The minuend is $12\frac{7}{8}$, and the subtrahend $7\frac{3}{4}$; what is $\frac{2}{3}$ of their difference?

5. What number divided by $\frac{5}{12}$ will give a quotient of $3\frac{5}{6}$?

6. What is the product of $2\frac{4}{5}$ and $\frac{1}{4}$ of $1\frac{3}{5}$?

7. The product of two factors is $\frac{4}{5}$ of 35, and one of the factors is $4\frac{2}{3}$; what is $\frac{1}{2}$ of 3 times the other factor?

8. The divisor is $4\frac{5}{7}$, and the quotient $3\frac{1}{2}$; what is 4 times $\frac{1}{2}$ of the dividend?

9. Find the sum of $\frac{4}{5}$ of $3\frac{1}{3}$ and $\frac{5}{6}$. Their difference. Their product. The quotient of the greater by the less.

10. Add 3 to both terms of the fraction $\frac{4}{5}$. Is its value increased, or diminished, and how much?

11. A farmer sold $\frac{3}{7}$ of his sheep and had 80 left; how many sheep had he at first?

12. Sold a sleigh for \$75, which was $\frac{5}{8}$ of what it cost; what was the loss?

13. How much less than \$5 will 12 pounds of coffee cost, at \$$\frac{3}{10}$ a pound? How much more than $\frac{3}{4}$ of 5 times \$$\frac{3}{5}$?

14. If $\frac{7}{10}$ of a ton of coal cost \$$4\frac{2}{3}$, what will 3 tons cost?

15. At \$$14\frac{3}{5}$ a ton, what will $\frac{3}{4}$ of a ton of hay cost? $\frac{4}{5}$ of a ton?

16. When flour is \$$9\frac{1}{2}$ a barrel, what part of a barrel can be bought for \$6? For \$7? For \$$4\frac{3}{4}$?

17. At \$$\frac{3}{8}$ a bushel, how many bushels of apples can be bought for \$$5\frac{1}{2}$? For \$$7\frac{5}{8}$? For \$$9\frac{1}{4}$?

18. By selling a harness for \$60, I gained $\frac{3}{7}$ of its cost; what did it cost?

19. A man bought $\frac{1}{2}$ of $\frac{3}{5}$ of an acre of land, and his neighbor bought $\frac{3}{4}$ of the remainder; which bought the more, and how much?

20. Add 3 to both terms of the fraction $\frac{5}{4}$. Will its value be increased or diminished, and how much?

21. At \$$4\frac{1}{3}$ a yard, what would $\frac{5}{6}$ of a yard of silk cost?

22. Three men bought a pile of wood for \$56. The first was to have $\frac{3}{8}$ of it, the second $\frac{1}{3}$ of the remainder, and the third what was left; what was each man's share worth?

23. If 3 yards of shirting cost \$$\frac{5}{8}$, what will 12 yards cost?

24. If $\frac{7}{16}$ of a bushel of plums cost \$$\frac{7}{8}$, what will $\frac{3}{4}$ of a bushel cost? $2\frac{1}{2}$ bushels? $3\frac{2}{3}$ bushels?

25. If $\frac{9}{10}$ of a bin of coal is worth \$63, what is $\frac{3}{8}$ of it worth?

26. Paid \$50 for a harness, and $\frac{4}{5}$ the cost of the harness was $\frac{3}{10}$ of the cost of my horse; what was the cost of both?

27. John, after spending $\frac{2}{5}$ of his money, found that \$6 was $\frac{2}{3}$ of what he had left; how much had he at first?

28. If $\frac{2}{3}$ of a ton of coal is worth \$$4\frac{4}{5}$, what are $7\frac{2}{3}$ tons worth? $\frac{1}{4}$ of $12\frac{1}{2}$ tons? $\frac{1}{2}$ of 15 tons?

Written Exercises.

192. 1. What number diminished by $\frac{2}{5}$ and $\frac{3}{7}$ of itself leaves a remainder of 240?

2. What number multiplied by $14\frac{1}{4}$ will produce $1684\frac{1}{2}$? Divided by $15\frac{2}{3}$ will give a quotient of $49\frac{5}{6}$?

3. If $\frac{5}{7}$ of a yacht is valued at $3840\frac{1}{2}$, what is the value of the whole? Of $\frac{3}{4}$? Of $\frac{3}{8}$? Of $\frac{2}{5}$?

4. A has 125 acres of land, which is $1\frac{2}{5}$ times as much as B has; how many acres has B?

5. Multiply the sum of $\frac{1}{2}$, $1\frac{2}{7}$, and $\frac{5}{6}$, by the difference of $\frac{4}{15}$ and $\frac{3}{20}$, and divide the product by $\frac{11}{18}$ of $1\frac{14}{15}$.

6. What is the sum of $\frac{5}{8}$ and its reciprocal?

7. Two men working the same number of days earned $76\frac{9}{16}$, one receiving $2\frac{5}{8}$ and the other $3\frac{1}{2}$ a day; how many days did they work?

8. What is the product of $1\frac{8}{9}$ and its reciprocal?

9. If $3\frac{3}{4}$ tons of hay cost $60, how much can be bought for $39? For $75? $120?

10. What is the quotient of $\frac{9}{14}$ divided by its reciprocal?

11. A merchant deposited $\frac{1}{6}$ of his money in one bank, $\frac{2}{7}$ of it in another, loaned $\frac{1}{3}$ of it, and put the remainder, $6142, in his safe; how much money had he?

12. If $\frac{5}{8}$ of a pound of tea cost $.50, what will $16\frac{3}{4}$ pounds cost?

13. A and B contract to do a job of work. A does $\frac{7}{15}$ and B $\frac{8}{15}$, and B receives $25 more than A; how much did each receive?

14. If $\frac{5}{9}$ of a mill is worth $10000, what is $\frac{1}{2}$ of the remainder worth? What is $\frac{2}{3}$ of the whole worth?

15. If $\frac{3}{5}$ of a barrel of flour cost $6\frac{3}{5}$, how many barrels can be bought for $104\frac{1}{2}$? For $52\frac{1}{4}$?

16. How many yards of cloth $\frac{7}{8}$ of a yard wide will line $23\frac{1}{2}$ yards that is $1\frac{1}{4}$ yards wide? That is $\frac{3}{4}$ of a yard wide?

17. The difference between $\frac{5}{9}$ and $\frac{3}{7}$ of the value of an estate is \$2432. What is $\frac{1}{3}$ of $\frac{1}{2}$ of the estate worth?

18. What is $\frac{3}{4}$ of an acre of land worth, if $\frac{5}{6}$ is worth \$60?

19. A and B can build a house in 30 days; B can do the work alone in 45 days. In how many days can A do it alone?

20. If $\frac{3}{15}$ of the rental of a store is paid for taxes, and the owner saves \$880, what rent does he get?

21. A man bought peaches at $\$\frac{3}{5}$ a basket, and sold them for $\$1\frac{1}{4}$ a basket, and gained \$52. How many baskets did he sell?

22. If $\frac{2}{3}$ of $\frac{6}{7}$ of a piece of work can be done in 16 days, in what time can $\frac{3}{4}$ of the whole work be performed?

23. Find the sum, difference, and product of $19\frac{3}{8}$ and $10\frac{5}{8}$; also the quotient of their sum divided by their difference.

24. A man bequeathed $\frac{7}{13}$ of his estate to his elder son, and the remainder to his younger son, who received \$1345.50 less than his brother. What was the estate worth?

25. If a tank, whose capacity is 420 gallons, is $\frac{5}{8}$ full, what part of it would be filled if $87\frac{1}{2}$ gallons were added?

26. A grocer bought 3 boxes of soap, each containing 75 pounds, at $6\frac{1}{3}$ cents a pound, and kept it until it dried away in weight $\frac{1}{3}$, and then sold it at $9\frac{2}{3}$ cents a pound. What did he gain or lose?

27. If $\frac{4}{5}$ of 8 acres of land cost $\$1011\frac{1}{3}$, what cost 4 times $1\frac{1}{2}$ acres?

28. How many vessels, holding $\frac{3}{8}$ of a gallon, can be filled from $\frac{1}{4}$ of a barrel of $31\frac{1}{2}$ gallons?

29. If $\frac{7}{8}$ of an acre of land cost $\$122\frac{1}{2}$, what will $2\frac{4}{5}$ acres cost?

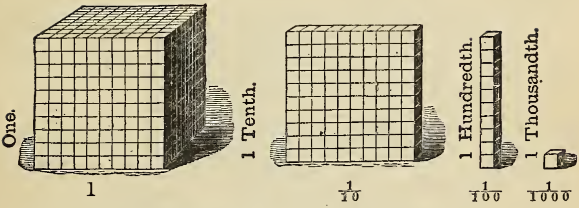
Reduce to the simplest form:

30. $(\overline{2\frac{3}{4} + 2\frac{1}{2}} \times \overline{7 \div 3\frac{4}{5}} - 1\frac{2}{3} \div 2\frac{1}{2}) \div 1\frac{77}{228}$.

31. $\frac{3}{4}$ of $\dfrac{4\frac{5}{9}}{6\frac{1}{8}} \times \dfrac{6\frac{8}{11}}{11\frac{5}{7}}$.

32. $\dfrac{\frac{1}{9} \text{ of } 1\frac{1}{4} \times 4\frac{1}{2}}{\frac{5}{56} \times 1\frac{1}{3} \text{ of } 3\frac{1}{2}} - \dfrac{3\frac{1}{4} + 4\frac{1}{3}}{6\frac{1}{2} + 1\frac{1}{12}}$.

DECIMALS.



INDUCTIVE EXERCISES.

193. 1. If a unit is divided into *ten* equal parts, what is *one* part called? 3 parts? 6 parts?

2. What is the fractional unit of 3 tenths? Of $\frac{7}{10}$? Of any number of tenths?

3. If 1 tenth of 1 is divided into *ten* equal parts, what is *one* part called? 7 parts? 15 parts? 43 parts?

4. What is $\frac{1}{10}$ of $\frac{1}{10}$ of 1? $\frac{3}{10}$ of $\frac{1}{10}$ of 1? $\frac{7}{10}$ of $\frac{1}{10}$?

5. What is the fractional unit of 7 hundredths? Of 27 hundredths? Of $\frac{31}{100}$? Of any number of hundredths?

6. If 1 hundredth of 1 is divided into *ten* equal parts, what is *one* part called? 9 parts? 50 parts? 125 parts?

7. What is $\frac{1}{10}$ of $\frac{1}{10}$ of $\frac{1}{10}$ of 1? $\frac{3}{10}$ of $\frac{1}{100}$ of 1? $\frac{9}{10}$ of $\frac{1}{100}$ of 1?

8. What is the fractional unit of 9 thousandths? Of 47 thousandths? Of any number of thousandths?

9. What part of 1 is 1 tenth? What part of 1 tenth is 1 hundredth? How many hundredths in 1 tenth? How many tenths in 1?

NOTATION AND NUMERATION.

194. A **Decimal Fraction** expresses one or more of the *decimal* divisions of a unit.

Thus, $\frac{1}{10}$, $\frac{3}{100}$, $\frac{45}{1000}$, and $\frac{126}{10000}$, etc., are *decimal* fractions. Hence,

The denominator of a decimal fraction is either *ten*, or the product of two or more tens, the *fractional units* being *tenths, hundredths, thousandths,* etc.

195. Decimal fractions, commonly called *Decimals,* are usually expressed by writing the numerator only. Thus,

$\frac{5}{10}$	is written	.5,	and is read		5 tenths.
$\frac{25}{100}$	"	.25,	"	"	25 hundredths.
$\frac{125}{1000}$	"	.125,	"	"	125 thousandths.
$\frac{7}{100}$	"	.07,	"	"	7 hundredths.
$\frac{16}{1000}$	"	.016,	"	"	16 thousandths.
$\frac{45}{10000}$	"	.0045,	"	"	45 ten-thousandths.
$\frac{245}{100000}$	"	.00245,	"	"	245 hundred-thous'ths.

196. The **Decimal Sign** (.) or point determines, by its position, the *denominator* of the fraction, and, in a number composed of an integer and a decimal, it shows where the decimal part begins.

It will be seen in the above examples that there must be as many places in the decimal as there are ciphers in the denominator of the fraction; and that every decimal has for its denominator 1, with as many ciphers annexed as there are places in the given decimal.

When the numerator does not contain as many places as there are ciphers in the denominator, prefix ciphers to the numerator until the number of places is equal to the number of ciphers in the denominator, and prefix the decimal-point. Thus, $\frac{5}{100} = .05$; $\frac{7}{1000} = .007$; $\frac{26}{10000} = .0026$; etc.

197. Express in decimal form, and read:

$\frac{9}{10}$	$\frac{125}{1000}$	$\frac{43}{1000}$	$\frac{215}{1000}$	$\frac{200}{1000}$
$\frac{4}{10}$	$\frac{8}{1000}$	$\frac{6}{1000}$	$\frac{324}{10000}$	$\frac{650}{10000}$
$\frac{12}{100}$	$\frac{17}{1000}$	$\frac{350}{1000}$	$\frac{1206}{10000}$	$\frac{324}{100000}$
$\frac{7}{100}$	$\frac{50}{100}$	$\frac{3}{100}$	$\frac{36}{10000}$	$\frac{18}{100000}$

Express in the form of a fraction and read:

.8	.27	.105	.08	.4320
.06	.006	.0105	.060	.00415
.25	.450	.0035	.240	.00084
.025	.03	.0009	.1250	.0010
.007	.010	.0320	.0008	.00005

198. When an integer and a decimal are united, or written together, the expression is called a *Mixed Number.* **(159.)**

The word "*and*" is always read between the integer and the decimal. Thus, 9.25 is read, 9 *and* 25 hundredths; 12.045 is read, 12 and 45 thousandths.

199. The notation of decimals does not differ from that of integers. It is but the extension of the same system, and the application of the same principles in a *descending* scale. Hence, having learned the scale of integers, and observed the tenfold increase and decrease of its orders, the pupil need only add *th* to the name of each order already known, to carry the descending scale to any point or order desired, since the same uniform law of value runs through the integral and decimal parts of the expression; that is, a unit of any order is *ten times* the value of a unit of the next lower order on the *right,* and *one-tenth* the value of the next higher order on the *left.*

Hereafter, the terms *fraction* and *decimal* will be used to distinguish the *common* from the *decimal form* of expression.

200. The relation of integral and decimal orders of the scale is clearly shown in the following

TABLE.

NAMES.	Hund.-millions.	Ten-millions.	*Millions.*	Hund.-thousands.	Ten-thousands.	*Thousands.*	Hundreds.	Tens.	UNITS.	Tenths	Hundredths.	*Thousandths.*	Ten-thousandths.	Hund.-thousandths.	*Millionths.*	Ten-millionths.	Hund.-millionths.
ORDERS.	9th.	8th.	7th.	6th.	5th.	4th.	3d.	2d.	1st.	2d.	3d.	4th.	5th.	6th.	7th.	8th.	9th.
UNITS.	2	2	2	2	2	2	2	2	2.	2	2	2	2	2	2	2	2
	INTEGERS.									DECIMALS.							

The number is read, 222 million 222 thousand 222, and 22 million 222 thousand 222 *hundred-millionths.*

201. Since the unit 1 is the basis of all numbers, integral and fractional, we make the order of units the starting-point or centre of the system of notation and numeration of decimals, designating it by the decimal point (.), placed *under*, or at the *right* of it.

All integral orders on the left are *multiples* of this unit, and all decimal orders on the right are *decimal parts* of this unit, and the values of the units, equally distant from this *fundamental* unit, are *reciprocals* of each other.

Thus, the 2d order on the left is 1 *ten*, and the 2d order on the right is 1 *tenth*, and $\frac{1}{10}$ or .1 is the reciprocal of 10; the 3d order on the left is 1 *hundred*, and the 3d order on the right is 1 *hundredth*, and $\frac{1}{100}$ or .01 is the reciprocal of 100, etc.

For convenience, the point is placed at the *right* of units instead of *under* it; so when the expression is a pure decimal, the point may be supposed to occupy units' place.

202. Decimals, abstractly considered, express but one kind of unit, and that of the highest order given, the lower denominations being represented in decimals of the higher order.

Thus, in the expression .32465, the point occupies the *hundred's* order of the *thousandth's* period; hence we know at once that 1 one hundred-thousand is the *decimal unit,* and that *hundred-thousandths* is the *name* of the decimal.

In writing decimals, as in writing integers, all *vacant orders* must be filled with ciphers. (**19.**)

203. Since the *value* of a figure in a decimal expression depends upon its *position* with reference to the decimal point, removing a figure one place to the left by removing the point one place to the right increases its value tenfold, or *multiplies* it by 10; removing it 2 places, *multiplies* it by 100; three places, by 1000, etc.

Thus, $.007 \times 10 = .07$; $.07 \times 10 = .7$; $.7 \times 10 = 7$. Or, $.007 \times 1000 = 7$.

The number of decimal units remains the same, but their *value* is increased 10 times, by each removal one place to the left.

204. Conversely, removing a decimal figure one place to the right, by removing the point one place to the left, diminishes its value tenfold, or divides it by ten; removing it two places, divides it by 100; three places, by 1000, etc.

Thus, $.5 \div 10 = .05$; $.05 \div 10 = .005$; $.005 \div 10 = .0005$. Or, $.5 \div 1000 = .0005$.

The *number* of decimal units remains the same, but their *value* is made 10 times less by each removal one place to the right.

205. Annexing a cipher to a decimal multiplies both terms of the fraction by *ten;* and removing a cipher from the right divides both terms by *ten.* Neither operation affects the *value* of the decimal, since the place or distance of the significant figures, in relation to the *fundamental unit,* is not changed.

206. Principles. I. *The laws of notation as applied to integers and to decimals are the same. Hence,*

II. *Ten units of any decimal order make one unit of the next higher order.*

III. *The value of any decimal order of units depends upon its position at the right of the decimal point.* **(199.)**

IV. *Each removal of a decimal order one place to the left increases its value ten-fold.* **(14, III.)**

V. *Each removal of a decimal order one place to the right diminishes its value ten-fold.* **(14, IV.)**

VI. *Annexing or rejecting ciphers at the right of any decimal does not change its value.*

Written Exercises.

207. Express as decimals:

1. Forty-five thousandths. $\frac{45}{1000} = .045$.

2. Thirty-four hundredths. Six thousandths. Three hundredths.

3. One hundred forty thousandths. Twelve ten-thousandths.

4. Eighty-two ten-thousandths. Ninety-six thousandths.

5. 84 ten-thousandths.	9. 12406 ten-millionths.
6. 2034 ten-millionths.	10. 934 hundred-millionths.
7. 450 ten-thousandths.	11. 36 ten-thousandths.
8. 642 millionths.	12. 104 ten-millionths.

13. $\frac{105}{1000}$.	16. $9\frac{11}{1000}$.	19. $327\frac{9}{100}$.	22. $75\frac{75}{10000}$.
14. $\frac{4060}{100000}$.	17. $57\frac{16}{10000}$.	20. $1240\frac{27}{1000}$.	23. $120\frac{156}{1000}$.
15. $\frac{10034}{1000000}$.	18. $146\frac{325}{100000}$.	21. $108\frac{1262}{100000}$.	24. $1000\frac{482}{1000}$.

25. Six hundred ten thousand and 249 millionths.

26. Ten million 5 hundred thousand and 105 hundred-thousandths.

27. Eighty thousand 142 and 13601 millionths.

Rules for Decimals.

I. To *write* a decimal.

Write the numerator of the decimal as an integral number, filling the vacant places, if any, with ciphers, and place the decimal point before tenths.

II. To *read* a decimal.

Read the decimal as if an integral number, and give it the name of the right-hand order.

Express in decimal form the following:

28. Three hundred twenty and five hundred seven ten-thousandths.
29. Two million 25 thousand and 631 ten-millionths.
30. Seven thousand and 3204 hundred-thousandths.

Copy and read the following:

31.	.1272.	36.	7.032.	41.	.000384.
32.	.00425.	37.	29.1037.	42.	.7062005.
33.	.60300.	38.	40.00036.	43.	96.003007.
34.	.000572.	39.	2.20075.	44.	8.03140.
35.	.0040071.	40.	16.00009.	45.	247.8640.

Decimal Currency.

208. Currency is a term applied to coin, notes, bills of exchange, or to any substitute for money, employed in trade and commerce.

209. United States Money is the *legal currency* of the United States, and as its denominations increase by the *decimal scale*, it is a *Decimal Currency*.

It is sometimes called *Federal Money* because issued by the Federal Government.

210. The denominations and scale of United States Money are shown in the following

TABLE.

10 Mills (*m.*)	= 1 Cent.	c. or ct.
10 Cents	= 1 Dime.	d.
10 Dimes	= 1 Dollar.	$
10 Dollars	= 1 Eagle.	E.

$1 = 10 d. = 100 ct. = 1000 m.

Thousands.	Hundreds.	Tens.	Units.		Tenths.	Hundredths.	Thousandths.
1	1	1	1	.	1	1	1
Dolls.					Cts.		M.

In business operations *eagles* are regarded as *tens of dollars,* and *dimes,* as *tens of cents.* Thus, 5 *eagles* is written as $50, and 5 dimes, as 50 cents.

211. Since the dollar is the *unit* of United States Money, dollars are written as *integers,* with the sign ($) prefixed; and the lower denominations are written as *decimals*—that is, as *tenths, hundredths,* and *thousandths* of the dollar.

Thus, 12 dollars 4 dimes 6 cents 7 mills is written $12.467. Hence, dimes are *tenths* of a dollar; cents are *tenths* of a dime, or *hundredths* of a dollar; and mills are *tenths* of a cent, or *hundredths* of a dime, or *thousandths* of a dollar.

212. In business transactions, *cents* are often written as fractions of a dollar; and the *half-cent* either as a fraction ($\frac{1}{2}$), or as 5 mills.

Thus, $7.42 may be written $\$7\frac{42}{100}$; $44\frac{1}{2}$ cents, $\$.44\frac{1}{2}$, or $.445.

Generally, in business calculations, if in the final result the mills are less than 5 they are not regarded; if 5 or more than 5, they are considered a *cent.* Thus, $9.234 would be called $9.23, and $9.236 would be called $9.24. This usage will be adopted in the answers to examples in this book.

213. PRINCIPLE.—*The system of notation, and all the operations in decimal currency, are the same as corresponding operations in integers and decimals.*

REDUCTION OF DECIMALS.

INDUCTIVE EXERCISES.

214. 1. How many tenths in 2 units? In 3? 4? 5?

2. How many hundredths in 1 unit? In 2? 5? 6?

3. How many tenths in 50 hundredths? In 60?

4. How many hundredths in 3 tenths? In 4 tenths?

5. In 1 dollar how many *tenths* of a dollar? How many *hundredths?* How many *thousandths?*

6. In 2 dollars how many dimes? How many cents?

7. How many thousandths in 2 units? In .2? In .25?

8. In 40 hundredths how many tenths? Thousandths?

9. In 40 cents how many dimes? How many mills?

10. In 5000 mills how many cents? Dimes? Dollars?

11. How many thousandths of a dollar are 7 mills? Are 8 cents? 45 cts. 8 mills? $12\frac{1}{2}$ cts.? $37\frac{1}{2}$ cts.?

12. What decimal part of a dollar is 7 cents? 27 cents?

13. Express as cents $4; $5.36; $14.25; $10.20.

14. Express as mills 35 cts.; $37\frac{1}{2}$ cts.; 75 cts.; $87\frac{1}{2}$ cts.

15. How are dollars changed to cents? Cents to mills? Dollars to mills?

16. How are mills changed to cents? Cents to dollars?

17. In .900 how many hundredths? How many tenths?

18. How are thousandths changed to hundredths? Hundredths to tenths? Tenths to units?

19. How are units changed to tenths? Tenths to hundredths? Hundredths to thousandths?

20. How is any integer or decimal changed to higher orders of units? To lower orders? (**206**, IV, V.)

21. Change .6 to hundredths. To thousandths.

22. Change .4 and .03 to thousandths. .05 and .012.

23. Change .2, .03, and .025 to equivalent *decimals* having a common denominator of 1000; of 10000.

24. Reduce .8, .250, and .400 to equivalent *fractions* having the least common denominator.

25. How are decimals reduced to a common denominator? State the principle. (**206**, VI.)

26. Express .5 as a fraction in *halves;* .25 in *fourths.*

27. How many halves in $\frac{50}{100}$? In $\frac{500}{1000}$?

28. In .50 how many halves? Fourths? Tenths?

29. Express as a fraction in its lowest terms .20; .8; .40.

30. How is a decimal changed to the form of a fraction?

31. How many tenths in 1? In $\frac{1}{2}$ of 1, or $\frac{1}{2}$?

32. How many hundredths in 1? In $\frac{1}{2}$? In $\frac{1}{4}$? In $\frac{3}{4}$?

33. How many cents in \$$\frac{1}{2}$? In \$$\frac{1}{4}$? In \$$\frac{3}{4}$?

34. How many tenths in $\frac{1}{5}$? In $\frac{3}{5}$? In $\frac{4}{5}$?

35. Express $\frac{7}{10}$ decimally; $\frac{4}{5}$; $\frac{1}{4}$ as hundredths; $\frac{3}{4}$; $\frac{7}{20}$.

36. In $\frac{2}{5}$ how many tenths? Hundredths?

37. Change to the form of a decimal $\frac{1}{2}$; $\frac{4}{5}$; $\frac{1}{4}$; $\frac{1}{8}$; $\frac{3}{4}$.

38. How is a fraction changed to the form of a decimal?

Written Exercises.

215. 1. Change \$575 to cents; \$84.62 to cents. (**41**, 4.)

2. Change 2684 cents to dollars; 68250 mills to dollars.

3. Change \$4.875 to mills; \$.926 to mills.

4. Change 842 cents to dollars; 785 mills to cents.

5. Change 30256 mills to dollars; 65 cents to mills.

6. Reduce 3.5, .225, and 4.0015 to equivalent decimals having a common denominator.

7. Reduce 4 tenths, 28 ten-thousandths, and 156 hundred-thousandths to their least common denominator.

Reduce to the least common denominator:

8. 42.04, 125.126, .0036, .108, and 5.00306.

9. .60034, 325.06, 4.00074, .0861, and 25.8.

The Inductive Exercises and the principles taught, are sufficient to enable the pupil to frame all rules necessary for the Written Exercises in decimals.

10. Change .625 to an equivalent fraction.

OPERATION.— $.625 = \frac{625}{1000} = \frac{5}{8}$.

Change to equivalent fractions in their lowest terms:

11. .04.	15. .48.	19. .068.	23. .0065.	27. .00032.
12. .12.	16. .64.	20. \$.375.	24. .9375.	28. .59375.
13. .80.	17. .35.	21. \$.875.	25. .3125.	29. .00060.
14. .45.	18. .125.	22. \$.1875.	26. .0024.	30. .6875.

31. Change $.16\frac{2}{3}$ to an equivalent fraction.

OPERATION.— $.16\frac{2}{3} = \dfrac{16\frac{2}{3}}{100} = \dfrac{\frac{50}{3}}{100} = \dfrac{50}{300} = \dfrac{1}{6}$.

Change to fractions in their lowest terms:

32. $\$.06\frac{1}{4}$; $\$.33\frac{1}{3}$.	35. $\$.62\frac{1}{2}$; $.312\frac{1}{2}$.	38. $.0008\frac{1}{4}$.
33. $\$.08\frac{1}{3}$; $\$.37\frac{1}{2}$.	36. $\$.18\frac{3}{4}$; $.833\frac{1}{3}$.	39. $.1666\frac{2}{3}$.
34. $\$.12\frac{1}{2}$; $\$.31\frac{1}{4}$.	37. $\$.87\frac{1}{2}$; $.007\frac{3}{4}$.	40. $.1428\frac{4}{7}$.

41. Express by an integer and a fraction $\$14.12\frac{1}{2}$.

OPERATION.— $\$14.12\frac{1}{2} = \$14\frac{25}{200} = \$14\frac{1}{8}$.

In like manner express:

42. 15.775.	44. $150.37\frac{1}{2}$; $9.007\frac{3}{4}$.	46. $\$125.00\frac{4}{5}$.
43. $\$87.444\frac{4}{9}$.	45. $705.41\frac{2}{3}$; $26.05\frac{5}{9}$.	47. $\$8.083\frac{1}{3}$.

216. 1. Change $\frac{3}{8}$ to an equivalent decimal.

OPERATION.

$$\frac{3}{8} = \frac{3000}{8000} = \frac{375}{1000} = .375. \quad \text{Or, } \frac{3}{8} = \frac{3.000}{8} = .375.$$

Reduce to equivalent decimals:

2. $\frac{5}{8}$; $\frac{7}{8}$.	5. $\frac{2}{25}$; $\frac{14}{20}$.	8. $\frac{21}{32}$; $\frac{33}{250}$.	11. $\frac{91}{1250}$; $\frac{26}{512}$.
3. $\frac{1}{16}$; $\frac{3}{16}$.	6. $\frac{11}{16}$; $\frac{3}{40}$.	9. $\frac{97}{128}$; $\frac{201}{625}$.	12. $\frac{548}{800}$; $\frac{18}{3200}$.
4. $\frac{1}{40}$; $\frac{5}{80}$.	7. $\frac{14}{25}$; $\frac{15}{16}$.	10. $\frac{47}{125}$; $\frac{27}{800}$.	13. $\frac{1603}{2000}$; $\frac{291}{480}$.

217. When the numerator, with ciphers annexed, is exactly divisible by the denominator, the result is called a **Perfect Decimal.** Thus, .25, .375 are *perfect decimals.*

218. When the denominator of a fraction in its lowest terms contains any other prime factor than 2 or 5, the division of the numerator by the denominator cannot be exact, and the result is called a **Circulating Decimal**; and the figure or set of figures repeated is called the **Repetend.**

Thus, .333 + and .4545 + are *circulating decimals*, and the repeating figures, 3 and 45, are the *repetends*.

219. A repetend is usually written but once, and a point (.) is placed over the single figure, or the first and last of the set of figures.

Thus, .66 + and .216 + are written, $.\dot{6}$ and $.\dot{2}1\dot{6}$.

220. A **Pure Circulating Decimal** contains no figures besides the repetend; as, $.\dot{5}$, $.\dot{1}2\dot{5}$, $.\dot{1}06\dot{7}$, etc.

221. A **Mixed Circulating Decimal** contains other figures *before* the repetend, called the *finite* part of the decimal; as, $.2\dot{2}\dot{7}$, $.04\dot{8}$, $.18\dot{2}4\dot{3}$, etc.

Change

1. To perfect decimals: $\frac{8}{25}$; $\frac{15}{128}$; $\frac{11}{3125}$; $\frac{3}{125}$ of $2\frac{3}{4}$; $\frac{7}{512}$.
2. To pure circulating decimals: $\frac{2}{3}$; $\frac{4}{9}$; $\frac{11}{37}$; $\frac{39}{41}$; $\frac{65}{101}$; $\frac{178}{333}$.
3. To mixed circulating decimals: $\frac{5}{6}$; $\frac{7}{15}$; $\frac{5}{36}$; $\frac{34}{75}$; $\frac{59}{330}$.

222. Reducing $\frac{1}{9}$ to a decimal, the result is .111 +, or $.\dot{1}$; and since $.\dot{1}$ is the decimal equivalent for $\frac{1}{9}$, $.\dot{2}$ must be the equivalent for $\frac{2}{9}$; $.\dot{3}$, for $\frac{3}{9}$; and so on to $.\dot{9}$, equal to $\frac{9}{9}$, or 1.

Again, $\frac{1}{99}$ reduced to a decimal is .0101 +, or $.\dot{0}\dot{1}$; and since $\frac{1}{99} = .\dot{0}\dot{1}$, $\frac{2}{99} = .\dot{0}\dot{2}$; $\frac{3}{99} = .\dot{0}\dot{3}$, etc.; $\frac{1}{999} = .001001 +$, or $.\dot{0}0\dot{1}$; and $\frac{2}{999} = .\dot{0}0\dot{2}$; $\frac{145}{999} = .\dot{1}4\dot{5}$, etc. Hence,

A repetend is changed to a fraction by writing for a denominator as many *nines* as there are figures in the repetend. Thus, $.\dot{3} = \frac{3}{9}$; $.\dot{2}\dot{7} = \frac{27}{99}$; $.4\dot{0}\dot{9} = .4\frac{9}{99} = 4\frac{1}{11}$.

1. Change $.\dot{2}3\dot{1}$ to the form of a fraction.

OPERATION.

$$.\dot{2}3\dot{1} = \tfrac{231}{999} = \tfrac{77}{333}.$$

In like manner change to the form of fractions:

2. $.\dot{3}$; $.\dot{7}$.	4. $.\dot{3}\dot{1}$; $.\dot{4}\dot{5}$.	6. $.\dot{0}4\dot{5}$; $.\dot{7}2\dot{9}$.	8. $.\dot{4}15\dot{8}$.
3. $.\dot{0}\dot{3}$; $.\dot{2}\dot{7}$.	5. $.\dot{6}\dot{6}$; $.\dot{0}\dot{1}$.	7. $.\dot{9}7\dot{2}$; $.\dot{3}2\dot{4}$.	9. $.\dot{9}512\dot{1}$.

10. Change $.13\dot{8}$ to the form of a fraction.

OPERATION.

$$.13\dot{8} = .13\tfrac{8}{9} = \frac{13\frac{8}{9}}{100} = \frac{125}{900} = \frac{5}{36}.$$

11. Change $4.3\dot{2}\dot{8}$ to a mixed number.

OPERATION.

$$.3\dot{2}\dot{8} = .3\tfrac{28}{99} = \frac{3\frac{28}{99}}{10} = \frac{325}{990} = \frac{65}{198}.$$

Hence, $4.3\dot{2}\dot{8} = 4\tfrac{65}{198}$.

Change to fractions and mixed numbers:

12. $.0\dot{7}$; $.57\dot{2}$.	15. $2.0\dot{8}\dot{1}$.	18. $3.0\dot{6}\dot{5}$.	21. $.02\dot{2}\dot{7}$.
13. $.08\dot{3}$; $5.2\dot{7}$.	16. $.5\dot{9}2\dot{5}$.	19. $7.18\dot{9}\dot{3}$.	22. $.35\dot{1}3\dot{5}$.
14. $.3\dot{0}\dot{9}$; $4.3\dot{7}$.	17. $3.45\dot{6}$.	20. $.00\dot{1}8\dot{5}$.	23. $.011\dot{3}\dot{6}$.

ADDITION AND SUBTRACTION OF DECIMALS.

INDUCTIVE EXERCISES.

223. 1. What is the sum of 5 *tenths* and 4 *tenths?* Of $\frac{3}{10}$ and $\frac{5}{10}$? Of .4 and .7? Of .6, .5, and .2?

2. What is the difference between $\frac{9}{10}$ and $\frac{3}{10}$? .7 and .4? $\frac{25}{100}$ and $\frac{15}{100}$? .18 and .12? .36 and .05?

3. Read .56 in *tenths* and *hundredths*. Read .325 in *tenths*, *hundredths*, and *thousandths*.

4. What is the sum of $\frac{15}{100}$ and $\frac{8}{100}$? Of .07 and .28? .3, .05, and .24?

5. What is the sum of 23 *tenths* and 7 *tenths*? The difference? What is the sum of 36 *hundredths* and 2 *tenths?* The *difference?*

6. How many tenths and hundredths in .7 less .55?

7. What is the sum of $\frac{1}{10}$, $\frac{3}{100}$, and $\frac{5}{1000}$? Of .2, .05, and .012?

8. What is the sum of 5 and $\frac{1}{2}$, expressed decimally? Of 9 and $\frac{1}{4}$? Of 6 and $\frac{3}{4}$?

9. What is the difference between 3 and $\frac{1}{4}$, expressed decimally? Between 26 cts. and 12 cts.? $.44 and $.30?

10. What is the sum of $2, $.12, and $.005? Of 50 cents and $.25?

11. What is the sum of .3 and .045? The difference between .43 and .3?

12. How many decimal places in the *sum* of tenths and hundredths? Of hundredths and thousandths? Of tenths and thousandths? Of hundredths and ten-thousandths?

13. How many places in the *difference* between tenths and hundredths? Between tenths and thousandths? Between thousandths and ten-thousandths?

Find the sum and difference of

14. 4 dollars and .5 of a dollar.
15. .75 of a ton and .3 of a ton.
16. .9 of a gallon and $\frac{1}{2}$ of a gallon.
17. $3\frac{1}{4}$ yards and .25 of a yard.
18. .07 of a foot and $1\frac{4}{5}$ feet.
19. 10 rods and 5.6 rods.
20. $2\frac{3}{4}$ days and .6 of a day.
21. $.75 and $\frac{1}{2}$ of $.8.

224. Principle. *Only decimals of like orders, expressing parts of like units, can be added or subtracted.*

WRITTEN EXERCISES.

225. 1. Find the sum of 18.7, .3027, 136.415, and .08.

OPERATION.

18.7	Or,	18.7000
.3027		.3027
136.415		136.4150
.08		.0800
155.4977		155.4977

2. From 7.43 subtract .6214.

OPERATION.

7.43	Or,	7.4300
.6214		.6214
6.8086		6.8086

EXPLANATION. The arrangement of the given numbers must be such that the decimal points shall stand in the same vertical line.

Then, since decimals are written upon the same scale as integers, they are added and subtracted in the same manner, and the proofs are the same.

The number of decimal places in the answer will be the same as in that one of the given numbers which contains the greatest number of places.

Find the sum

3. Of 7.36, 1.24, .0346, 5.00164, .570036, and 14.09.
4. Of \$24.035, \$150.10, \$.965, \$3.50, and \$225.
5. Of 48.7$\frac{3}{4}$, 5.384$\frac{1}{8}$, .4726$\frac{3}{6}$, 124.56$\frac{7}{35}$, and 17.0$\frac{7}{8}$.
6. Find the sum of 46 hundred, 46 tens, and 46 tenths.
7. From 715 and 45 hundredths subtract 715 hundredths.
8. Find the sum of 900 dollars, 800 cts., and 500 mills.
9. From 827 thousandths subtract 32468 ten-millionths.
10. Find the sum of 250 dollars 24 cts. 6 mills, 28 dollars 10 cts., 5 dollars 8 cts. 5 mills, and 62 cts.
11. A man sold a house for \$9840.50, which was \$542$\frac{3}{5}$ more than it cost him. What did it cost him?
12. A lady bought a shawl for \$35$\frac{1}{2}$, a bonnet for \$9.37$\frac{1}{2}$, silk for a dress for \$42.75, and a pair of gloves for 1 dollar and a half, and gave in payment a \$100 bill. How much change ought she to receive?

Find the sum, and the difference, expressed decimally, of

13. 32.0624 and 7.34.
14. 175.85 and 16.00156.
15. 4.0602 and .000314.
16. 984.721 and 1500.008.
17. 400 tons and 108.75 tons.
18. 25.025 rods and $120\frac{4}{5}$ rods.
19. $\$187.87\frac{1}{2}$ and \$50.125.
20. $\$256\frac{7}{8}$ and $\$26.37\frac{1}{2}$.
21. \$300 and $\$35\frac{15}{16}$.
22. 225 and 225 hundredths.
23. \$1200 and $\$500\frac{7}{8}$.
24. 84 and 84 tenths.
25. 10 and 6 millionths.
26. $.2\frac{5}{8}$ and $.02\frac{7}{25}$.
27. $\$5\frac{14}{20}$ and \$.57.
28. $.6\frac{1}{18}$ and $.06\frac{5}{9}$.
29. $1\frac{1}{2}$ and 45 tenths.
30. \$.625 and $\$\frac{3}{8}$.
31. $\$27\frac{7}{8}$ and \$40.
32. .8 and $.08\frac{2}{3}$.

33. If a barrel of flour cost $\$10\frac{7}{8}$, a ton of coal $\$6\frac{5}{16}$, a ton of hay $\$19\frac{4}{5}$, and a barrel of sugar $\$21.37\frac{1}{2}$, what is the cost of all?

Find the value of

34. $200 + .02 - 1.0002$.
35. $\$75.10 - (\$.05 + \$25\frac{9}{10})$.
36. $4\frac{17}{100} - .9 - (.25 + \frac{14}{25})$.
37. $\$750 - (\$476.06\frac{1}{4} - \$87\frac{7}{25})$.
38. $.95\frac{3}{4} - \frac{7}{16} + .16\frac{1}{2} + .08\frac{4}{5}$.
39. $\$\frac{7}{8} + \$1.18\frac{3}{4} - \$1\frac{31}{100}$.

MULTIPLICATION AND DIVISION OF DECIMALS.

INDUCTIVE EXERCISES.

226. 1. What is 4 times $\frac{1}{10}$? 2 times .4? $2 \times .4$?

2. What is $\frac{1}{2}$ of 6 tenths? $\frac{1}{4}$ of $\frac{8}{10}$? $\frac{1}{3}$ of .9? $.6 \div 3$? $.8 \div 4$?

3. What is $\frac{1}{5}$ of .25 hundredths? $\frac{1}{6}$ of $\frac{30}{100}$? $\frac{1}{8}$ of .40?

4. What is 6 times $\frac{8}{100}$? 5 times $\frac{9}{1000}$? $.06 \times 4$? $.008 \times 5$?

5. What is $\frac{1}{10}$ of 60 thousandths? $\frac{1}{5}$ of .025? $.064 \div 8$?

6. What is $\frac{4}{10} \times 3$? $6 \times .4$? $.8 \times 7$? $12 \times .5$? $.9 \times 9$?

7. What is $\frac{3}{10} \times \frac{4}{10}$? $.5 \times .6$? $.7 \times .4$? $.9 \times .6$? $.5 \times .11$?

8. What is $\frac{5}{10} \times \frac{6}{100}$? $.04 \times 5$? $\frac{7}{100} \times \frac{6}{100}$? $.06 \times .08$?

9. Divide .36 by 6; .48 by 8; .63 by 9; .84 by 12.

10. What is $\frac{9}{10} \times 3$? $\frac{9}{10} \times \frac{3}{10}$? $.9 \times .4$? $\frac{15}{100} \times 3$?

11. What is 9 times \$.4? 6 times \$.09? \$.12 × 7?

12. Divide .9 by 3; .84 by 7; .096 by 12; .040 by 8.

13. What is $.8 \times .9$? $.72 \div 8$? $.72 \div 9$? $.96 \div 12$?

14. The product of two factors is .120, one of which is .8; what is the other? If one is .06, what is the other?

15. Divide 5.6 by 8.

OPERATION. 5.6 equals 56 *tenths,* and $\frac{1}{8}$ of 56 tenths is 7 *tenths,* or .7.

16. What is $\frac{1}{4}$ of \$3.2? $\frac{1}{6}$ of \$.66? \$.063 divided by 9?

17. Multiply 3 by 5; .3 by 5; .03 by .5; .003 by .05.

18. Divide .45 by 5; .45 by .5; .45 by .05; .045 by .005.

19. The product of two factors is .0064, one of which is .08; what is the other?

20. How much is .12 of a ton multiplied by .7?

21. What is .125 of a ton divided by .5? \$.75 ÷ .25?

22. How many decimal places in the product of any two decimals?

23. Of what two factors is the dividend the product?

24. Then how many places must the dividend contain?

25. What is the product of units by tenths? Of tenths by tenths? Hundredths by tenths? Hundredths by hundredths? etc.

26. What is the quotient of tenths divided by units? Of units by tenths? Hundredths by tenths? Hundredths by hundredths? Thousandths by tenths? etc.

27. How many decimal places in the product, if one of the factors contain *two* places, and the other *one?* If there are *three* places in one factor, and *two* in the other?

28. How many decimal places in the quotient, if there are *three* in the dividend and *one* in the divisor? If *two* in the divisor and *two* in the dividend? If *none* in the divisor and *four* in the dividend? If *two* in the divisor and *none* in the dividend?

227. PRINCIPLES. 1st. For Multiplication.

I. *The number of decimal places in the product of two decimals is equal to the number of decimal places in both factors.*

2d. For Division.

II. *The number of decimal places in the quotient is equal to the excess of the number in the dividend over that in the divisor.* Hence,

III. *If the number of decimal places in the dividend is the same as in the divisor, the quotient will be an integer.*

The number of decimal places in the dividend must first be made at least equal to those in the divisor before division is possible.

WRITTEN EXERCISES.

228. The operations and proofs of Multiplication and Division of Decimals are the same as of Integers, with the exception of locating the decimal point.

1. What is the product of 8.25 by .9?

OPERATION.—1st. As in fractions: $8.25 \times .9 = \frac{825}{100} \times \frac{9}{10} = \frac{7425}{1000} =$ 7.425. (**181**, RULE II.)

2d. As in integers: $8.25 \times .9 = 7.425$. (**227**, I.)

2. Multiply .035 by .06.

OPERATION.

$$\begin{array}{r} .035 \\ \underline{.06} \\ .00210 \end{array}$$

Since hundredths into thousandths produces hundred-thousandths, the product requires 5 decimal places. Hence, 2 ciphers must be prefixed.

3. What is the quotient of .215 divided by .5?

OPERATION.—1st. As in fractions: $.215 \div .5 = \frac{215}{1000} \div \frac{5}{10} = \frac{215}{1000} \times \frac{10}{5} = \frac{43}{100} = .43$. (**183**, II.)

2d. As in integers: .215 divided by $.5 = .43$. (**227**, II.)

4. Divide 1.025 by 25.

OPERATION.

$$25\overline{)1.025}$$
$$.041$$

Since the dividend contains 3 decimal places and the divisor none, the quotient must have 3 decimal places. (**227**, II.)

5. Divide 3 by .25.

First make the decimal places in the dividend *equal* to those in the divisor, then the quotient will be an integer. (**227**, III.)

OPERATION.

$$.25)\overline{3.00}$$
$$12$$

If there is a remainder the division may be continued, each cipher annexed being a decimal of the dividend.

If the quotient is carried to three or four places, it is sufficiently exact for most business transactions.

Multiply

6. .4 by 400.
7. 125 by .65.
8. 3.26 by .019.
9. $17.4 by 2.3.
10. 6.4 by .075.
11. .006 by .08.
12. 98.21 by 1.515.
13. 5.0204 by 40.2.
14. 3.0701 by 70.01.
15. $146.75 by 2.04.
16. $150.7 by ½ of 2.5.
17. 96 tenths by .12½.
18. 5.85 by 9.
19. 1.44 by 18.
20. .096 by 32.
21. 23.4 by .009.
22. 45.3 by 302.
23. $63.36 by .132.

24. Multiply 21.65 by .042; 9.325 by 2.6; $14.75 by 15.

25. Multiply $276½ by .085; .25 by 3.02.

26. Divide 196.2 by .6; .327 by .6; 19.62 by .06; $46.95 by .125.

27. Divide .1728 by .0144; 49000 by .007; $35.91 by $.285.

28. Find the product of 9000 by 9 thousandths.

29. Find the quotient of 16.578 divided by 5.4; by .54; by .054.

30. Multiply 4 millionths by 6 hundredths.

31. Multiply $84¾ by .25; 6⅘ tenths by .031¼; $⅞ by 2⅗.

32. Divide 2 and 22 hundredths by 74 ten-thousandths.

33. Divide .00005 by 2.5; by .25; by .0000025.

34. Divide 72.36 by 36; by .0036; .003 by 1.6.

35. Divide 456 thousandths by 6 hundredths; by 12 ten-thousandths.

a. To multiply a decimal by 10, 100, 1000, etc., move the decimal point as many places to the *right* as there are ciphers in the *multiplier*. **(203.)**

b. To divide a decimal by 10, 100, 1000, etc., move the decimal point as many places to the *left* as there are ciphers in the *divisor*. **(204.)**

Find the value of

36. $4.65 \times .025 \times .08$.
37. $7.6 \times .045 \times 13 \times .0002$.
38. $6.45 \times 100 \times \frac{3}{4}$ of $.8 \times 10$.
39. $27\frac{9}{25} \times 1000 \times .009\frac{3}{8}$.
40. $\$320\frac{3}{16} \times 10 \times 2\frac{7}{10} \times 100$.
41. $(137.05 \div 100) \times .0\frac{3}{4}$.
42. $(\$425 \times .006) \div 1000$.
43. $(7.6875 \div 187.5) \times 5\frac{1}{8}$.
44. $(\$61.376 \div \$2.74) \times 10$.
45. $(11.7 \div 1300) \times \frac{1}{3}$ of $.027$.

46. If $1 in gold is worth $1.07⅛ in paper currency, what are $1000 in gold worth? $100? $10000? $1500?

47. What is the cost of 246.04 acres of land, at $25.50 an acre? At $37½ an acre? At $56⅝? At $100?

48. How many lots, each containing .87½ of an acre, can be made from 12¼ acres? From 19.25 acres?

49. Bought 20 pounds of sugar, at 11¾ cts., 25 pounds of coffee, at 28½ cts., and 15 pounds of tea, at $.87½ a pound. What was the cost of all?

What is the cost of

50. 75 sheep, at $7.62½ a head? At $6¾?
51. 136½ barrels of flour, at $9⅛ a barrel? At $8.12½?
52. 327¼ pounds of sugar, at 10½ cents a pound? At 8¾ cts.?
53. 23⅝ tons of coal, at $6.37½ a ton? At $5⅞?
54. 100 cords of wood, at $3.87½ a cord? At ½ of $6¾?
55. 1000 barrels of apples, at $2⅞ a barrel? At $3.10?
56. 105½ yards of cloth, at $4⅘ a yard?
57. 5000 bushels of wheat, at $1⅞ a bushel? At $1.37½?
58. .625 of an acre of land, at $100 an acre?
59. ⅝ of 456 bushels of oats, at $.62½ a bushel?
60. $\frac{3}{7}$ of 327.6 acres of land, at $\frac{2}{5}$ of $124 an acre?

What is the price of each,

61. If 894 pounds of sugar cost \$80.46 ? If \$58.11 ?

62. If 64 bushels of wheat cost \$136 ? If \$88 ?

63. If 100 acres of land cost \$3325.50 ? If \$2675 ?

64. If 2.7 tons of hay cost $\$32\frac{2}{5}$? If $\$33\frac{3}{4}$?

65. If $352\frac{1}{2}$ pounds of coffee cost $\$66.09\frac{3}{8}$?

66. If $842\frac{3}{4}$ tons of railroad iron cost \$55992.31 ?

67. If $25\frac{1}{8}$ yards of cloth cost $\$37.68\frac{3}{4}$? If \$72.36 ?

68. What is the cost of $5\frac{7}{8}$ bales of sheeting, each bale containing $41\frac{3}{4}$ yards, at $12\frac{1}{2}$ cents a yard ? At $16\frac{2}{3}$ cents?

69. If $10\frac{1}{2}$ barrels of apples cost \$22.75, what will $40\frac{1}{4}$ barrels cost ?

70. If $\frac{1}{4}$ of one hundred pounds of coffee cost $\$7.81\frac{1}{4}$, how many pounds can be bought for \$20 ? For \$100 ?

71. How many tons of coal, at $\$7\frac{13}{16}$ a ton, will pay for $6\frac{1}{2}$ barrels of flour, at $\$8\frac{3}{4}$ a barrel ? At $\$9\frac{1}{2}$?

72. Three men bought a piece of land containing 645 acres, and divided it so that the first had $\frac{3}{16}$, the second had .375, and the third, the remainder. How much did the third receive ?

73. If $\frac{1}{2}$ of .5 of a barrel of sugar is worth $\$5\frac{1}{2}$, what is $\frac{3}{4}$ of .125 of a barrel worth ? $\frac{7}{10}$ of $.62\frac{1}{2}$ of a barrel ?

74. A man bought at one time $826\frac{1}{2}$ bushels of wheat, at $\$1\frac{3}{4}$ a bushel, and at another time $281\frac{1}{2}$ bushels at $\$1.87\frac{1}{2}$ a bushel ; then sold the whole at a profit of $\$398.01\frac{1}{4}$. At what price did he sell it ?

75. A farmer sold a merchant $25\frac{3}{4}$ cords of wood, at $\$3\frac{1}{2}$ a cord, and received in payment 4 barrels of flour, at $\$8\frac{3}{8}$ a barrel ; 1 barrel of sugar, containing 204 pounds, at $12\frac{1}{2}$ cents a pound ; some dry goods, worth $\$17.32\frac{1}{2}$; and the balance in coffee, at 30 cents a pound. How many pounds of coffee did he receive ?

76. $(.025075 \div 1.003) + (\overline{12.6 \div .0012} \times \frac{1}{8}) - 1000\frac{7}{8} = ?$

77. $(\$3250 \times 2\frac{4}{5} \div \$56\frac{7}{8} - \overline{\$83.12\frac{1}{2} \div \$.875}) \div 1000 = ?$

229. ***To find the cost, when the thing or quantity is sold by the 100 or 1000.***

1. What is the cost of 360 oak posts, at $8.75 per hundred?

EXPLANATION.—Multiplying the price of *one hundred* posts by the number of posts, gives a product 100 times too large; hence, the product must be divided by 100. (**104.**) Had the price been by the *thousand*, then the product should be divided by 1000. Hence,

OPERATION.

$$\frac{\$8.75 \times 360}{100} = \$31.50.$$

The product of the quantity by the price, divided by 100 or 1000, as the case may require, equals the cost.

The letter *C* is sometimes used for *hundred* and *M* for *thousand.*

2. What is the cost of 1672 feet of boards, at $1.75 per C.?

3. What will be the freight on 675 pounds of merchandise, at $.62½ per C.? At $2.40 per M.?

4. What will be the cost of 40000 laths, at $.28 per C., and 1570 feet of boards, at $18.75 per M.?

Find the cost of

5. 948 pounds of beef, at $12½ per hundred. At $10⅜.
6. 1368 feet of square timber, at $28 per M. At $26.50.
7. 5680 cedar rails, at $6¼ per 100. At $56¾ per 1000.
8. 27890 bricks, at $10.40 per 1000. At $9¼ per M.
9. 349 pineapples, at $9.17 per 100. At $75½ per 1000.

230. ***To find the cost when the thing or quantity is sold by the ton of 2000 pounds.***

1. At $6.50 a ton, what will 1680 pounds of coal cost?

EXPLANATION.—Since $6.50 is the price of 2000 pounds, $3.25 is the price of 1000 pounds. We obtain the cost of 1680 pounds as in **229.** Hence,

OPERATION.

$$\frac{\$6.50 \div 2 \times 1680}{1000} = \$5.46.$$

The product of the quantity in pounds by one half the price, or one half the quantity by the price, divided by 1000, equals the cost.

2. Find the cost of 2760 pounds of hay, at $\$18\frac{1}{2}$ a ton.

3. A potter bought 5680 pounds of porcelain clay, at \$17.50 a ton. What did it cost him?

4. Find the freight on 7950 pounds of iron, at $\$2\frac{5}{8}$ a ton.

5. What will a load of plaster weighing 1784 pounds cost, at \$2.75 a ton? At \$2.50? At $\$4\frac{1}{4}$? At $\$3.62\frac{1}{2}$?

6. What is the cost of 100 sacks of bone-dust, each weighing $112\frac{1}{2}$ pounds, at \$26 a ton? At $\$31\frac{1}{4}$?

7. What is the freight on 9860 pounds of iron, at \$1.75 a ton? On 18456 pounds, at \$2.05 a ton?

231. Let all the operations and results in the following examples be in decimals.

First, add the *four* numbers on each and every line of the two double columns; thus, $417 + .417 + 56.875 + .0144 = 474.3064$, etc.

Next, take the first set of two numbers, 417 and .417, find their *difference,* their *product,* and then the quotient of the *first* divided by the *second.*

Treat each set of numbers in both double columns in the same manner.

1. 417, .417.	16. 56.875, .0144.
2. 2.052, .0031.	17. 1.32, .7614948.
3. .314, .0021.	18. 40.2, .82008.
4. .00281, .002.	19. .08748, .0081.
5. 72.36, .0036.	20. .90804, .7567.
6. .00005, 2.5.	21. 1.0665, .00135.
7. 512.1, 56.9.	22. $3\frac{6}{25}$, .014904.
8. 10.005, .345.	23. .2219904, .3854.
9. 47.75, $\frac{7}{8}$.	24. $6\frac{17}{64}$, 2.075.
10. $7\frac{3}{4}$, $\frac{1}{25}$.	25. $\frac{629}{625}$, .00352.
11. $\frac{3}{32}$, .005.	26. $.05\frac{3}{16}$, $\frac{8}{25}$.
12. $3\frac{1}{4}$, $\frac{81}{125}$.	27. .7504, 1000.
13. $56\frac{1}{4}$, .075.	28. .0016, 10000.
14. 3.5, .016.	29. 998.4375, $17\frac{3}{4}$.
15. 11, 2.50.	30. $56.3\frac{3}{4}$, $163\frac{1}{16}$.

ACCOUNTS and BILLS.

232. An **Account** is a record of items of debt and credit between parties.

A person who owes money, goods, or services, is called a *Debtor;* and the person to whom the money, goods, or services are due is called the *Creditor*.

233. A **Bill** is a written statement of goods sold or delivered, services rendered, or work done, with the price, quantity, and cost annexed to each item.

A bill should also state the names of the buyer and seller, the place and time of the transaction, and any special terms agreed upon by the parties.

A bill is *receipted* when the words "Received Payment," or "*Paid*," are written at the bottom, and the creditor, or some one acting for him, affixes his name.

234. In bills various characters and abbreviations are in use. The following are some of the most common:

@,	At.	Exch.,	Exchange.	No. or #,	Number.
Acc't, %,	Account.	Exps.,	Expenses.	Pay't,	Payment.
Am't,	Amount.	Fr't,	Freight.	Pd.,	Paid.
Ass'd,	Assorted.	Fol.,	Folio.	Per,	By.
Bal.,	Balanced.	Fw'd,	Forwarded.	Pk'gs,	Packages.
Bo't,	Bought.	Inst.,	This month.	Ps.,	Pieces.
Co.,	Company.	Int.,	Interest.	Prem.,	Premium.
Cr.,	Creditor.	Ins.,	Insurance.	Rec'd,	Received.
Dr.,	Debtor.	Invt,	Inventory.	Ship't,	Shipment.
Dft.,	Draft.	Mdse.,	Merchandise.	S. S.,	Steamship.
Disc't,	Discount.	Mo.,	Month.	Sunds.,	Sundries.
Do. or Ditto,	The same.	Net,	Without disc't.	Ult.,	Last month.

When the character @ is used, it is always followed by the *price* of *a unit*. Thus, 6 hats @ \$5.50 signifies 6 hats at \$5.50 *each*.

235. Required the footings of the following bills:

(1.)

COLUMBIA, April 10, 1880.

MR. JAMES S. ROLLINS,

Bought of MOSS & SON.

26 yds. Silk,	@	\$2.75	\$
$44\frac{1}{2}$ " Sheeting,	"	.18	
2 ps. Muslin, 32 yds.,	"	$.12\frac{1}{2}$	
6 pair Kid Gloves,	"	$1.37\frac{1}{2}$	
		Am't,	\$

(2.)

ST. LOUIS, Sept. 4, 1881.

MESSRS. HAYES & GLENN,

Bo't of EDGAR & COSTE.

140 lbs. Java Coffee,		@	28¢.	\$
216 " Coffee Sugar A,		"	$12\frac{1}{2}$¢.	
6 boxes Soap, Babbitt's best,	486 lbs.,	"	$7\frac{1}{2}$¢.	
1 " Duryea's Starch,	40 lbs.,	"	$8\frac{1}{4}$¢.	
				\$

Received Payment,

EDGAR & COSTE.

(3.)

CHICAGO, Jan. 14, 1880.

MESSRS. J. B. HOYT & CO.,

Bo't of FARWELL & SONS.

1 case Cassimeres,	176 yds.,	@	$\$1.87\frac{1}{2}$		
2 cases Prints,	850 "	"	$.09\frac{1}{4}$		
1 bale Drilling.	578 "	"	.14		
5 ps. Shaker Flannel,	216 "	"	$.62\frac{1}{2}$		
12 doz. Coates' Thread,		"	.70		
50 gross Silk Buttons,		"	$.87\frac{1}{2}$		
15 M. Milward's Needles,		"	$2.37\frac{1}{2}$		
				\$	

Rec'd Pay't by note at 4 mo.,

FARWELL & SONS.

(4.)

NEW ORLEANS, Nov. 7, 1879.

MR. GEORGE LANE,

Bo't of FRANCIS & BAKER.

10 bales Cotton, ea. 516 lb.,	Texas Middlings,	@	14$\frac{3}{4}$¢.	$
14 " " " 495 "	Good Ordinary,	"	11$\frac{1}{4}$¢.	
8 " " " 520 "	Alabama Fair,	"	13$\frac{5}{8}$¢.	
6 hhd., 414 gal., N. O. Molasses (choice),		"	66¢.	
Rec'd Payment by draft on N. York,				$

T. MORONEY,

FOR FRANCIS & BAKER.

(5.)

LOWELL, April 6, 1881.

MR. BENJ. F. TAYLOR,

To AMES KENT, *Dr.*

Feb.	12	For building Grape Arbor as per contract,	$25	00
"	25	" 6$\frac{1}{2}$ days work of self, @ 2\frac{1}{2}$.		
"	"	" 3 " " " man, " 1.75		
Mar.	6	" 320 feet siding, " 2.25 per C.		
"	"	" 950 " plank, " 1.50 " M.		
			$	

Rec'd Payment,

AMOS KENT.

(6.)

SEDALIA, Jan. 3, 1880.

MR. JOSEPH NORWOOD,

To G. S. BRYANT, *Dr.*

1879				
Oct.	5	To 1 caddy Oolong Tea, 18 lb., @ $ .87$\frac{1}{2}$		
"	"	" 25 lb. Coffee Sugar, " .12		
"	15	" 1 box soap, 65 lb., " .10$\frac{1}{2}$		
Nov.	10	" 6 gal. Silver Drip Syrup, " 1.08		
Dec.	14	" 1 cheese, 48 lb., " .18$\frac{3}{4}$		
"	24	" 6 doz. eggs, " .22		

Rec'd Payment,

G. S. BRYANT.

(7.)

NEW YORK, Oct. 24, 1881.

MESSRS. DODGE, PHELPS & CO.,

In Acct. with LANE BROS., *Dr.*

July	12	To 27640 ft. Clapboards, @ \$22.50 per M.		
"	"	" 3644 Cedar Posts, " 9.37 " C.		
"	"	" 15729 Railroad Ties, " 12.75 " C.		
Sept.	22	" 75680 ft. Pine Boards, " 18.87½ " M.		
"	"	" 40000 Shingles, " 4.65 " M.		
"	"	" 84560 Fence Pickets, " 6.62½ " M.		
		Cr.		
Aug.	18	By 12 tons Common Bar Iron, @ \$62.		
"	"	" 3 " Horseshoe Iron, " \$91½.		
"	"	" 1260 lb. Spring Steel, " 7½¢.		
Sept.	4	" Cash,	500	
Oct.	6	" 4 doz. Simmons' Light Axes, " \$10½.		
"	"	" 228 lb. Manilla Rope, " 14½¢.		
			\$	
		Balance due,	\$	

Rec'd Payment,

LANE BROS.,

per SMITH.

8. Henry Jones bought of Hill & Bro., Springfield, Ill., May 15, 1879, 42 lb. of sugar @ 13¢; 3 lb. Y. H. tea @ \$.90; 4 gal. molasses @ \$.63; 48 yd. sheeting @ 16¢; 1 box starch, 14 lb., @ 9½¢; and 8 doz. eggs @ 21¢. Make the bill, and find the amount due.

9. The following items were sold in Cleveland, O., by George Bliss & Co., to Mrs. S. C. Barr, Sept. 25, 1879: 16 yd. silk @ \$3½; 6 pair hose @ 62½¢; 2 pr. kid gloves @ \$1.75; 8 yd. ribbon @ 26¢; 6 handkerchiefs @ 42¢; 10 yd. cambric @ 11¢; 1 umbrella @ \$3⅜; 3¼ yd. elastic @ 24¢; 2½ yd. ruching @ 60¢; 1½ yd. silk velvet @ \$4; and 6 doz. buttons @ 37½¢. Make out a receipted bill.

Review.

Written Exercises.

236. 1. At $.75 a bushel, how many sacks of corn, each containing $2\frac{1}{2}$ bushels, will $$67\frac{1}{2}$ purchase?

2. At $$.31\frac{1}{4}$ a bushel, how many bushels of apples can be bought for $18? $27? $$31\frac{1}{2}$? $63?

3. What is the cost of 1475 pounds of guano, at $$37\frac{1}{2}$ a ton? Of 3840 pounds, at $.48 a ton?

Find the cost of

4. $17\frac{1}{2}$ yards of cloth, if $14\frac{1}{4}$ yards cost $$21.37\frac{1}{2}$.
5. $23\frac{3}{8}$ cords of wood, if 9.55 cords cost $$21.48\frac{3}{4}$.
6. $56\frac{1}{4}$ pounds of butter, if 252 pounds cost $84.
7. .875 of a ton of hay, if $4\frac{5}{8}$ tons cost $59.57.
8. $\frac{3}{4}$ of a ton of plaster, if 1560 pounds cost $$2\frac{1}{2}$.
9. 78 pounds of wool, if 175 pounds cost $69.65.
10. 1650 pineapples, if 100 cost $$16\frac{1}{4}$.
11. 26780 bricks, if 1000 cost $$8\frac{7}{8}$.
12. 975 pounds of bone dust, if a ton cost $$36\frac{1}{5}$.
13. 100 pounds of cheese, if .84 of 100 pounds cost $$15\frac{3}{4}$.

14. Bought $13\frac{3}{4}$ barrels of sugar for $$226.87\frac{1}{2}$, and sold it for a profit of $$3.37\frac{1}{2}$ a barrel. For how much was it sold?

15. Find the value of 36 head of cattle, of an average weight of 1135 pounds each, at $9.4 per 100 pounds.

16. The cheese made of the milk of 26 cows during a season was sold for $634.92, at $18\frac{1}{2}$ cts. a pound. What was the number of pounds sold, and the average per cow?

17. Sold 5225 feet of pine boards for $$169.81\frac{1}{4}$, and gained by the sale $$39.18\frac{3}{4}$. What did it cost per C.?

18. At $.65 a bushel for barley, $.42 for oats, $$.87\frac{1}{2}$ for corn, and $$1.62\frac{1}{2}$ for wheat, how many bushels, of each an equal number, can be bought for $453.39?

19. Bought a quantity of coal, and sold .25 of it to one man, .4$\frac{1}{2}$ to another, .09 to another, and kept 13$\frac{1}{8}$ tons. What was the quantity bought, and its cost, at \$4$\frac{5}{8}$ per ton?

20. If I own .3 of a steamship, and sell .5$\frac{1}{2}$ of my share for \$3300, what is the value of $\frac{3}{4}$ of the ship at the same rate?

21. If a man, on an average, drink 3 glasses of beer a day, costing 5 cents a glass, and smokes 3 cigars, at 10 cents each, for 20 years, allowing 365 days to the year, how many acres of land, worth \$15 an acre, would the money buy?

22. If 11.5 acres of land cost \$362$\frac{1}{4}$, what is .875 of an acre worth?

23. Bought 500 barrels of apples, at \$2$\frac{5}{8}$ a barrel, and sold 150 barrels at \$2$\frac{7}{8}$, 100 barrels at \$3.12$\frac{1}{2}$, 132 barrels at \$3$\frac{1}{6}$ a barrel, and the remainder at \$2.96 a barrel. What was my whole gain?

24. Shipped to Cuba 1500 barrels of flour, at \$10$\frac{3}{4}$, and received in payment 90 hogsheads of sugar, each weighing 640 pounds, worth 7$\frac{1}{2}$ cents a pound; 75 bales of cotton, each weighing 512 pounds, worth 18$\frac{1}{4}$ cents a pound; and the remainder in coffee, at 28 cts. a pound. How many sacks of coffee, each weighing 50 pounds, were received?

25. A lumberman bought 325000 feet of lumber, at \$16$\frac{1}{2}$ per M. and retailed it, at \$2$\frac{1}{8}$ per C. What was his gain?

26. A man bought 214.56 acres of land, at \$47.625 an acre, and gave in payment 12 reapers, at \$125$\frac{1}{2}$ each; 15 horses, at \$153$\frac{3}{4}$ each; some patent rights, worth \$1275; 175 barrels of flour, at \$9$\frac{7}{8}$ a barrel; and the remainder in cash. How much money did he pay?

27. A fire destroyed $\frac{3}{8}$ of a pile of lumber, valued at \$10000. What would be the loss to a man who owned .35 of the entire pile?

28. Reduce ($\frac{3}{25}$ of 2.45 — $\frac{1}{100}$ of .02) $\div$ 1000, to the form of a decimal.

29. $(124 \times 10 + \frac{1}{10}$ of $.01 + .001$ of $10) \div .03$ of $1000 = ?$

MEASURES.

237. **A Measure** is a *standard unit*, established by law or custom, by which *quantity* is measured or estimated.

Thus, *length* is ascertained by applying the *foot* or *yard* measure; *capacity*, by the use of the *quart*, the *gallon*, or the *bushel* measure; *weight*, by the use of the pound, etc.

MEASURES OF EXTENSION.

238. **Extension** is that which has one or more of the dimensions, *length, breadth,* and *thickness,* and may be a *line*, a *surface,* or a *solid.*

239. **A Line** has only *one* dimension, *length.*

The **Standard Unit** in linear, surface, and solid measure is the *yard,* which is subdivided into feet and inches.

240. **Linear Measures** are used in measuring lines and distances.

Table of Units.				Equivalents.
12 Inches (*in.*)	=	1 Foot,	*ft.*	1 *Mi.* = 63360 *in.*
3 Feet	=	1 Yard,	*yd.*	5280 *ft.*
$5\frac{1}{2}$ Yards, or $16\frac{1}{2}$ Feet	=	1 Rod,	*rd.*	1760 *yd.*
320 Rods	=	1 Mile,	*mi.*	320 *rd.*

In measuring goods sold by the *yard*, the linear yard is divided into *halves, fourths, eighths,* and *sixteenths.* In estimating duties in the Custom House, the yard is divided into *tenths* and *hundredths.*

241. Surveyors' Linear Measures are used by land surveyors in measuring *roads* and *boundaries* of land.

Table of Units.			Equivalents.
7.92 Inches	= 1 Link, ...	*l.*	1 *Mi.* = 63360 *in.*
25 Links	= 1 Rod, ...	*rd.*	8000 *l.*
4 Rods	= 1 Chain, ...	*ch.*	320 *rd.*
80 Chains	= 1 Mile, ...	*mi.*	80 *ch.*

A Gunter's Chain is the *unit* of measure, consisting of 100 links, and is 4 rods or 66 feet long.

242. Mariners' Measures are used by seamen in measuring distances, the depth of the sea, etc.

Table of Units.			Equivalents.
9 Inches	= 1 Span, ...	*sp.*	1 *Mi.* = 63360 *in.*
8 Spans, or 6 ft.	= 1 Fathom, . . .	*fath.*	5280 *ft.*
120 Fathoms	= 1 Cable's Length,	*c. l.*	880 *fath.*
$7\frac{1}{3}$ C. Lengths	= 1 Com. Mile, . .	*mi.*	$7\frac{1}{3}$ *c. l.*

The *Nautical Mile* (or *Knot*) is the same as the *Geographic* Mile, and is equal to 6086.7 feet, or about $1.15\frac{1}{5}$ statute miles.

243. In **Geographical** and **Astronomical** calculations, and for other purposes, the following denominations are used.

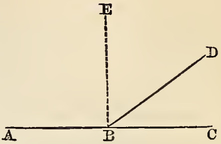
Table of Units.

60 Geographic, or 69.16 Statute Miles	= 1 Degree	of Latitude on a Meridian, or of Longitude on the Equator.
360 Degrees	= the Circumference of the Earth.	
$1.15\frac{1}{5}$ Common Miles	= 1 Geog. Mi.	Used to meas. distances at sea
3 Geographic Miles	= 1 Nautical League.	
3 Barley-corns, or sizes	= 1 Inch.	Used by shoemakers.
4 Inches	= 1 Hand.	" to measure the height of horses at the shoulder.

The *average* length of a degree of latitude is 69.16 statute miles, and is the standard adopted by the U. S. Coast Survey.

244. Surface or Square Measures are used in computing areas.

245. An Angle is the opening between two right lines that proceed from a common point, called the *Vertex*.



Thus, ABD and DBC are *angles*, and B is their *vertex*.

When one line meets another line so as to make two *adjacent* angles equal, each angle is a *Right Angle*, and the lines are said to be *perpendicular* to each other.

Thus, ABE and EBC are right angles.

246. A Surface has *two* dimensions, *length* and *breadth*.

247. A Square is a plane figure, bounded by four equal sides, and having four right angles.

A *Square Inch* is a square, each side of which is 1 *in.* in length.

SQUARE INCH.

TABLE OF UNITS.

144	Sq. Inches	=	1 Sq. Foot,	*sq. ft.*
9	Sq. Feet	=	1 Sq. Yard,	*sq. yd.*
$30\frac{1}{4}$	Sq. Yards	=	1 Sq. Rod,	*sq. rd.*
160	Sq. Rods	=	1 Acre,	*A.*
640	Acres	=	1 Sq. Mile,	*sq. mi.*

EQUIVALENTS.

1 *Sq. Mi.* =	4014489600 *sq. in.*
	27878400 *sq. ft.*
	3097600 *sq. yd.*
	102400 *sq. rd.*
	640 *A.*

Roofing and flooring are usually estimated by the *square*, which contains 100 sq. ft.

248. Surveyors' Square Measure is used by surveyors in computing the area of *land*.

TABLE OF UNITS.

625	Sq. Links	=	1 Pole, . .	*P.*
16	Poles	=	1 Sq. Chain,	*sq. ch.*
10	Sq. Chains	=	1 Acre, . .	*A.*
640	Acres	=	1 Sq. Mile, .	*sq. mi.*
36	Sq. Miles	=	1 Township,	*Tp.*

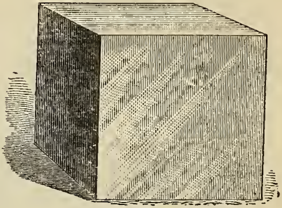
EQUIVALENTS.

1 *Tp.* =	2304000000 *sq. l.*
	3686400 *P.*
	230400 *sq. ch.*
	23040 *A.*
	36 *sq. mi.*

249. Solid Measures are used in computing the contents of solids.

250. A Solid is a body, volume, or space, that has three dimensions, *length*, *breadth*, and *thickness*.

251. A Cube is a body bounded by six equal squares, called *Faces*. The sides of these squares are called the *Edges* of the cube.



CUBIC INCH.

A *Cubic Inch* is a cube, each edge of which is 1 *in.* in length.

TABLE OF UNITS.

1728 Cubic In. (*cu. in.*)	= 1 Cu. Foot,	*cu. ft.*
27 Cubic Feet	= 1 Cu. Yard,	*cu. yd.*
16 Cubic Feet	= 1 Cord Ft.,	*cd. ft.*
8 Cord Feet, 128 Cubic Feet	= 1 Cord,	*Cd.*

EQUIVALENTS.

1 *Cu. yd.* = { 46656 *cu. in*, 27 *cu. ft.* }

1 *Cd.* = { 128 *cu. ft.*, 8 *cd. ft.* }

MEASURES OF CAPACITY.

252. Capacity signifies extent of *room* or *space*.

Measures of capacity are of two kinds: *Measures of Liquids*, and *Measures of Dry Substances*.

253. Liquid Measures are used in measuring *liquids*.

The Standard Unit is the *gallon*, containing 231 cubic inches.

TABLE OF UNITS.

4 Gills (*gi.*)	= 1 Pint, ...	*pt.*
2 Pints	= 1 Quart, . . .	*qt.*
4 Quarts	= 1 Gallon, . . .	*gal.*

EQUIVALENTS.

1 *Gal.* = { 32 *gi.*, 8 *pt.*, 4 *qt.* }

In estimating the capacity of *cisterns, reservoirs*, etc.

$31\frac{1}{2}$ Gal.	= 1 Barrel, . . .	*bbl.*
63 Gal.	= 1 Hogshead, . .	*hhd.*

	bbl.	*gal.*	*qt.*	*pt.*
1 *Hhd.* =	2 =	63 =	252 =	504.

The *barrel* and *hogshead* are not fixed measures, but vary when used for commercial purposes.

254. Apothecaries' Fluid Measures are used by physicians in prescribing, and by apothecaries in compounding *liquid* medicines.

TABLE OF UNITS.			EQUIVALENTS.
60 Minims (♏)	= 1 Fluidrachm,	*f*ʒ.	♏ 61440.
8 Fluidrachms	= 1 Fluidounce,	*f*℥.	1 *Cong.* = *f*ʒ 1024.
16 Fluidounces	= 1 Pint,	*O.*	*f*℥ 128.
8 Pints	= 1 Gallon,	*Cong.*	*O.* 8.

Cong. stands for the Latin *Congius*, a gallon; and *O.* for *Octarius*, *one eighth*, a pint being one eighth of a gallon.

A common teaspoon holds about one fluidrachm.

The symbols precede the numbers to which they refer.

255. Dry Measures are used in measuring grain, fruit, and other articles not liquid.

The Standard Unit is the *bushel* of 2150.42 cubic inches.

TABLE OF UNITS.			EQUIVALENTS.
2 Pints (*pt.*)	= 1 Quart,	*qt.*	64 *pt.*
8 Quarts	= 1 Peck,	*pk.*	1 *Bu.* = 32 *qt.*
4 Pecks	= 1 Bushel,	*bu.*	4 *pk*

MEASURES OF WEIGHT.

256. Weight is the measure of the force of gravity, and varies as the quantity of matter in a body.

The Standard Unit of weight is the *Troy Pound of the Mint*, and contains 5760 grains.

257. Troy Weight is used in weighing gold, silver, and jewels.

TABLE OF UNITS.			EQUIVALENTS.
24 Grains (*gr.*)	= 1 Pennyweight,	*pwt.*	5760 *gr.*
20 Pennyweights	= 1 Ounce,	*oz.*	1 *lb.* = 240 *pwt.*
12 Ounces	= 1 Pound,	*lb.*	12 *oz.*

258. In weighing diamonds and gems, the unit generally employed is the *Carat,* which is about 3.2 Troy grains.

Diamond Weight.			Assayers' Weight.		
16 Parts	= 1	Carat Grain.	1 Carat	= 10	pwt.
4 Carat Gr.	= 1	Carat.	1 Carat Gr.	= 60	Troy gr.
1 Carat	= $3\frac{1}{6}$	Troy gr., nearly.	24 Carats	= 1	Troy lb.

The term *Carat* is also used to express the *fineness* of gold.

259. Apothecaries' Weight is used in prescribing and in compounding *dry* medicines.

Table of Units.			Equivalents.	
20 Grains (*gr.* xx)	= 1 Scruple, ...	℈.	℔ 1 =	*gr.* 5760.
3 Scruples (℈ iij)	= 1 Dram, ...	ʒ.		℈ 288.
8 Drams (ʒ viij)	= 1 Ounce, ...	℥.		ʒ 96.
12 Ounces (℥ xij)	= 1 Pound, ...	℔.		℥ 12.

The pound, ounce, and grain of this weight are the same as those of Troy weight. The pound in each contains 5760 grains.

Medicines are bought and sold in quantities by Avoirdupois weight.

260. Avoirdupois Weight is used for all the ordinary purposes of weighing.

Table of Units.		Equivalents.	
16 Ounces (*oz.*)	= 1 Pound, . . . *lb.*	1 *T.* =	32000 *oz.*
100 Pounds	= 1 Hundredweight, *cwt.*		2000 *lb.*
20 Hundredweight, or 2000 Pounds	= 1 Ton, ... *T.*		20 *cwt.*

261. In weighing some of the coarser articles, as iron and coal at the mines, and goods on which duties are paid at the U. S. Custom Houses, the *long ton* of 2240 pounds is still used.

Table of Units.		Equivalents.	
28 Pounds	= 1 Quarter, . . . *qr.*	1 *T.* =	2240 *lb.*
4 Qr., or 112 lb.	= 1 Hundredweight, *cwt.*		80 *qr.*
20 Hundredweight, or 2240 Pounds	= 1 Ton, ... *T.*		20 *cwt.*

262. COMPARISON OF WEIGHTS.

		Troy.		Apothecaries.		Avoirdupois.
1 Pound	=	5760 Grains	=	5760 Grains	=	7000 Grains.
1 Ounce	=	480 "	=	480 "	=	437.5 "
		175 Pounds	=	175 Pounds	=	144 Pounds.

263. The number of Avoirdupois pounds in a bushel, as fixed by law in the States named:

Commodities.	Cal.	Conn.	Del.	Ill.	Ind.	Iowa.	Ky.	La.	Me.	Mass.	Mich.	Minn.	Mo.	N. C.	N. H.	N. J.	N. Y.	Ohio.	Or.	Penn.	Vt.	W. T.	Wis.
Barley,	50			48	48	48	48	32		46	48	48	48	48		48	48	48	46	47	46	45	48
Buckwheat,	40	45		40	50	52	52			46	42	42	52	50		50	48		42	48	46	42	42
Clover Seed,				60	60	60	60				60	60	60			64	60	60	60			60	60
Indian Corn,	52	56	56	52	56	56	56	56		56	56	56	56	56		56	58	56	56	56	56	56	56
Oats,	32	28		32	32	35	$33\frac{1}{3}$	32	30	30	32	32	32	32	30	30	32	32	34	32	32	36	32
Rye,	54	56		54	56	56	56	32		56	56	56	56			56	56	56	56	56	56	56	56
Timothy Seed,				45	45	45	45						45				44						46
Wheat,	60	56	60	60	60	60	60	60		60	60	60	60	60		60	60	60	60	60	60	60	60

Peas, beans, and potatoes are usually weighed 60 lb. to the bushel.

The following denominations are also in common use:

100 lb. of Grain or Flour = 1 Cental.
100 " Dry Fish = 1 Quintal.
100 " Nails = 1 Keg.
196 lb. of Flour = 1 Barrel.
200 " Beef or Pork = 1 Barrel.
240 " Lime = 1 Cask.

280 lb. of Salt at N. Y. Salt Works = 1 Barrel.

MEASURES OF TIME.

264. The Standard Unit of Time is the *mean solar day.*

Table of Units.

60 Seconds (*sec.*)	= 1 Minute,	. .	*min.*
60 Minutes	= 1 Hour,	. . .	*hr.*
24 Hours	= 1 Day,	. . .	*da.*
7 Days	= 1 Week,	. . .	*wk.*
365 Days, or 12 Calendar Mo.	= 1 Common Year,		*yr.*
366 Days	= 1 Leap Year,	.	*yr.*
100 Years	= 1 Century,	.	*Cen.*

Equivalents.

Common Year.

1 *Yr.* = 525600 *min.*, 8760 *hr.*, 365 *da.*, 52 *wk.*, 12 *mo.*

The *months* in the year, and the number of *days* in each.

Months.		No. Days.	Months.		No. Days.
1. January,	*Jan.*,	*31.*	7. July,	*July*,	*31.*
2. February,	*Feb.*,	*28* or *29.*	8. August,	*Aug.*,	*31.*
3. March,	*Mar.*,	*31.*	9. September,	*Sept.*,	*30.*
4. April,	*Apr.*,	*30.*	10. October,	*Oct.*,	*31.*
5. May,	*May*,	*31.*	11. November,	*Nov.*,	*30.*
6. June,	*June*,	*30.*	12. December,	*Dec.*,	*31.*

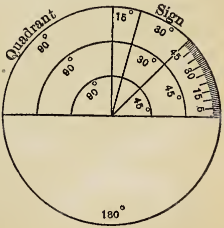
In most business transactions, 30 days are counted as a *month*, and 12 months a year.

A *true year* is the time of one revolution of the earth *around the sun*, which is 365 da. 5 hr. 48 min. 46.15 sec. If we consider 365 da. as a *common year*, the time lost in the calendar in 4 years will lack only 44 min. 55.4 sec. of a day. Hence, we add 1 day to February every fourth year, calling it *Leap Year*, which contains 366 days.

In 100 years, this difference of 44 min. 55.4 sec. amounts to 18 hr. 43 min. 5 sec.; hence, at the end of 100 years we omit to add 1 day, thus losing 5 hr. 12 min. 55 sec., which is corrected by adding 1 day at the end of 400 years. Hence,

When the number denoting the year is divisible by 4, and not by 100, and also when it is divisible by 400, it is a LEAP YEAR.

CIRCULAR MEASURES.



265. A **Circle** is a plane figure bounded by a curved line, all points of which are equally distant from a point within called the *Centre*.

266. A **Circumference** is the line that bounds a circle.

267. A **Degree** is one of the 360 equal parts into which the circumference of a circle is supposed to be divided.

268. Circular or Angular Measure is used in measuring angles, determining latitude and longitude, etc.

The **Standard Unit** of Circular Measure is the *degree.*

TABLE OF UNITS.

60 Seconds (″)	= 1 Minute, . . .	′.
60 Minutes	= 1 Degree, . . .	°.
30 Degrees	= 1 Sign, ...	*S.*
12 Signs, or 360°	= 1 Circumference	*C.*

EQUIVALENTS.

$$1\ C. = \begin{cases} 1296000''. \\ 21600'. \\ 360°. \\ 12\ S. \end{cases}$$

The length of a *degree* varies with the size of the circle; thus, a degree of long. at the Equator is 69.16 stat. mi., at 30° of latitude it is 59.89 mi., at 60° of latitude it is 34.58 mi., and at 90° it is nothing.

A *minute* of the earth's circumference is called a *geographical* or *nautical mile,* and is equal to 1.15⅓ common miles.

269. In **Counting** certain articles, use the following:

TABLE OF UNITS.

12 Units	= 1 Dozen, . . .	*doz.*
12 Dozen	= 1 Gross, ...	*gro.*
12 Gross	= 1 Great Gross, .	*G. gro.*
20 Units	= 1 Score, ...	*sc.*

EQUIVALENTS.

$$1\ G.\ gro. = \begin{cases} 1728\ units. \\ 144\ doz. \\ 12\ gro. \end{cases}$$

270. Used by **Stationers** and the **Paper Trade.**

TABLE OF UNITS.

24 Sheets	= 1 Quire, ...	*qr.*
20 Quires	= 1 Ream, ...	*rm.*
2 Reams	= 1 Bundle, . . .	*bun.*
5 Bundles	= 1 Bale, ...	*B.*

EQUIVALENTS.

$$1\ B. = \begin{cases} 4800\ \text{Sheets}. \\ 200\ \text{Quires}. \\ 10\ \text{Reams}. \\ 5\ \text{Bundles}. \end{cases}$$

271. A **Book** formed of sheets folded

In 2	Leaves is called	a Folio,	and makes	4 pp. (pages).
In 4	" "	a Quarto, 4to,	"	8 pp.
In 8	" "	an Octavo, 8vo,	"	16 pp.
In 12	" "	a Duodecimo, 12mo,	"	24 pp.
In 16	" "	a 16mo,	"	32 pp.
In 18	" "	an 18mo,	"	36 pp.
In 24	" "	a 24mo,	"	48 pp.

MEASURES OF VALUE.

272. Money is the measure of *value*.

273. United States Money is the legal currency of the United States.

The **Standard Unit** of United States Money is the *Gold Dollar*, weighing 25.8 gr.

TABLE OF UNITS.			
10 Mills (*m.*)	= 1 Cent,	*ct.*	
10 Cents	= 1 Dime,	*d.*	
10 Dimes	= 1 Dollar,	$.	
10 Dollars	= 1 Eagle,	*E.*	

EQUIVALENTS.

$$1\ E. = \begin{cases} 10000 & m. \\ 1000 & cts. \\ 100 & d. \\ 10 & \$. \end{cases}$$

The legal **Coin** of the United States is as follows: **Gold**, the double-eagle, eagle, half-eagle, quarter-eagle, three-dollar, and one-dollar pieces; **Silver**, the trade-dollar, new currency-dollar, half-dollar, quarter-dollar, and ten cent pieces; **Nickel**, the five-cent and three-cent pieces; **Bronze**, the one-cent piece.

The Trade-dollar, used for commercial purposes, weighs 420 grains. The *New Currency-dollar* of 1878 weighs 412½ grains.

274. English or Sterling Money is the legal currency of Great Britain.

The **Standard Unit** of Eng. Money is the *Sov.*, or *Pound Sterling*.

TABLE OF UNITS.				U. S. VALUE.
4 Farthings (*far.*)	=	1 Penny,	*d.*	$.02027 +
12 Pence	=	1 Shilling,	*s.*	$.2433 +
2 Shillings	=	1 Florin,	*fl.*	$.48665
5 Shillings	=	1 Crown,	*cr.*	$1.2166 +
20 Shillings	=	1 Sovereign, or 1 Pound,	*sov.* £	$4.8665
21 Shillings	=	1 Guinea,	*G.*	$5.11

The **Coin** of Great Britain in general use is as follows: **Gold**, the sovereign and half-sovereign; **Silver**, the crown, half-crown, florin, shilling, six-penny, and three-penny pieces; **Copper**, the penny, half penny, and farthing.

275. Canada Money is the legal currency of the *Dominion* of Canada.

The Coin of the Dominion of Canada in use is—**Gold**, the sovereign and half-sovereign ; **Silver**, the fifty-cent, twenty-five cent, ten-cent, and five-cent pieces ; **Bronze**, the one-cent piece.

The silver and bronze coins have the same *nominal* value as the corresponding denominations of U. S. Money.

The *intrinsic* value of the 50-cent piece in United States coin is about $46\frac{1}{5}$ cents ; of the 25-cent piece, $23\frac{1}{10}$ cents.

276. French Money is the legal currency of France, and is *decimal.*

The Standard Unit of French Money is the *Silver Franc.*

10 Millimes (*m.*)	=	1 Centime,	*ct.*	$0.00193
10 Centimes	=	1 Decime,	*dc.*	.0193
10 Decimes	=	1 Franc,	*fr.*	.193

The Coin of France consists of the following: **Gold**, 100, 40, 20, 10, and 5 francs ; Silver, 5, 2, and 1 franc ; **Bronze**, 10, 5, 2, and 1 centime pieces.

277. The **New Empire of Germany** has adopted a new and uniform system of coinage.

The Standard Unit of the new coinage is the "Mark" (Reichsmark), subdivided into 100 pennies (*Pfennige*).

The value of the "*Mark*" in United States money is $.238\frac{1}{2}$.

The Coin of the New Empire is as follows: **Gold**, 20, 10, and 5 marks ; **Silver**, 2 and 1 mark, and 20 penny ; **Nickel**, 10 and 5 penny, and pieces of less value.

The silver thaler, equal to $.746, and the silver groschen, equal to $2\frac{1}{2}$ cents, are in common use.

278. Japan has a new and *decimal* system of coinage.

The Standard Unit of the new coinage is the gold *Yen*, valued at $.997 United States Money.

The Coin of Japan consists of *five gold* coins, valued at $20, $10, $5, $2, and $1. Also *five silver* coins, valued at 5, 10, 20, 50, and 100 cents respectively.

DENOMINATE NUMBERS.

279. **A Denominate Number** is a number whose unit is named (5), and may be either Simple or Compound.

Thus, 40 inches, and 3 feet 5 inches are denominate numbers.

280. **A Simple Denominate Number** expresses a quantity in units of but one denomination.

Thus, 24 hr., 10 bbl., 50 cts., are simple denominate numbers.

281. **A Compound Denominate Number** expresses a quantity in two or more denominations of the same kind.

Thus, 5 yd. 2 ft., 7 lb. 5 oz., are compound denominate numbers.

282. **A Denominate Fraction** is one which expresses one or more of the equal parts of a denominate unit.

Thus, $\frac{7}{8}$ of a pound, .9 of a mile, are denominate fractions.

REDUCTION.

283. **Reduction of Denominate Numbers** is the process of changing the *denomination* of a number without changing the *value.*

Thus, 3 yards may be expressed as 9 feet, or 108 inches; 36 pints as 18 quarts, or 4 gallons 2 quarts.

284. ***To change denominate numbers to lower denominations.***

MENTAL EXERCISES.

How many

1. Inches in 4 ft.? In 6 ft.? Feet in 5 yd.?
2. Quarts in 5 gal.? In 3 gal. 2 qt.? Pecks in 10 bu.?
3. Rods in 10 ch.? In 4 ch. 3 rd.? Feet in $8\frac{1}{2}$ fath.?

How many

4. Sq. ft. in 6 sq. yd.? In 12 sq. yd.? Drams in 8 oz.?
5. Days in 12 wk.? In 4 wk. 5 da.? In $6\frac{3}{7}$ wk.?
6. Ounces in 3 lb. 6 oz.? In $4\frac{3}{4}$ lb.? In $5\frac{3}{8}$ lb.?
7. Pounds in $\frac{1}{2}$ ton? In 7 cwt.? In $4\frac{1}{2}$ cwt.?
8. Min. in 2 hr. 20 min.? In $\frac{1}{2}$ hr.? In $1\frac{1}{4}$ hr.?
9. Sq. ch. in 5 A.? In 3 A. 5 sq. ch.?
10. How many miles in 12 leagues? In 20 lea. 2 mi.?
11. How many inches high is a horse that measures 16 hands?
12. At 4 cts. a foot, what will 10 yards of wire cost?
13. What is the cost of a cable 8 fath. long, at $1 a foot?
14. What part of an acre is 40 sq. rd.? 5 sq. ch.?
15. How many sq. rods in $\frac{1}{2}$ of an acre? In $\frac{1}{4}$? In $\frac{3}{4}$?
16. How many cubic feet in $\frac{1}{4}$ of a cord? In $\frac{1}{2}$?
17. What will 10 pounds of clover-seed cost, at $10 a bushel?
18. What will a ton and a half of hay cost, at 1 cent a pound?
19. At $3 a bushel, what must be paid for a bag of beans, weighing 100 pounds?
20. At 10 cts. a quart, what will 1 peck of peanuts cost?
21. How many quart boxes will 3 pk. 4 qt. of cherries fill?
22. How many spoons, weighing 2 oz. each, can be made of 1 lb. 10 oz. of silver? Of $2\frac{1}{4}$ lb.?
23. Which is heavier, an ounce of calomel, or an ounce of silver? An ounce of butter, or an ounce of gold?
24. At 10 cts. a pound, what will 2 cwt. of sugar cost?
25. What will $\frac{1}{2}$ ton of hay cost, at $1\frac{1}{2}$ cts. a pound?
26. How many pounds in $\frac{1}{2}$ bbl. of pork? In $\frac{1}{2}$ bbl. of flour? In $\frac{3}{10}$ of a cental of wheat?
27. In 1 T. 8 cwt. how many pounds? In $\frac{1}{4}$ of a ton?
28. How many days from April 1 to Aug. 10, inclusive?
29. How many degrees in a quadrant? In a sextant?

30. In 2 reams how many quires? Reams in a bale?

31. What cost 1 gross of pens, at 10 cents a dozen? Half a ream of paper, at 25 cents a quire?

285. PRINCIPLE. *A denominate number is changed to lower denominations by MULTIPLICATION.*

WRITTEN EXERCISES.

286. 1. Reduce 6 bu. 2 pk. 6 qt. 1 pt. to pints.

OPERATION.

$$\begin{array}{l} 6 \text{ bu. } 2 \text{ pk. } 6 \text{ qt. } 1 \text{ pt.} \\ \underline{\quad 4} \\ 26 \text{ pk.} \\ \underline{\quad 8} \\ 214 \text{ qt.} \\ \underline{\quad 2} \\ 429 \text{ pt.} \end{array}$$

EXPLANATION. Since in 1 bu. there are 4 pk., in 6 bu. 2 pk. there are 6 times 4 pk., plus 2 pk.; $\overline{6 \times 4}+2 = 26$, the number of pecks in 6 bu. 2 pk.

Since in 1 pk. there are 8 qt., in 26 pk. 6 qt. there are 26 times 8 qt., plus 6 qt.; $\overline{26 \times 8} + 6 = 214$, the number of quarts in 6 bu. 2 pk. 6 qt.

And since in 1 qt. there are 2 pt., in 214 qt. 1 pt. there are 214 times 2 pt., plus 1 pt.; $\overline{214 \times 2}+1 = 429$, the number of pints in 6 bu. 2 pk. 6 qt. 1 pt.

2. In 8 wk. 3 da. 10 hr. 30 min., how many minutes?

3. Change 12 cwt. 22 lb. 9 oz. to ounces.

4. How many inches in 9 yd. 2 ft. 7 in.?

5. How many gills in 14 gal. 3 qt. 1 pt. 1 gi.?

RULE. 1. *Multiply the number of the highest denomination given by the number of units of the next lower denomination required to make one of that higher, and to the product add the given number of the lower denomination, if any.*

2. *Proceed in like manner with this result and each successive denomination obtained, until the given number is reduced to units of the required denomination.*

Reduce

6. 18 rd. 9 ft. 8 in. to inches.
7. £9 15s. 9d. to pence.
8. 21 lb. 6 oz. 14 pwt. to pwt.
9. 3 mi. 26 ch. 2 rd. 10 l. to l.
10. ℔ 3 ℥ 6 ʒ 4 ℈ 2 gr. 7 to gr.
11. 6 mi. 45 rd. to yards.
12. 125 A. to square feet.
13. 36 gal. 1 pt. to gills.
14. 142 T. 7 cwt. 26 lb. to lb.
15. 1 cwt. 6 oz. to oz.
16. 2 mi. 5 ch. 3 rd. to links.
17. 21° on the Equator to mi.
18. 26 fathoms to inches.
19. 12 gross 5 doz. to dozens.
20. 16 A. 4 sq. ch. to sq. rd.

Reduce

21. 6 cu. yd. 18 cu. ft. to cu. in.
22. 14 cords to cubic feet.
23. 27 bu. 3 pk. to quarts.
24. 4 da. 5 hr. 45 min. to min.
25. 2 common years to hours.
26. 245 eagles to cents.
27. 120 rods to inches.
28. Cong. 6 O. 5 to f ʒ.
29. 7 cd. ft. $11\frac{3}{4}$ cu. ft. to cu. in.
30. 6 mi. 96 rd. 4 yd. to yd.
31. 44 bbl. of flour to pounds.
32. $28\frac{1}{2}$ yd. of cloth to qr.
33. $6\frac{1}{4}$ yd. of silk to eighths.
34. 16° 24′ 26″ to seconds.
35. 2 B. 3 bun. of paper to qr.

How many pounds

36. In 28 bbl. of flour?
37. In 9 bbl. of beef?
38. In $12\frac{1}{2}$ quin. of fish?
39. In 47 kegs of nails?
40. In 5 casks of lime?
41. In 14.5 cen. of grain?
42. In $56\frac{1}{2}$ bu. of wheat?
43. In 65 bu. of oats?
44. In 36.5 bu. of corn?
45. In $10\frac{3}{4}$ bu. of clover-seed?
46. In 120 bu. of potatoes?
47. In 36.25 bu. of rye?

Value in U. S. money:

48. Of 45 sovereigns?
49. Of £16 12s.?
50. Of 60 crowns?
51. Of 150 francs?
52. Of 75 marks?
53. Of 26 guineas?

54. What is the value of 10 lb. 7 oz. 16 pwt. of old gold, at $.75 a pwt.?

55. How many barrels will be required to hold 1677 bu. 2 pk. of potatoes, each containing 2 bu. 3 pk.?

56. At 9 cts. each, what would be the cost of 5 gross of writing books?

57. How many pages in an 8vo book containing 18 printed sheets?

58. What will be the cost of 12 hhd. of sugar, each containing 5 cwt. 18 lb., at 8 cents a pound?

59. How many sacks, each holding 2 bu. 3 pk., will be required to hold 20002 bu. of wheat?

60. Bought 90 gal. of molasses, at $.84 a gallon, and sold it at $.28 a quart; what was the gain?

What is the cost

61. Of a gold chain, weighing 2 oz. 6½ pwt., at $1.10 a pwt.? At $1.12½ a pwt.?

62. Of 7 cwt. 45 lb. of rice, at 6½ cents a pound?

63. Of 2¾ bbl. of flour, at 5 cents a pound? At 6¼ cts.?

64. Of building a wall ¼ of a mile long, at 10 cents a foot?

65. Of a gross of pencils, at $.40 a doz.? At 4½ cts. ea.?

66. Of 2 bu. of chestnuts, at 12 cents a quart?

287. ***To change denominate numbers to higher denominations.***

MENTAL EXERCISES.

How many

1. Feet in 144 in.? Yards?
2. Sq. yd. in 18 sq. ft.?
3. Quarts in 112 pt.? Pecks?
4. Gallons in 56 qt.? 72 pt.?
5. Acres in 120 sq. ch.?
6. Bushels in 64 qt.? 41 pk.?
7. Shillings in 240d.? cr.?
8. Francs in 500 centimes?
9. Marks in 600 pennies?

How many

10. Tons are 45 cwt.? 63 cwt.?
11. Weeks are 74 da.? 86 da.?
12. Dozens in 144 pens?
13. Pounds are 96 oz. butter?
14. Pounds are 96 oz. silver?
15. Pounds are 60 oz. quinine?
16. Reams are 140 quires?
17. Bu. are 600 lb. wheat?
18. Pounds are 240 pwt.?

19. Change O. 84 to Cong.; f ʒ 96 to f ℥; f ℥ 64 to O.
20. Change ℈ 60 to oz.; ʒ 120 to ℔; ℥ 100 to ℔.
21. How many degrees in 150′? Signs in 180°?
22. How many cubic yards in 81 cu. ft.?
23. In 144 in. how many hands? Spans? Feet?
24. How many bbl. in 1200 lb. of beef? In 600 lb. of pork?
25. When pork is worth $20 a barrel, what is 25 lb. worth?
26. At 6 cents a pint, how many quarts of cherries can be bought for $.72?
27. What will 3 pk. of apples cost, at $.80 a bushel?
28. What will 8 sheets of paper cost, at $.18 a quire?
29. At $.36 a yard, what will 2 ft. of ribbon cost? 12 in.?
30. What part of a pound Avoir. are 8 oz.? 4 oz.? 12 oz.?
31. In a field containing 320 sq. rods, how many acres?

288. PRINCIPLE. *A denominate number is changed to higher denominations by* DIVISION.

WRITTEN EXERCISES.

289. 1. Change 3095 gills to gallons.

OPERATION.

$$\begin{array}{r|l} 4 & 3095 \text{ gi.} \\ \hline 2 & 773 \text{ pt.} + 3 \text{ gi.} \\ \hline 4 & 386 \text{ qt.} + 1 \text{ pt.} \\ \hline & 96 \text{ gal.} + 2 \text{ qt.} \end{array}$$

EXPLANATION. Since 4 gi. equal 1 pt., 3095 gi. equal as many pints as 4 is contained times in 3095, or 773 pt. + 3 gi.

Since 2 pt. equal 1 qt., 773 pt. equal as many quarts as 2 is contained times in 773, or 386 qt. + 1 pt.

Since 4 qt. equal 1 gallon, 386 qt. equal as many gallons as 4 is contained times in 386, or 96 gal. + 2 qt. Hence, in 3095 gi. there are 96 gal. 2 qt. 1 pt. 3 gi.

In like manner, change

2. 6320 pennyweights to lbs.
3. 4346 pence to pounds.
4. 64800 minutes to weeks.
5. 5260 pints to bushels.
6. 42465 ounces to tons.
7. 2876 farthings to pounds.

RULE. 1. *Divide the given number by the number of units of the given denomination required to make a unit of the next higher denomination.*

2. *In the same manner, divide this and each successive quotient, until the required denomination is reached. The last quotient, with the remainders annexed, will be the required result.*

Change the following to units of higher denominations:

8. 42621 gr. to pounds.
9. 107624 sq. in. to sq. yds.
10. 157060 pounds to tons.
11. 12627 min. to days.
12. 15672 inches to rods.
13. 26000 links to chains.
14. 4276 sq. ch. to acres.
15. 364220″ to degrees.
16. 13264 farthings to £.
17. 3648 doz. to gross.
18. 26382 sheets to reams.
19. ℈4580 to pounds.
20. 182642 cu. in. to cd. ft.
21. 10724 links to miles.
22. 6728 cu. in. to gallons.
23. 2078420 cu. in. to bushels.
24. 36840 lb. of wheat to bu.
25. $389.32 to sovereigns.

What will be the cost

26. Of 256 pints of chestnuts, at $1.75 a bushel?

27. Of $\frac{1}{2}$ an acre of land, at $.16 a square foot?

28. Of 8000 lb. of feed, at $.75 a hundredweight?

29. Of 6000 lb. of wheat, at $1.80 a bushel?

30. Of 480 cord feet of wood, at 3\frac{1}{2}$ a cord?

31. How many peanuts can be bought for $12.56, at 8 cts. a pint?

32. How many acres in a field containing 7200 sq. rd.?

33. What is the cost of a load of barley in New York, weighing 2400 lb., at $.62$\frac{1}{2}$ a bushel?

34. How much time will a person gain in 4 yr. by rising $\frac{1}{2}$ an hour earlier, and retiring 20 min. later every day?

35. How many reams of paper will be required to supply 2500 subscribers with a weekly paper 1 year?

290. ***To change a denominate fraction to integers of lower denominations.***

MENTAL EXERCISES.

1. Change $\frac{3}{4}$ of a peck to quarts.

ANALYSIS. Since 1 peck equals 8 qt., $\frac{3}{4}$ of a pk. is equal to $\frac{3}{4}$ of 8 qt.; $\frac{3}{4} \times 8 = \frac{24}{4} = 6$. Hence, $\frac{3}{4}$ pk. $= 6$ qt.

2. Change .5 of a pound Avoir. to lower denominations.

ANALYSIS. Since 1 pound equals 16 oz., .5 of a pound equals .5 of 16 oz.; $.5 \times 16 = 8$. Hence, .5 lb. $= 8$ oz.

3. Change to hours, $\frac{1}{3}$ da.; $\frac{1}{4}$ da.; $\frac{5}{6}$ da.; $\frac{3}{8}$ da.
4. How many pecks in $\frac{1}{4}$ bu.? In .5 bu.? In $\frac{3}{4}$ bu.?
5. Reduce to minutes .6 hr.; .25 hr.; .8 hr.; $\frac{4}{5}$ hr.
6. How many inches in $\frac{2}{3}$ ft.? In $\frac{7}{12}$ ft.? In .25 ft.?
7. How many cwt. in $\frac{3}{5}$ T.? How many pounds?
8. Change $\frac{3}{4}$ lb. Troy to oz.; .3 oz. to pwt.; .45 cwt. to lb.

WRITTEN EXERCISES.

291. 1. Change .645 lb. Troy, and $\frac{4}{5}$ bu. each to units of lower denomination.

OPERATION.

$$\begin{array}{rl}
.645 & \text{lb.} \\
12 & \\ \hline
7.740 & \text{oz.} \\
20 & \\ \hline
14.800 & \text{pwt.} \\
24 & \\ \hline
19.200 & \text{gr.}
\end{array}$$

$$.645 \text{ lb.} = \left\{ \begin{array}{l} 7 \text{ oz. } 14 \text{ pwt.} \\ \quad 19.2 \text{ gr.} \end{array} \right.$$

OPERATION.

$$\begin{array}{rrl}
 & 4 & \\
 & 4 & \\ \hline
5\,) & 16 & \text{pk. } (3 \text{ pk. } 1 \text{ qt. } 1\tfrac{1}{5} \text{ pt.} \\
 & 15 & \\ \hline
 & 1 & \\
 & 8 & \\ \hline
5\,) & 8 & \text{qt.} \\
 & 5 & \\ \hline
 & 3 & \\
 & 2 & \\ \hline
5\,) & 6 & \text{pt.} \\
 & 5 & \\ \hline
 & 1 &
\end{array}$$

$\frac{4}{5}$ bu. $= 3$ pk. 1 qt. $1\frac{1}{5}$ pt.

The *Analysis* and *Rule* for the preceding examples are essentially the same as in Art. **286**.

Change the following to integers of lower denominations:

2. $\frac{5}{8}$ lb. Troy.
3. $\frac{3}{4}$ of a yd.
4. .325 of a £.
5. $\frac{5}{9}$ of a ch.
6. .125 of a sq. yd.
7. $\frac{11}{16}$ of a fath.
8. $\frac{4}{9}$ of a week.
9. .125 of a bbl.
10. ℔ $\frac{11}{16}$.
11. $\frac{4}{15}$ of a min.
12. $\frac{5}{7}$ of a bu.
13. .55 lb. Avoir.
14. $\frac{7}{8}$ of a gal.
15. $\frac{1}{32}$ of a ream.
16. .016 of a mi.
17. $\frac{4}{7}$ of a gross.
18. .7575 of a mi.
19. .076 of a cu. yd.
20. $\frac{3}{7}$ of $\frac{5}{8}$ of a hhd.
21. $\frac{5}{7}$ of 4 lb. Avoir.
22. .024 of a ton.
23. $\frac{2}{25}$ of 3 cwt.
24. .4375 of a Cd.
25. $\frac{4}{5}$ of .45 mi.
26. .07 of 210 lb.
27. .54 of 90°.
28. .21675 of a T.

292. ***To change integers of lower denominations to fractions of a higher.***

MENTAL EXERCISES.

1. What part of a gallon is 2 qt. 1 pt.?

ANALYSIS. Since there are 8 pt. in 1 gal., and 5 pt. in 2 qt. 1 pt., 5 pt. are $\frac{5}{8}$ of a gallon = .625 gal. Hence, etc.

2. What part of 2 yd. is 4 ft. 6 in.? 1 yd. 2 ft.?
3. What part of 3 lb. Troy is 1 lb. 6 oz.? 2 lb. 4 oz.?
4. Change 3 pk. to the decimal of 3 bu.; of 2 bu. 3 pk.?
5. What fraction of 2 da. are 12 hr.? 18 hr.? 36 hr.?
6. What part of 3 cords is 1 Cd. 4 cd. ft.?

WRITTEN EXERCISES.

293. 1. What part or fraction of 1 pound is 10 oz. 10 pwt.?

FIRST OPERATION.

20	10.0 pwt.
12	10.500 oz.
	.875 lb., or $\frac{7}{8}$ lb.

SECOND OPERATION.

10 oz. 10 pwt. = 210 pwt.

1 lb. = 240 pwt.

$\frac{210}{240} = \frac{7}{8} = .875$

The fraction in the result may be expressed in either the common or decimal form, by Art. **215**, or **216**.

2. What part of 5 yd. 1 ft. is 2 yd. 2 ft.?

To find *what part* one compound number is of another, they must be *like* numbers, and reduced to the *lowest* denomination in *either*.

OPERATION.

2 yd. 2 ft. = 8 ft.
5 yd. 1 ft. = 16 ft.
$\frac{8}{16} = \frac{1}{2} = .5$

If there is a fraction in either of the given numbers, it must be regarded as the lowest denomination.

The pupil may be required to write or give rules, when omitted, for all operations.

What part of

3. 1 hhd. is 39 gal. 1 qt. 1 pt.?
4. 1 cwt. is 41 lb. 8.96 oz.?
5. 1 lb. Avoir. is 14.4 oz.?
6. 1 da. is 10 hr. 40 min.?
7. 10s. is 2s. 6d.?
8. 5 yd. 2 in. is 2 yd. 1 ft.?
9. 2 A. is 96 P.?
10. 1 ream is 15 sheets?
11. 1 Cd. is 3 cd. ft. 13 cu. ft.?
12. 1 T. is 5 cwt. 64 lb.?
13. 1 lb. Troy is 7 oz. 4 pwt.?
14. £5 is 7s. 6d.?
15. 4 bu. is $1\frac{3}{4}$ pk.?

What fraction of

16. 1 T. is 11 cwt. 11 lb. $1\frac{7}{9}$ oz.?
17. 1 rd. is 14 ft. $5\frac{1}{4}$ in.?
18. 3 bu. is 3 pk. 7 qt. $1\frac{1}{2}$ pt.?
19. 1 bbl. is 3 gal. 3 qt. 1 pt.?
20. £1 is 14s. 3d. $1\frac{5}{7}$ far.?
21. 1 bu. is 1 pk. 7 qt.?
22. 1 cwt. is 96 lb. 8 oz.?
23. 31 yd. 1 ft. 6 in. is 4 rd.?
24. 1 cir. is 8 S. 7° 30′?
25. 1 A. is 133 P. 11 sq. yd.?
26. 2 T. 7 cwt. 28 lb. is 5 cwt.?
27. f℥1 is fʒ5 ♏36?
28. 1 wk. 3 da. is 4 da. 9 hr.?

294. ***To find the sum of two or more denominate numbers, or denominate fractions.***

The processes of *adding*, *subtracting*, *multiplying*, and *dividing* denominate numbers are based on the same principles that govern similar operations in simple numbers; the principal difference being, that denominate numbers have an *irregular scale* of increase and decrease, while simple numbers have a *uniform decimal scale*.

The *Operations* and *Explanations* given will enable the pupil readily to form a rule for each.

Written Exercises.

295. 1. Find the sum of 9 yd. 2 ft. 10 in., 4 yd. $1\frac{1}{2}$ ft., and $5\frac{1}{2}$ yd.

OPERATION.

yd.	ft.	in.
9	2	10
4	1	6
5	1	6
19	2	10

Explanation. Write the numbers so that units of the same denomination stand in the same column. Commencing with the lowest denomination, add as in simple numbers. The sum of the inches is 22 in., equal to 1 ft. 10 in. Write the 10 in. under the column of inches, and add the 1 ft. to the column of feet.

In like manner, add the columns of feet and yards.

2. What is the sum of $\frac{4}{9}$ wk., $1\frac{3}{5}$ da., .3 da., and .325 hr.?

OPERATION.

	da.	hr.	min.	sec.
$\frac{4}{9}$ wk. =	3	2	40	00
$1\frac{3}{5}$ da. =	1	14	24	00
.3 da. =		7	12	00
.325 hr. =			19	30
	5	0	35	30

Explanation. First find the value of each denominate fraction in integers of a lower denomination (**291**); then add the resulting compound numbers.

3. Add 7 T. 14 cwt. 25 lb., 14 T. 9 cwt. 16 lb. 8 oz., 36 cwt. 17 lb., 4 T. 12 cwt., and 5 cwt. 10 lb. 14 oz.

4. Find the sum of 12 wk. 3 da. 5 hr. 20 min. 42 sec., 4 da. $12\frac{1}{2}$ hr., 3 wk. 1 da. 10 hr. 40 min., and 16 hr. $36\frac{1}{2}$ min.

5. Add 6 Cd. 5 cd. ft., 3 Cd. 6 cd. ft. 9 cu. ft., 4 cd. ft. 14 cu. ft., and $5\frac{5}{8}$ Cd.

6. What is the sum of 3 hhd. 26 gal., 42 gal. 3 qt. 1 pt., $32\frac{3}{4}$ gal., 12 gal. 1 pt., and 18.75 gal.?

7. How many acres in 3 fields, the first containing 16 A. 75 P., the second 26 A. 45 P., and the third 32 A. 120 P.?

8. How many units in $2\frac{1}{2}$ gross $5\frac{1}{3}$ doz., 6 gross $2\frac{3}{4}$ doz., $\frac{3}{4}$ of 12 gross, 8.25 doz., and 7 doz. 9 units?

Find the sum of

9. 14.45 lb., $8\frac{3}{4}$ oz., $3\frac{3}{8}$ lb., and 5 lb. 9 oz. 8.5 pwt.
10. $\frac{1}{4}$ wk., .8 da., .75 hr., $\frac{2}{3}$ min., .75 da., and 35 sec.
11. 6.75 T., 4.7 cwt., $\frac{1}{2}$ cwt., 24 lb., and $21\frac{5}{8}$ lb.
12. ℔6 ℥9 ʒ4, ℔7 ℥$5\frac{1}{2}$ ʒ4 ℈$2\frac{3}{4}$, and ℔$5\frac{3}{8}$.
13. .282 T. 96 cwt. 325 lb., and $4\frac{7}{20}$ cwt.
14. .9 mi. $\frac{7}{8}$ rd., $\frac{2}{3}$ mi., .25 mi., and $25\frac{3}{4}$ rd.
15. 4.8 bu., $2\frac{5}{6}$ bu. .8125 pk., 3 bu. $2\frac{3}{4}$ pk., and $\frac{3}{8}$ bu.
16. What cost 3 loads of hay, the first weighing 1.375 T., the second $1\frac{4}{5}$ T., and the third 2625 lb., at $18 a ton?
17. A farmer received $1.75 a bushel for 4 loads of wheat. The first contained 36.8 bu., the second $42\frac{2}{3}$ bu., the third 40 bu. 45 lb., and the fourth 2860 lb. What did he receive for the whole?
18. Three coal cars contain respectively 6.375 T., $5\frac{7}{8}$ T., and 9520 lb. What was the value of the coal, at $\$6\frac{1}{2}$ per long ton?

Written Exercises.

296. ***To find the difference between any two denominate numbers, or denominate fractions.***

1. Find the difference between 8 lb. 9 oz. 12 pwt. 14 gr. and 5 lb. 2 oz. 15 pwt. 12 gr.

OPERATION.

lb.	oz.	pwt.	gr.
8	9	12	14
5	2	15	12
3	6	17	2

Explanation. Write the numbers so that units of the same denomination stand in the same column, and begin at the right to subtract.

Subtract 12 gr. from 14 gr., and write the difference, 2 gr., under the column of grains.

Since 15 pwt. cannot be taken from 12 pwt., take 1 oz. from the 9 oz., leaving 8 oz., and add it to the 12 pwt., making 32 pwt. 15 pwt. from 32 pwt. leaves 17 pwt., which write under the pennyweights.

Since 1 oz. was taken from 9 oz., subtract 2 oz. from 8 oz., and write the difference, 6 oz., under the column of ounces. 5 lb. from 8 lb. leave 3 lb., which write under the column of pounds.

2. From $\frac{4}{5}$ mo. subtract .659 wk.

EXPLANATION. First find the value of each denominate fraction in integers of lower denomination (**291**); then find the difference between the resulting compound numbers.

OPERATION.

	wk.	da.	hr.	min.	sec.
$\frac{4}{5}$ mo. =	3	1	9	36	00
.659 wk. =		4	14	42	43.2
	2	3	18	53	16.8

3. From 3 T. 15 cwt. 18 lb. take 1 T. 7 cwt. 9 lb. 6 oz.

4. From 340 bu. 2 pk. 4 qt. take 116 bu. 3 pk. 6 qt.

5. From a cask containing 36 gal. 2 qt. of vinegar, 18 gal. 3 qt. 1 pt. was drawn. How much remained?

6. From 25 mi. 150 rd. 12 ft. take 16 mi. 120 rd. $14\frac{1}{2}$ ft.

7. From 145 A. 96 P. subtract 120 A. 105 P.

Find the difference between

8. 5.45 T. and 15.6 cwt.
9. $\frac{5}{8}$ wk. and $\frac{2}{15}$ da.
10. $3\frac{1}{2}$ lb. Troy and $\frac{8}{9}$ lb. Troy.
11. $\frac{2}{3}$ mi. and .7 rd.
12. ℔ 5.75 and ℥ $10\frac{5}{8}$.
13. .375 hr. and $44\frac{11}{15}$ min.
14. 18° 33′ 16″ and .715°.
15. $\frac{3}{4}$ rd. and 5 ft. 6 in.
16. .976 A. and $\frac{3}{5}$ A.
17. .8 bu. and 6 qt. 1 pt.
18. .625 gross and $\frac{2}{3}$ doz.
19. 5.5 bbl. and $\frac{4}{7}$ hhd.

20. A merchant tailor sold cloth that cost him £228 10s. $10\frac{1}{2}$d. for £300 6s. 10d. What was his profit?

21. From a piece of land containing .75 A., a piece was sold, containing 80 sq. rd. 140 sq. ft. 96 sq. in.; how much remained?

22. From a bin containing 130 bu. 1 pk. 6 qt. of corn, 75 bu. $2\frac{1}{4}$ pk. were sold; how much remained unsold?

23. From 1 gross of steel pens, 7 doz. 10 pens were sold; how many were left?

24. From a pile of wood containing $124\frac{5}{8}$ cords, was sold at one time $32\frac{1}{2}$ Cd., at another 24 Cd. 5 cd. ft., and at another $28\frac{9}{16}$ Cd.; how much remained?

297. *To find the time between two dates.*

1. What length of time elapsed from 2 o'clock P. M., Sept. 4, 1862, to 10 o'clock A. M., April 15, 1880?

OPERATION.

yr.	mo.	da.	hr.
1880	4	15	10
1862	9	4	14
17	7	10	20

EXPLANATION. Write the later date for the *minuend*, and the earlier for the *subtrahend*, giving the month its *number* instead of the *name*. Usually, 12 mo. are reckoned as a year, and 30 da. a month. But to be *exact*, the *true* number of days in each month and parts of a month must be reckoned when the time is less than a year. When hours are to be obtained, reckon from 12 o'clock, night.

Find the time

2. From May 12, 1848, to June 1, 1860.
3. From June 1, 1861, to Oct. 20, 1872.
4. From April 1, 1875, to August 16, 1879.
5. From May 10, 1878, to 3 o'clock P. M., Nov. 4, 1881.
6. Money borrowed July 9, 1877, was paid June 26, 1879. How long was it kept?
7. The American civil war began April 11, 1861, and closed April 9, 1865. How long did it continue?
8. How long has a note to run that is dated May 20, 1876, and made payable Sept. 12, 1880?
9. How many years, months, and days from your birthday to this date; or what is your age?
10. A note dated July 15, 1879, was paid May 21, 1882; how long did it run?
11. I started on a tour around the world at 10 o'clock A. M., Aug. 8, 1879, and returned to the same depot at 4 o'clock P. M., May 10, 1881. How long was I absent?
12. The construction of the Brooklyn Suspension Bridge was commenced Jan. 3, 1870, and opened for travel July 4, 1882. How long was it in building?

298. ***To multiply a denominate number by an abstract number.***

1. Multiply 7 bu. 2 pk. 6 qt. 1 pt. by 6.

OPERATION.

7 bu.	3 pk.	6 qt.	1 pt.
			6
47 bu.	2 pk.	7 qt.	0 pt.

EXPLANATION.—Write the multiplier under the lowest denomination of the multiplicand, and multiply as in simple numbers. Thus, 6 times 1 pt. are 6 pt., equal to 3 qt. Write a cipher under the pints, and reserve the 3 qt. to be added to the product of quarts. 6 times 6 qt. are 36 qt., and 3 qt. added make 39 qt., equal to 4 pk. and 7 qt. Write the 7 qt. in the product under the quarts, and reserve the 4 pk. to be added to the product of pecks. Proceed in the same manner with each denomination until the entire product is found.

When the multiplier is large and a *composite* number, multiply successively by its factors. (**81.**)

2. Multiply 5 lb. 8 oz. 10 pwt. by 6; by 7; by 8.

3. Multiply 7 T. 12 cwt. 10.5 lb. by 8; by 9; by 7.

4. How many bushels of corn will 12 A. produce, if 1 A. yield 38 bu. 2 pk. 6 qt.?

5. What is the weight of 2 doz. spoons, if each spoon weighs 3 oz. 12 pwt. 15 gr.?

6. How much land in 5 farms, each farm divided into 8 fields, and each field containing 12 A. 120 P.?

7. Find the weight of 32 carloads of coal, by the long ton, each weighing 5 T. 7 cwt. 2 qr. 24 lb. 10 oz.

8. If 1 hhd. of sugar weighs 6 cwt. 28 lb., what is the weight of 8 hhd., and their value, at 9 cts a pound?

Find the product in integers of lower denominations of

9. $5\frac{7}{8}$ A. by 16.
10. $\frac{5}{9}$ mi. by 9.
11. 8.125 pk. by 24.
12. $6\frac{1}{5}$ lb. Troy by 36.
13. 8.84 T. by .9.
14. 2 da. 9.48 hr. by 42.
15. 7.125 cwt. by 1.6.
16. $4\frac{5}{8}$ hhd. by 15.
17. 5.2 wk. by 28.
18. 2 bu. 2.45 pk. by 10.

299. ***To divide a denominate number into equal parts.***

1. Divide 47 bu. 3 pk. 6 qt. by 7.

OPERATION.

7) 47 bu. 3 pk. 6 qt.
6 bu. 3 pk. $3\frac{1}{7}$ qt.

EXPLANATION.—Write the divisor at the left of the dividend, and proceed to find *one seventh* of the given number. Thus, $\frac{1}{7}$ of 47 bu. is 6 bu. and a remainder of 5 bu. Write the 6 bu. in the quotient, and reduce the 5 bu. to pecks, which added to 3 pk. make 23 pk. $\frac{1}{7}$ of 23 pk. is 3 pk. and a remainder of 2 pk. Write the 3 pk. in the quotient, and reduce the 2 pk. to quarts, which added to 6 qt. make 22 qt. $\frac{1}{7}$ of 22 qt. is $3\frac{1}{7}$ qt., which write in the quotient.

2. Divide 15 lb. 9 oz. 12 pwt. 17 gr. by 4; by 6; by 8.
3. Divide 3 T. 12 cwt. 14 lb. 13 oz. by 3; by 9; by 7.
4. Divide ℔69 ℥6 ʒ2 ℈0 gr.18 by 9; by 4; by 12.
5. Divide 1946 gal. 3 qt. 1 pt. by 35; by 42.

When the divisor is large and a *composite* number, the work may be shortened by dividing successively by its *factors*. (**103.**)

Find the result

6. Of 31 hhd. 4 gal. 3 qt. ÷ 5.
7. Of £66 14s. $11\frac{1}{2}$d. ÷ 7.
8. Of 336 A. 144 P. ÷ 12.
9. Of 635 Cd. 2 cd. ft. ÷ 8.
10. Of 53° 16′ 12″ ÷ 18.
11. Of 70 mi. 40 rd. 3 yd. ÷ 6.
12. Of 23 cu. yd. 7 cu. ft. ÷ 4.
13. Of 12 wk. 3 da. 21 hr. ÷ 11.
14. Of 6 yd. 2 ft. 3 in. ÷ 9.
15. Of £98 16s. ÷ 24.

16. Bought 12 spoons, weighing 1 lb. 10 oz. 6 pwt.; what was the weight of each spoon?

17. A town containing 16 sq. miles was equally divided into 120 farms. What was the size of each farm?

18. From the sum of 61 gal. 3 qt. 1 pt., and 36 gal. 1 pt., take 28 gal. 2 qt., and divide the result by 18; by 24; by 30.

19. In excavating a cellar 3240 cu. ft. of earth was removed by 3 men in 8 days. How many cubic yards did each man remove daily?

20. How many times is £5 1s. 8d. contained in £25 8s. 4d.?

Reduce both dividend and divisor to the same denomination; then divide as in simple numbers.

21. How long will 5 bu. 2 pk. 4 qt. of oats feed a horse, if he eats 1 pk. 4 qt. a day? If 1 pk. 6 qt.?

LONGITUDE and TIME.

300. The **Meridian** of any place is an imaginary line passing from the North Pole to the South Pole through that place.

301. The **Longitude** of any place is its distance east or west from some assumed meridian.

The English and Americans usually reckon longitude from the meridian of Greenwich, England.

Since the earth turns upon its axis *once* in 24 *hours*, it follows that $\frac{1}{24}$ of 360°, or 15° of longitude passes under the sun in 1 hr., and $\frac{1}{60}$ of 15°, or 15′, passes under the sun in 1 min., and $\frac{1}{60}$ of 15′, or 15″, passes under the sun in 1 sec. of time. Hence, the following

TABLE OF EQUIVALENTS.

A difference of			A difference of	
15°	in Long.	produces	1 hr.	in Time.
15′	"	"	1 min.	"
15″	"	"	1 sec.	"
1°	"	"	4 min.	"
1′	"	"	4 sec.	"

As the earth revolves from west to east, places east of a given meridian have midday sooner, and are said to have earlier, or faster time, because the sun appears to them *earlier*; and for similar reasons places west are said to have later or *slower time*.

MENTAL EXERCISES.

1. When it is noon in New York, what is the hour 15° west of N. Y.? 30° east of N. Y.? 30° 30′ west of N. Y.?

2. The difference of time between New York and Washington is 12 min. 15 sec. What is their difference of longitude?

3. A man left Chicago, and traveled until the local time was 12 min. slow by Chicago time. Through how many degrees of longitude did he travel? Was he then east or west of the meridian of Chicago?

4. A man left Columbia, Mo., and traveled until the local time was 1 hr. 12 min. fast by Columbia time. Through how many degrees of longitude did he travel? Was he then east or west of the meridian of Columbia?

5. What is the difference of time between two places whose difference of longitude is 95°?

6. When it is noon at Chicago, is it before noon or after noon at places eastward? At places westward?

7. A and B, located at different points, observe the beginning of an eclipse of the moon. By A's time the eclipse begins at 10 P. M., and by B's at 12 hr. 15 min. A. M. What is their difference of longitude, and which is west of the other?

302. *The longitudes in the following table are estimated from the meridian of Greenwich:*

Albany, 73° 44′ 48″ W.	New York, 73° 59′ 9″ W.
Berlin, 13° 23′ 44″ E.	Omaha, 96° W.
Boston, 71° 3′ 30″ W.	Paris, 2° 20′ 9″ E.
Bombay, 72° 54′ 0″ E.	Pekin, 116° 0′ 28″ E.
Columbia, Mo., 92° 19′ 31″ W.	Philadelphia, 75° 9′ 37″ W.
Chicago, 87° 36′ 42″ W.	Rome, 12° 28′ 26″ E.
London, 0° 5′ 0″ W.	San Francisco, 122° 24′ 40″ W.
Mexico, 99° 6′ 39″ W.	St. Louis, 90° 12′ 17″ W.
Montreal, 73° 25′ 0″ W.	St. Petersburg, 30° 18′ 23″ E.
New Orleans, 90° W.	Washington, 77° 3′ 1″ W.

Written Exercises.

303. ***To find the difference of time in two places, when their longitudes are given.***

1. Find the difference in the time of Boston and St. Louis.

OPERATION.

	90°	12′	17″	Long. of St. Louis.
	71°	3′	30″	" " Boston.
15)	19°	8′	47″	Difference of longitude.
	1 hr.	16 min.	35 sec.	Difference of time.

Explanation. Since 15° of longitude correspond to 1 hr. of time, 15′ of longitude to 1 min. of time, and 15″ of longitude to 1 sec. of time, it follows that there are $\frac{1}{15}$ as many *hours*, *minutes*, and *seconds* of *time* as there are *degrees*, *minutes*, and *seconds* of *longitude*.

2. When it is 2 o'clock P. M. at New York, what is the time at Berlin?

OPERATION.

	73°	59′	9″	W. Long. of New York.
	13°	23′	44″	E. " " Berlin.
15)	87°	22′	53″	Difference of longitude.
	5 hr.	49 min.	32 sec.	Difference of time.

Explanation. The time at Berlin being faster, the difference must be added to New York time. Hence, the time at Berlin will be 49 min. 32 sec. past 7 o'clock P. M.

If the given places are both in east, or both in west longitude, the difference of longitude is found by subtracting the less from the greater; if one is in east, and the other in west longitude, the difference is found by adding the longitudes.

Rule. *Divide the difference of longitude, expressed in degrees, etc., by 15. The quotient will express the difference in time in hours, minutes, and seconds.*

Using the table (**302**), find the difference in time between

3. Albany and Boston.	5. Washington and Rome.
4. N. Y. and San Francisco.	6. London and New York.

When it is noon at Washington, what is the time

7. In Chicago?	9. In Mexico?	11. In London?
8. In San Francisco?	10. In Paris?	12. In Pekin?

304. ***To find the difference of longitude between any two places, when the difference in time is known.***

1. When it is 9 o'clock at Boston, it is 34 min. 34 sec. past 5 o'clock at San Francisco. Find the difference in longitude.

EXPLANATION. There are 15 times as many degrees, min., and sec. in the difference of longitude as there are hr., min., and sec. in the difference of time.

OPERATION.

9 hr.	0 min.	0 sec.	Time in B.
5 "	34 "	34 "	" " San F.
3 hr.	25 min.	26 sec.	Diff. of time.
		15	
51°	21′	30″	

RULE. *Multiply the difference of time between the two places, expressed in hours, minutes, and seconds, by 15; the product will be the difference in longitude in degrees, etc.*

Find the difference in longitude between two places, the difference in their time being

2. 4 hr. 46 min.	4. 2 hr. 25 min. 30 sec.
3. 1 hr. 25 min.	5. 3 hr. 10 min. 25 sec.

In what longitude from Washington is a place whose time compared with that of Washington is

6. 1 hr. 20 min. earlier?	8. 2 hr. 30 min. later?
7. 56 min. later?	9. 3 hr. 15 min. earlier?

Review

305. 1. What cost 45 bu. 3 pk. 1 qt. of wheat, at $1.75 a bushel?

2. How many centals of wheat can I purchase for $150, at $1.25 a bushel? At $1.60? At $1.84?

3. How many bushels of barley in California are equal to 540 bu. in New York? To 350 bu. in Mass.?

4. If I exchange hay at $16 a ton, for flour at $7.60 a barrel, how many barrels of flour will 3 T. of hay purchase?

5. A ship sailed due north 31.4 degrees. How far did she sail in statute miles?

6. From a pile of wood containing 960 cu. ft., was sold at one time $3\frac{1}{4}$ Cd., at another $2\frac{5}{8}$ Cd. What was the remainder worth, at $\$4\frac{1}{4}$ a cord?

7. What cost 5 T. 80 lb. of hay, if 3 T. 12 cwt. 20 lb. cost $\$15\frac{3}{4}$?

8. If $1052.10 was paid for 7 T. of cheese, and the same was retailed at 10 cents a pound, what was the profit?

9. At 10 o'clock A. M. in New York, what is the time in Paris? In London? In New Orleans? In Mexico?

10. What is the cost of 35000 lb. of salt at the New York salt works, at $1.84 a barrel?

11. Wishing to travel in Great Britain, I exchange $1500 for English money. How many pounds should I receive?

12. What will 25 T. 6 cwt. 3 qr. 10 lb. of coal cost, at $6.40 the long ton?

13. At the rate of 17 mi. 300 rd. a day, in what time would a man walk 188 mi. 110 rd.?

14. How many duodecimo volumes, using 10 sheets to a volume, can be printed on 33 reams 7 quires 12 sheets of paper?

15. At the rate of 45 bu. 3 pk. 6 qt. 1 pt. of grain to the acre, what will 48 acres produce?

16. Received from the mint a bag of gold, weighing 2 lb. 6 oz. 17 pwt. 12 gr. What was its value, the weight of $1 being 25.8 gr.?

17. From $\frac{4}{5}$ lb. + $4\frac{5}{6}$ oz. + $31\frac{1}{3}$ pwt. take ($\frac{3}{5}$ oz. — $\frac{7}{8}$ pwt.).

18. Bought 24 T. 4 cwt. 1 qr. 18 lb. of English iron, at 3d. a pound, long ton weight, and sold the same at $142, by the short ton; what was gained by the transaction?

Find the result

19. Of $\frac{7}{8}$ mi. + $\frac{2}{3}$ yd. + $\frac{3}{4}$ ft. — $\frac{2}{3}$ rd.

20. Of 5 × ($\frac{4}{5}$ mo. + $\frac{3}{5}$ da. — .659 wk.).

21. Of $\frac{7}{13}$ hhd. — .125 bbl.

22. Of .31 bu. — $\frac{7}{12}$ pk.

23. Of $\frac{4}{5}$ of $8\frac{3}{8}$ Cd. — $\frac{5}{16}$ Cd.

24. Of $\frac{3}{8}$ of $3\frac{2}{3}$ A. — .625 A.

Find the cost of the following *quantities*, at the *price* per *unit* named:

Ex.	Quantity.	Price.	Per.	Ex.	Quantity.	Price.	Per.
25.	125 T.	$12.00	cwt.	40.	5 hhd. 20 gal.	$.20	gal.
26.	26 cwt.	.25	lb.	41.	4 Cd. 6 cd. ft.	.54	cu. ft.
27.	432 bu.	.40	pk.	42.	12 bbl. 16 gal.	36.00	hhd.
28.	650 qt.	1.20	bu.	43.	16 cwt.	75.00	T.
29.	328 qt.	.375	gal.	44.	$2\frac{1}{8}$ sq. mi.	.40	sq. rd.
30.	96 cd. ft.	3.125	cord.	45.	270 sq. ft.	12.00	sq. yd.
31.	84 cords.	.24	cu. ft.	46.	54 gal.	62.50	hhd.
32.	640 sq. rd.	45.00	A.	47.	4 bbl. 10 gal.	.06	pt.
33.	℔ 75.	$.12\frac{1}{2}$	℥.	48.	$14\frac{2}{5}$ reams.	.30	quire.
34.	112 A.	4.50	sq. rd.	49.	$15\frac{1}{2}$ doz.	1.10	gross.
35.	140 gross.	.25	doz.	50.	124.3 yd.	2.75	rd.
36.	860 sheets.	3.00	ream.	51.	12.75 lb.	.45	pwt.
37.	114 sq. rd.	.75	sq. ft.	52.	3 pk. 6 qt.	4.00	bu.
38.	1040 pwt.	$21.62\frac{1}{2}$	lb.	53.	℔ 4 ℥ 6.	.25	ʒ.
39.	Cong. 5.	$.87\frac{1}{2}$	f℥.	54.	36.25 sq. rd.	50.00	A.

MEASUREMENTS.

306. *Practical Applications* of measures to the mechanic arts, and to the common business of life.

MEASURES OF RECTANGULAR SURFACES.

307. A **Surface** has length and breadth, without thickness.

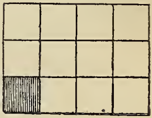
308. A **Rectangle** is any plane figure bounded by four straight lines, and having four right-angles.

When all its sides are *equal* it is called a **Square.**

309. The **Unit of Measure** of any surface is a *square,* whose side is some known unit.

Thus, the unit of measure for square inches is 1 sq. in.; for square feet, 1 sq. ft.; for square yards, 1 sq. yd., etc.

310. The **Area** of a rectangle is expressed by the *product* of the length and the breadth, or by the number of times it contains a given *unit of measure.*



Thus, the figure represents a rectangle 4 inches long and 3 inches wide, and 1 square inch is the *unit of measure.* The *area* is 3 times 4 square inches, or 12 *square inches.*

Formulas for Rectangles.

311.
1. *Length* $\times$ *Breadth* = *Area.*
2. *Area* $\div$ *Length* = *Breadth.*
3. *Area* $\div$ *Breadth* = *Length.*

The two given dimensions must be expressed in units of the same denomination.

WRITTEN EXERCISES.

312. Find the area of rectangles of the following dimensions:

1. 26 in. by 18 in.
2. 44 ft. by 20 ft.
3. $12\frac{1}{2}$ feet square.
4. 45 ft. 6 in. by 12 ft. 9 in.
5. 9.5 ch. by 4 ch. 2 rd.
6. 8.5 yd. by 9 feet.

7. How many square feet in a blackboard 22 ft. long and 3 ft. wide?

8. How many sq. rods in a garden 279 ft. long and 180 ft. wide?

9. How many square yards in the walls of a room 16 ft. long, 12 ft. 6 in. wide, and 10 ft. 4 in. high?

10. What is the width of a room 24 ft. long, the floor containing 444 square feet?

11. If the width of the room is 12 ft., what is the length?

Find the number of yards in length, and the cost, of carpeting sufficient to cover the floors of rooms of the following dimensions:

12. A room 18 ft. by 16 ft. 6 in.; carpet 1 yd. wide, at $.90.

13. A room 24 ft. by 36 ft.; carpet 28 in. wide, at $1.50.

14. A room $13\frac{2}{3}$ ft. square; carpet 30 in. wide, at $1.84.

15. A hall $42\frac{1}{2}$ ft. by 8.4 ft.; carpet $\frac{7}{8}$ yd. wide, at $\$2\frac{1}{8}$.

16. How many yards of Brussels carpet, $\frac{3}{4}$ yd. wide, will be required to carpet a parlor 38 ft. by 18 ft. 4 in., and what will be the cost, at $2.75 a yard?

17. What will it cost to cement a cellar bottom 42 ft. 6 in. long and 36 ft. wide, at $.38 a square yard?

18. What will be the cost of slating a roof 48 ft. long and 36 ft. wide, at $\$12.62\frac{1}{2}$ a square? A roof 32 ft. 6 in. long and 24 ft. wide, at $9.80 a square?

19. How many flagstones, 2 ft. by 1 ft. 3 in., will be required to flag a walk 250 ft. long and 4 ft. 6 in. wide; and what will be the cost, at \$2½ a sq. yard?

20. How many acres in a field that is 210 ch. long and 50 ch. wide?

21. A rectangular piece of land contains 20 A. 120 sq. yd., and the breadth is 8 ch. 25 l.; what is the length?

22. What part of an acre is a piece of land 121 yd. long and 75 ft. wide?

23. A field 160 rd. long contains 32 A.; what is its width?

24. How many acres in a field 130 rods square?

25. How many sods, 1 ft. 4 in. square, will cover a yard 50 ft. long and 21 ft. 4 in. wide?

26. How many planks, 12 ft. long by 1 ft. 6 in., will floor a room 30 ft. 6 in. long and 24 ft. wide?

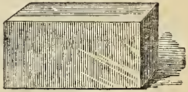
27. How many yards of silk, $\frac{7}{8}$ yd. wide, will line 18 yd. of velvet, $\frac{1}{2}$ yd. wide? $\frac{5}{8}$ yd. wide?

28. What will be the cost of glazing 6 windows, each 8 ft. 4 in. by 3 ft. 6 in., at \$.90 a square foot?

29. What will be the cost of plastering a room 21 ft. 6 in. by 16 ft., and 9 ft. high, at \$.35 a square yard, allowing 225 sq. ft. for doors, windows, etc.?

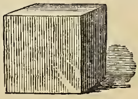
30. A field containing 16 A. 14 P. is 16.5 ch. long; what is its width, and how many rods of fence will enclose it?

MEASURES OF RECTANGULAR SOLIDS.



313. A **Solid** has length, breadth, and thickness, or height.

314. A **Rectangular Solid** is a body bounded by six rectangular surfaces, called *Faces*.

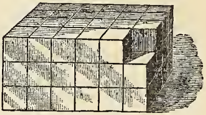


If all the faces of a rectangular solid are *equal*, it is called a **Cube**.

315. The **Unit of Measure** of any solid is a *cube* whose edge is some known unit.

Thus, the unit of measure for cubic inches is 1 cu. in.; for cubic feet, is 1 cu. ft.; for cubic yards, 1 cu. yd., etc.

316. The **Contents** or **Volume** of a rectangular solid is expressed by the *product* of the length, breadth, and height, or by the number of times it contains a given *unit of measure.*



Thus, the figure represents a rectangular solid 6 in. long, 4 in. wide, and 3 in. high, and 1 cu. in. is the *unit of measure.* The lower face or base contains 24 sq. in. (**310**); hence, if the solid is 1 in. high, it would contain 24 cu. in. But the solid is 3 in. high; hence, it contains 3 times 24 cu. in., or 72 cu. in.

FORMULAS FOR RECTANGULAR SOLIDS.

317. 1. *Length* × *Breadth* × *Height* = *Volume.*
2. *Volume* ÷ (*Length* × *Breadth*) = *Height.*
3. *Volume* ÷ (*Length* × *Height*) = *Breadth.*
4. *Volume* ÷ (*Breadth* × *Height*) = *Length.*

The three given dimensions must be expressed in units of the same denomination.

318. **Masonry** is ordinarily estimated by the *cubic foot,* or by the *perch;* also, by the *square foot* or *square yard* of surface.

1. *Excavations* and *embankments* are estimated by the *cubic yard,* called a *load.*

2. *Brickwork* is usually estimated by the *thousand bricks;* sometimes in *cubic feet.*

3. If the average size of a *common* brick is 8 × 4 × 2 in., for ordinary calculation it is sufficiently accurate to reckon 27 bricks to the cubic foot, laid dry, or 20 bricks laid in mortar.

4. An allowance of from $\frac{1}{10}$ to $\frac{1}{8}$ of the solid contents of a wall is made for the mortar.

5. In estimating *material*, allowance is made for doors, windows, and corners; but in estimating the *work*, the measure is usually taken on the outside of walls, and no allowance made for doors, windows, and corners, unless by special contract.

6. A *Perch* of masonry is $16\frac{1}{2}$ ft. long, $1\frac{1}{2}$ ft. wide, and 1 ft. high, and contains 24.75 cu. ft.

319. ***To find the number of perches in a wall.***

Divide the contents in cubic feet by $24\frac{3}{4}$.

320. ***To find the number of common bricks in a wall.***

Multiply the contents in cubic feet by 20.

WRITTEN EXERCISES.

321. 1. How many feet in a rectangular solid 8 ft. long, 3 ft. wide, and 2 ft. high?

2. What is the volume of a solid 6 ft. 4 in. long, 4 ft. 6 in. wide, and 3 ft. high?

3. How many cubic yards of earth must be removed in digging a reservoir 45 ft. long, 36 ft. wide, and $6\frac{1}{2}$ feet deep?

4. How many cubic feet of air in a room 18 ft. long, 16 ft. 8 in. wide, and 9.5 ft. high?

5. Find the volume of a solid 12.5 ft. long, the end of which is 3 ft. 9 in. square.

Find the contents of rectangular solids of the following dimensions:

6. 10 in. by 3 in. by $6\frac{1}{2}$ in.
7. 45 ft. by 6 ft. by 15 ft.
8. $8\frac{2}{3}$ yd. by $4\frac{1}{2}$ ft. by 3.25 ft.
9. 4 yd. by $6\frac{1}{4}$ ft. by 10 in.
10. Of a cube whose edge is 1 yd. 2 ft. 6 in.

Find the third dimension of rectangular solids, the volumes and two dimensions being as follows:

11. Volume, 108 ft.; length, 9 ft.; width, 4 ft.
12. Volume, 221 ft.; width, 6 ft.; height, 4 ft. 4 in.

13. Volume, 310 ft.; length, 40 ft., width, 31 ft.

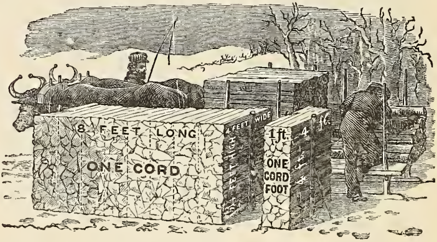
14. What will be the cost of digging a cellar 40 ft. long, 36 ft. wide, and 6 ft. 6 in. deep, at $.50 a cu. yd. or *load?*

15. How many perches of masonry in a wall 6 ft. high and 2 ft. thick, enclosing a garden 10 rods long and 8 rods wide?

16. What is the volume of a bin, the inside dimensions of which are 8 ft. 3 in. long, 4 ft. 6 in. wide, and 3 ft. deep?

1. A **Cord** of wood, stone, etc., is a pile 8 ft. long, 4 feet wide, and 4 feet high.

2. A **Cord Ft.** is 1 ft. in length of such a pile, or $\frac{1}{8}$ of a cord.



17. How many cords of wood in a pile 44 ft. long, 8 ft. wide, and 5 ft. high?

18. At $3.50 a cord, what is the value of a pile of wood 3 ft. long, 8 ft. wide, and 6 ft. high? If it were 8 ft. high?

19. How many perches of stone, laid dry, will build a wall 240 ft. long, 8 ft. 9 in. high, and 2 ft. 3 in. thick?

20. Find the cost of excavating and walling a cellar 42 ft. 6 in. long, 34 ft. wide, $6\frac{1}{2}$ ft. deep, the wall to be 18 in. thick; the excavating will cost 40 cents a load, and the mason work $4 a perch.

21. How many bricks will be required to build a wall 124 ft. long, 6 ft. high, and 1 ft. 6 in. thick?

22. How many bricks of average size will be required to build a house 54 ft. long, 27 ft. wide, and 24 ft. high, the wall being 13 in. thick, allowing 256 sq. ft. for doors and windows; and what will be their cost at $$7\frac{1}{2}$ per M.?

322. A **Board Foot** is 1 ft. long, 1 ft. wide, and 1 in. thick, and 12 such board feet make 1 cu. ft. Hence,

To change board feet to cubic feet, divide by 12; and to change cubic feet to board feet, multiply by 12.

1. In the lumber business, all boards are assumed to be 1 in. thick, and for every additional $\frac{1}{4}$ in. in thickness the price is increased *one fourth.* Thus,

1600 sq. ft., 1 in. thick or less	=	1600 ft. board measure.
1600 sq. ft., $1\frac{1}{4}$ in. thick	=	2000 ft. " "
1600 sq. ft., 2 in. thick	=	3200 ft. " "

2. *Lumber* and *sawed timber* are usually estimated in board measure, and quoted by the *hundred* or *thousand square feet.*

3. When a board tapers uniformly, the *average* width is equal to one half the sum of the two ends.

I. To find the contents of a board:

Multiply the length in feet by the width in inches, and divide the product by 12.

II. To find the contents of a plank, joist, etc.:

Multiply the length in feet by the width and thickness in inches, and divide the product by 12.

1. Find the number of board feet in a board 16 ft. long and 9 in. wide.

OPERATION. $\overline{16 \times 9} \div 12 = 12$ *board feet.*

2. How many board feet in a plank 14 ft. long, 12 in. wide, and $3\frac{1}{2}$ in. thick?

OPERATION. $\overline{14 \times 12 \times 3\frac{1}{2}} \div 12 = 49$ *board feet.*

Find the contents of boards, measuring:

3. 16 ft. by 14 in.	6. 23 ft. by 1 ft. 6 in.	9. 26 ft. by 2 ft.
4. 18 ft. by 15 in.	7. 24 ft. by $16\frac{1}{2}$ in.	10. 19 ft. by 1 ft.
5. 21 ft. by 20 in.	8. 16 ft. by $12\frac{3}{4}$ in.	11. 22 ft. by 2 ft.

12. Find the contents of a board 18 ft. long, 1 ft. 4 in. at one end, and 10 in. at the other.

Find the cost of the following:

13. Of 40 boards 14 ft. long, 9 in. wide, at \$2.75 per C.

14. Of 8 planks 12 ft. long, 14 in. wide, and 3 in. thick, at \$15 per M.

15. Of 36 scantling 9 ft. long, 4 in. by 3 in., at \$2$\frac{1}{4}$ per C.

16. Find the cost of flooring a two-story house, the floors being 48 ft. by 40 ft., and the plank 1$\frac{1}{2}$ in. thick, at \$35 per M., no allowance being made for waste.

17. How much lumber 1 in. thick will be required to make a bin which, on the outside, shall be 6 ft. long, 4 ft. 6 in. wide, and 3 ft. 3 in. high?

18. What is the cost of 356 fence boards, 16 ft. long, and 8 in. wide, at \$12 per M.?

19. What will be the cost of 4 sticks of timber 32 ft. long, 10 in. by 14 in., at \$1.60 per hundred feet, board measure?

CAPACITY OF CISTERNS, BINS, ETC.

The liquid and dry measures of the same denomination are of different capacities.

323. COMPARISON OF MEASURES OF CAPACITY.

	Cubic inches in			
	1 gallon.	1 quart.	1 pint.	1 gill.
Liquid measure, ...	231	57$\frac{3}{4}$	28$\frac{7}{8}$	7$\frac{7}{32}$
Dry measure ($\frac{1}{2}$ pk.), . . .	268$\frac{4}{5}$	67$\frac{1}{5}$	33$\frac{3}{5}$	8$\frac{2}{5}$

1. A *cubic foot* of pure water weighs 1000 oz., 62$\frac{1}{2}$ lb. Avoir.
2. The *Standard Bushel* of the United States contains 2150.42 cu. in.
3. In measuring grain, seeds, or small fruits, the measure must be *stricken*, or *even full*. In measuring bulky fruits and vegetables, as apples, potatoes, etc., the measure should be *heaped*.
4. Four *heaped* measures are considered as equal to five *stricken* measures.
5. Six *dry* quarts are considered equal to nearly 7 *liquid* quarts.
6. A *quarter*, containing 8 U. S. bushels, or a *ton*, containing 33$\frac{1}{3}$ U. S. bushels, is used in shipping grain from New York.

I. To find the capacity of a vessel in gallons:

Divide the contents in cubic inches by 231 for liquid, or by 268.8 for dry gallons.

II. To find the cubic inches in a given number of gallons:

Multiply the given number of liquid gallons by 231.

III. To find the capacity of a bin in bushels:

Divide the contents in cubic inches by 2150.42.

IV. To find the cubic contents in a given number of bu.:

Multiply the number of bushels by 2150.42; the product will be the number of cubic inches.

For all practical purposes, any number of cubic feet, diminished by $\frac{1}{5}$, will represent their equivalent in bushels; and any number of bushels, increased by $\frac{1}{4}$, will represent their equivalent in cubic feet.

Written Exercises.

324. 1. How many more cubic inches in $164\frac{1}{2}$ dry gallons than in $164\frac{1}{2}$ liquid gallons?

How many cubic feet in a space that holds

2. 1500 bushels?
3. 450 bu. 3 pk.?
4. 60 hhd. of water?
5. 150 barrels of water?
6. 14 T. 12 cwt. of pure water?
7. 464 dry gallons?

8. What is the capacity of a cistern in hhd., the depth being 5 ft. $7\frac{3}{8}$ in., and the bottom 8 ft. by 6 ft.?

9. How many barrels of water will a vat hold that contains 58212 cubic inches?

10. How many bushels of oats can be put in a bin 10 ft. by 6 ft. by 5 ft.?

11. A bin holds 148 bu. of wheat; what is its volume in cubic feet?

12. A bin 15 ft. by 6 ft. by 4 ft. 6 in. will hold how many bushels of barley? Of potatoes?

13. A tank in the attic of a house is 8 ft. by 6 ft. by 2 ft. How many gallons of water will it hold, and what will be the weight of the water?

14. A vat that will hold 3000 gallons of water will hold how many bushels of corn?

15. A man bought 8 bu. of hickory nuts, at $3 a bushel, dry measure, and retailed the same at 15 cents a quart, liquid measure. What was his gain?

16. How many bushels of wheat will fill an empty hhd.?

17. How many tons of ice can be packed in an ice-house, 30 ft. by 15 ft. by 12 ft., a cubic foot of ice weighing $58\frac{1}{8}$ lb.?

18. A California farmer put his entire harvest of barley into a bin 12 ft. by 8 ft. by 6 ft. What was its value, at $4.25 per cental?

19. A bin, 8 ft. 6 in. by 6 ft. by 3 ft. 9 in. is filled with wheat. If a bushel of 60 lb. make 48 lb. of flour, how many barrels of flour can be made from the contents of the bin?

20. The capacity of a tank is 126 cu. ft. How many gallons of water will it contain? How many bushels of wheat?

21. A farmer has a wagon, the box of which is 12 ft. long, 3 ft. 2 in. wide, and 2 ft. 3 in. deep; how many bushels of corn will it hold? How many bushels of apples?

22. What must be the length of a bin, whose width is 5 ft. and depth 5 ft., to hold 180 bushels of grain?

23. A cellar 40 ft. long and 30 ft. wide contains $2\frac{1}{2}$ ft. of water; what will be the cost of pumping it out, at 4 cents a hogshead?

24. What will be the cost of a pile of wood 57 yd. 1 ft. long, 4 ft. wide, and 6 ft. 8 in. high, at $3.75 a cord?

25. How many cords of stone will be required to enclose with a dry wall a lot 30 rods long, and 17 rods wide, the wall being 6 ft. high and 2 ft. 9 in. thick; and what will be the cost, at $$3\frac{1}{2}$ a cord?

PERCENTAGE.

INDUCTIVE EXERCISES.

325. 1. What is $\frac{1}{100}$ of 100 lb.? $\frac{3}{100}$ of \$100? $\frac{5}{100}$ of 100 ft.? .05 of \$100? .25 of \$100? .40? .75?

2. What is $\frac{2}{100}$ of 100 mi.? Of 200 ft.? Of \$500?

3. What part of 100 bu. is 1 bu.? 3 bu.? 5 bu.? 10 bu.?

4. How many hundredths of \$100 are \$7? Of \$300 are \$15? Of \$500 are \$20?

5. How many hundredths of any number equal $\frac{1}{2}$ of it?

6. In a quantity of apples 5 bbl. of every 100 bbl. are rotten; what part of the whole is bad? What part is good?

7. A grocer gains \$6 on every \$100 worth of groceries sold; what part of his sales is gain?

326. Per Cent. (from the Latin *per-centum*) means *by the hundred;* hence, any per cent. of a number or quantity is so many *hundredths* of that number or quantity.

Thus, 4 per cent. means 4 of every 100, or $\frac{4}{100}$, or .04, the 4 standing for the numerator, and the word "per cent" for the denominator 100. 12 per cent. of 50 lb. is .12, or $\frac{12}{100}$ of 50 lb.

327. The **Sign %** is generally used to denote *per cent.*

Thus, 6% is read 6 per cent.; 7% is read 7 per cent.; $\frac{1}{2}$%, $\frac{1}{2}$ per cent. 6 per cent., 6%, $\frac{6}{100}$, and .06 are equivalent expressions.

1. How many *hundredths* of a number is 6 per cent. of it? 8 per cent. of it? 15%? 25%? 40%? 75%?

2. What *per cent.* of a number is $\frac{3}{100}$ of it? $\frac{8}{100}$ of it? .07? .16? .35? .48? $.62\frac{1}{2}$?

3. What *per cent.* of a number is $\frac{1}{4}$ of it?

ANALYSIS.—Since the *whole* of any number or thing is $\frac{100}{100}$, $\frac{1}{4}$ of the same is $\frac{1}{4}$ of $\frac{100}{100}$, or $\frac{25}{100}$, equal to 25%.

4. What per cent. of a number is $\frac{1}{3}$ of it? $\frac{1}{5}$? $\frac{1}{8}$? $\frac{1}{10}$? $\frac{1}{12}$? $\frac{3}{4}$? $\frac{4}{5}$? $\frac{5}{8}$?

328. *Per cent.* may be expressed by a *common* or by a *decimal* fraction. Thus,

Per cent.	Dec.	Com. F.	Per cent.	Dec.	Com. F.
1%	.01	$\frac{1}{100}$	$2\frac{1}{2}$%	.025	$\frac{1}{40}$
4%	.04	$\frac{1}{25}$	$\frac{1}{2}$%	.005	$\frac{1}{200}$
6%	.06	$\frac{3}{50}$	$\frac{3}{4}$%	.0075	$\frac{3}{400}$
8%	.08	$\frac{2}{25}$	$12\frac{1}{2}$%	.125	$\frac{1}{8}$
25%	.25	$\frac{1}{4}$	$16\frac{1}{4}$%	.1625	$\frac{13}{80}$
$66\frac{2}{3}$%	$.66\frac{2}{3}$	$\frac{2}{3}$	125%	1.25	$1\frac{1}{4}$

EXERCISES.

1. Express in both forms, 7%; 10%; 18%; $16\frac{2}{3}$%; $8\frac{1}{3}$%.
2. Express decimally $7\frac{3}{5}$%; $10\frac{1}{2}$%; $\frac{3}{5}$%; $2\frac{3}{4}$%; $6\frac{3}{10}$%.
3. Express fractionally 16%; $\frac{3}{4}$%; $4\frac{1}{2}$%; $\frac{5}{8}$%; 75%.
4. Change to equivalent expressions having the sign %, .0825; 1.15; .755; $.00\frac{5}{8}$; 1.00; .01125; .50; .0925; .375.

DEFINITIONS

329. In the applications of percentage, the three principal elements or parts involved are, the *Base*, or Multiplicand; the *Rate per cent.*, or Multiplier; and the *Percentage*, or Product.

Any two of these being given, the third is readily found.

330. The **Base** is the number on which the percentage is computed.

331. The **Rate Per Cent.** is the number which denotes how many hundredths of the base are to be taken, and is usually expressed as a decimal.

332. **Percentage** has two significations—

1st. It is the process of computing by *hundredths.*

2d. It is the result obtained by taking as many *hundredths* of the base as are indicated by the rate.

Thus, in the statement 8% of 50 is 4, the *rate per cent.* is .08, the *base* 50, and the *percentage* 4.

333. The **Amount** is the sum of the base and the percentage.

334. The **Difference** is the remainder after deducting the percentage from the base.

Thus, if the base is 60, and the percentage 3, the *amount* is $60 + 3 = 63$; and the *difference* is $60 - 3 = 57$.

335. PRINCIPLE. *The* BASE *is either an abstract or a denominate number; the* RATE PER CENT. *is always abstract; and the* PERCENTAGE, AMOUNT, *and* DIFFERENCE *are always like the base.*

336. ***The Base and Rate % given to find the Percentage.***

MENTAL EXERCISES.

1. What is 10% of $120?

ANALYSIS.—10% of $120 is $\frac{10}{100}$, or $\frac{1}{10}$ of $120, which is $12. Hence, etc.

What is	Find
2. 6% of $200?	6. 20% of 500 men.
3. 8% of 25 lb.?	7. 25% of $800.
4. 9% of 60 bu.?	8. 40% of 300 miles.
5. 12½% of 400 A.?	9. 75% of 50 sheep.

Find the *amount*	Find the *difference*
10. Of \$60, at $33\frac{1}{3}\%$; at 25%.	13. Of \$200, less 3%; 7%.
11. Of 75 doz., at 20%; at 5%.	14. Of 800 lb., less 25%; $12\frac{1}{2}\%$.
12. Of \$150, at 4%; at 10%.	15. Of 100 lb., less 16%; 40%.

16. In a purchase of 150 doz. of eggs, 10% proved to be bad; how many were good?

17. A clerk receiving \$50 a month had his salary raised 40%. What did he then receive?

18. A man paid \$300 for a horse, and sold him for 5% less than he paid. What did he receive for him?

19. If a steamer running 12 miles an hour increases her speed 25%, how far does she then run in an hour?

WRITTEN EXERCISES.

337. 1. What is 18% of \$1450? Find the am't and diff.

OPERATION.

\$1450	Base.
.18	Rate %.
\$261.00	Percentage.
\$1450 + \$261 = \$1711	Am't.
\$1450 − \$261 = \$1189	Diff.

EXPLANATION.—Since 18% of any number is .18 of that number, 18% of \$1450 is \$1450 × .18 = \$261, the required percentage.

FORMULAS:
1. *Base* × *Rate %* = *Percentage.*
2. *Base* + *Percentage* = *Amount.*
3. *Base* − *Percentage* = *Difference.*

The pupil should be required to construct a rule in ordinary language from each *formula.* Thus,

RULES.
1. *Multiply the base by the rate %, for the percentage.*
2. *Add the percentage to the base, for the amount.*
3. *Subtract the percentage from the base, for the difference.*

What is

2. 4% of 940? 5%? 6%?
3. $6\frac{1}{2}\%$ of \$875? 7%? $12\frac{1}{2}\%$?
4. 10% of 2340 ft.? 15%?
5. 105% of 3462 rd.? $87\frac{1}{2}\%$?

Find

6. $1\frac{1}{2}\%$ of 50 A.; $\frac{3}{4}\%$; $2\frac{1}{4}\%$.
7. $33\frac{1}{3}\%$ of 420 yd.; $2\frac{1}{8}\%$; $\frac{4}{5}\%$.
8. 8% of £25 10s. 6d.; 15%.
9. 25% of 20 rd. 1 ft. 6 in.

10. Find $8\frac{1}{3}\%$ of 360 bu.; $\frac{1}{2}\%$ of \$450; $1\frac{3}{8}\%$ of 600 ft.; 26% of 48 T.

11. Find the value of 9% of \$350 + 14% of \$175 — $6\frac{1}{4}\%$ of \$500, multiplied by $3\frac{1}{2}\%$ of 40.

12. How much copper in 18 T. of ore, if it yields $66\frac{2}{3}\%$ of metal to the ton? If 45%? If $37\frac{1}{2}\%$?

13. A house and lot bought for \$3750 increased in value $112\frac{1}{2}\%$. What was it then worth?

14. From a bill of goods amounting to \$642 is deducted 5%; what is the percentage allowed, and the amount paid?

15. If the bread made from a barrel of flour weighs 30% more than the flour, what is the weight of the bread?

16. A man having \$5000, invested 20% of it in bonds and mortgages; 40% of the remainder in bank stock, and the remainder in real estate. What did the real estate cost him?

17. Bought 150 bbl. of flour at \$6.75 a barrel, and sold it at an advance of $12\frac{1}{2}\%$. What amount did it bring?

18. A man owning $\frac{4}{5}$ of an iron mine, sold $37\frac{1}{2}\%$ of his share for \$25600. What part did he then own? What was its value?

19. A farmer having 800 bu. of wheat, sold $12\frac{1}{2}\%$ of it to one man, and $16\frac{2}{3}\%$ of the remainder to another. How many bushels remained?

20. A man having an annual income of \$3500, spends 15% in traveling, 20% for board, $6\frac{1}{4}\%$ in donations, and $12\frac{1}{2}\%$ for clothing and incidentals. How much does he spend, and how much does he save?

338. ***The Base and Percentage given to find the Rate %.***

MENTAL EXERCISES.

1. What per cent. of 28 is 7?

ANALYSIS.—Since 7 is $\frac{7}{28}$ of 28, it is $\frac{7}{28}$ or $\frac{1}{4}$ of 100%, which is 25%. Hence, etc.

What per cent

2. Of 88 is 11?
3. Of 40 lb. are 30 lb.?
4. Of 90 A. are 15 A.?
5. Of $12\frac{1}{2}$ bu. are $2\frac{1}{2}$ bu.?
6. Of $12\frac{1}{2}$ ft. are $6\frac{1}{4}$ ft.?
7. Of $37\frac{1}{2}$ yd. are 5 yd.?
8. Of $2 are $.80?
9. Of $5 are $.25?
10. Of .75 are .15?
11. Of 1 cental are $8\frac{1}{3}$ lb.?
12. Of $\frac{3}{4}$ is $\frac{1}{4}$?
13. Of $\frac{5}{6}$ is $\frac{1}{3}$?
14. Of $1\frac{1}{2}$ is $\frac{3}{8}$?
15. Of $3\frac{3}{4}$ is $\frac{3}{4}$?

16. $\frac{3}{5}$ of $25 is what per cent. of $\frac{3}{4}$ of $40?

17. $2\frac{1}{4}$ times a number is what per cent. of it?

18. $\frac{3}{5}$ of 15% is what per cent. of $2\frac{1}{4}$ times 12%?

19. If a miller takes 2 qt. of every bushel he grinds, what per cent. does he take? If he takes 4 qt.?

20. If $6 is paid for the use of $72 one year, what is the rate per cent.?

21. A boy having $1.20, gave 90 cents for a knife. What per cent. of his money did he spend, and what per cent. had he left?

22. A farmer having 300 sheep sold $\frac{1}{4}$ of his flock to one man, and 20% of it to another. What per cent. remained?

23. A newsboy lost 15 papers out of 45 which he had bought. What per cent. did he lose?

24. In a school there are 100 boys and 150 girls. What per cent. more is the number of girls than of boys?

25. After 21 gal. had been drawn from a full hhd. of molasses, what per cent. remained?

WRITTEN EXERCISES.

339. 1. What per cent. of 125 is 35?

EXPLANATION.—35 is $\frac{35}{125}$ or $\frac{7}{25}$ of 125; and $\frac{7}{25}$ expressed in *hundredths*, or $\frac{7}{25}$ of 100%, is .28 or 28%. Or, since the percentage is the *product* of the base by the rate, the rate is the quotient of the percentage divided by the base, or $35 \div 125 = .28$ or 28%, the required rate.

OPERATION.
Base. Percentage.
$125\ )\ 35.00$
.28 Rate %.

FORMULA: *Percentage ÷ Base = Rate %.*

What per cent. of

2. 450 is 25? 30? 75?
3. \$80 is \$12? \$10? \$6⅔?
4. 6 lb. is 1½ lb.? ¾ lb.? 8 oz.?
5. 12.5 rd. is .75 rd.? 2.5 rd.?
6. 8 bu. 3 pk. 5 qt. is 3 bu. 2 pk.?
7. \$120 is \$6.60? \$20?
8. 30 lb. is 11 lb. 4 oz. Av.?
9. 3 hr. is 48 min.? 2¼ hr.?
10. 8 yd. is 7 ft.? 4 ft.?
11. £1 is 5d.?

12. If a barrel of flour makes 264.6 lb. of bread, what per cent more than the flour does the bread weigh?

13. A farmer raised 500 bu. of oats, and sold all but 77½ bu. What per cent. of his crop did he keep?

14. A bankrupt owes \$12750, and his assets are \$7968.75. What per cent. of his debts can he pay?

15. From a farm containing 320 A., ½ was sold at one time, and ½ of the remainder at another. What per cent. of the farm remained?

16. From a hogshead of sugar containing 640 lb., 160 lb. were sold at one time, and ⅔ of the remainder at another time. What per cent. of the whole remained?

17. A regiment went into battle with 1050 men, and came out with 588 men. What per cent. was lost?

18. A cask holding 51 gal. leaked so that but 19 gal. 1 pt. remained. What per cent. leaked out?

340. ***The Rate % and Percentage given to find the Base.***

MENTAL EXERCISES.

1. 16 is 8% of what number?

ANALYSIS.—Since 8%, or $\frac{8}{100}$ of some number is 16, 1% or $\frac{1}{100}$ of the number is $\frac{1}{8}$ of 16, or 2; and 100% is 100 times 2, or 200.

2. 12 is $\frac{1}{10}$ of what number? 10% of what number?
3. 15 is $\frac{3}{8}$ of what number? $37\frac{1}{2}\%$ of what number?
4. 25 is 6% of what number? 8%? $12\frac{1}{2}\%$? $33\frac{1}{3}\%$?
5. 4.9 is 7% of what number? $3\frac{1}{2}\%$? $\frac{1}{2}\%$?
6. Of what is 60 lb. 20%? 25 lb., $6\frac{1}{4}\%$? 16 doz., $8\frac{1}{3}\%$?
7. 10% of 50 oz. is 4% of how many ounces? 8%? 12%?
8. How many acres in a farm of which 10.5 A. is 15%?
9. The rent of a house is $450, which is 9% of its value. What is its value?
10. $12\frac{1}{2}\%$ of 96 bu. is 20% of how many bushels?

WRITTEN EXERCISES.

341. 1. 288 is 18% of what number?

EXPLANATION.—Since 288, the percentage, is the *product* of the base by the rate, the base is the quotient of the percentage divided by the rate, or 288÷.18=1600, the required base.

OPERATION.

Rate %. Percentage.

$$.18\,\overline{)\,288.00}$$

1600 Base.

FORMULA: *Percentage ÷ Rate % = Base.*

Of what number

2. Is 420, $12\frac{1}{2}\%$? 25%?
3. Is $18.75, $6\frac{1}{4}\%$? 15%?
4. Is 22.4, $\frac{1}{2}\%$? $2\frac{1}{2}\%$?
5. Is 1089, 125%? $8\frac{1}{3}\%$?

Of what

6. Is $$62\frac{1}{2}$, 8%? $12\frac{1}{2}\%$?
7. Is 15 mi. 8 rd., $\frac{3}{4}\%$? 3%?
8. Is 37 bu. 2 pk., 2%? $2\frac{1}{2}\%$?
9. Is 19.8 lb., $7\frac{1}{2}\%$? 15%?

10. 40% of 640 bbl. is 8% of how many barrels?

11. A merchant sold \$6300 worth of goods, and had 40% of his stock left. What was his entire stock?

12. If a man owns 75% of a factory, and sells $33\frac{1}{3}$% of his share for \$7400, what is the value of the whole factory?

13. A farmer raised 250 bu. of barley, which was $12\frac{1}{2}$% of his crop of wheat. How much wheat did he raise?

14. A clerk spends \$960, and has 30% of his salary left. What is his salary?

15. A farmer sold 42 A. 120 sq. rd. of land, which was 20% of his farm. What was the size of his farm?

16. A man owning 60% of a cotton mill, sold 25% of his share for \$7500. What was the value of the whole mill?

17. A man drew out 15% of his bank deposit to pay a debt of \$643.50. How much had he in bank?

342. ***The Amount, or Difference, and Rate given to find the Base.***

MENTAL EXERCISES

1. What number increased by 25% of itself amounts to 35?

ANALYSIS.—Since 35 is 25% more than the number, 35 is 125%, or $\frac{125}{100}$, equal to $\frac{5}{4}$ the number; hence, the number is 4 times $\frac{1}{5}$ of 35, which is 28.

2. 60 is 25% more than what? $12\frac{1}{2}$% more? $6\frac{1}{4}$%? $2\frac{1}{2}$%?

3. What number increased 15% amounts to 46? To 69?

4. My watch cost \$66, which was 10% more than the cost of the chain. What was the cost of the chain?

5. What number diminished by 20% of itself gives 36?

ANALYSIS.—Since 36 is 20% less than the number, 36 is 80%, or $\frac{80}{100}$, equal to $\frac{4}{5}$ the number; hence, the number is 5 times $\frac{1}{4}$ of 36, which is 45.

6. 70 rd. is $12\frac{1}{2}$% less than how many rods? $16\frac{2}{3}$% less?

7. I paid \$210 for a horse, which was 30% less than I paid for a carriage. What did the carriage cost me?

WRITTEN EXERCISES.

343. 1. What number increased by 24% of itself amounts to 1829?

EXPLANATION.—Since the number is increased by 24% of itself, 1829 is 124%, or 1.24 times the number. Hence, 1829 ÷ 1.24 = 1475, the required base.

OPERATION.

$$\begin{array}{l} 1+\text{Rate }\%.\quad \text{Amount.} \\ 1.24\,\overline{)\,1829.00} \\ \quad\quad 1475 \text{ Base.} \end{array}$$

2. What number diminished by 15% of itself gives 799?

EXPLANATION.—Since the number is diminished by 15% of itself, 799 is 85%, or .85 of the number. Hence, 799 ÷ .85 = 940, the required base.

OPERATION.

$$\begin{array}{l} 1-\text{Rate }\%.\quad \text{Difference.} \\ .85\,\overline{)\,799.00} \\ \quad\quad 940 \text{ Base.} \end{array}$$

$$\text{FORMULAS: } \left\{\begin{array}{l} 1.\ \textit{Amount} \div (1 + \textit{Rate }\%) \\ 2.\ \textit{Difference} \div (1 - \textit{Rate }\%) \end{array}\right\} = \textit{Base.}$$

What number increased	What number diminished
3. By 35% of itself is 168.75?	5. By 55%, is 42 mi. 60 rd.?
4. By $12\frac{1}{2}$%, is 1413?	6. By $7\frac{1}{2}$%, is 740 mi.?

7. A drover sold 40 head of cattle for \$3040, which was $33\frac{1}{3}$% more than the cost. Find the average cost of each.

8. A man, after paying $62\frac{1}{2}$% of his debts, finds that \$4500 will pay the remainder. What is his whole debt?

9. Sold two building lots for \$1200 each; for one I received 25% more than cost, and for the other 25% less than cost. What was the gain or loss on both?

10. A lot, 20 rd. by 30 rd., was sold for \$17.20, which was $9\frac{1}{5}$% less than it cost. What did it cost per square rod?

11. A farmer sold 25 sheep for \$115, which was 25% more than he paid for them. What did he pay per head?

12. A dealer sold 120 bbl. of flour for \$792, which was 12% less than he paid for it. What did he pay per barrel?

TRADE DISCOUNT.

344. Commercial or Trade Discount has no reference to *time*, but is simply a deduction of a certain *per cent.* from the *nominal* price or value of an article, or from the amount of debt, for cash payment or any other consideration, and expressed by the term "*per cent. off*," or so much *off*.

Thus, 15% off, or 15 off, means a discount of 15%, or 15% less than the nominal price, or the amount of debt.

Sometimes two or more discounts or percentages are deducted in succession.

Thus, 10 and 5% off means, first a discount of 10%, and then of 5% from the remainder; 20, 10, and 5% off means three successive discounts; 3 tens and 5 off means three successive discounts of 10% and 5% from the remainder.

When no discount is made, the price is called a **Net Price.**

WRITTEN EXERCISES.

1. Bought a bill of goods amounting to $675.50, at 20% discount, and 3% off for cash. Find the cash payment.

This means that the goods were bought at the regular rates on the usual time, at 20% off, but for cash down a further discount of 3% from this would be made.

2. Bought a lot of envelopes marked $6.50 per M., at 10 and 5% off; what was paid for them?

3. From a bill of books amounting to $150, a discount of 10 and 2½% was made. What was the discount, and the amount paid?

4. What is the difference between a discount of 40%, and 10% taken 4 times? Between 20% off, and 10 and 10% off?

5. What is the difference between 20, 2 tens, and 5% off, and 50% off? Between 10 and 5% on, and 10 and 5% off?

6. School slates, marked to sell at $9.50 a case, were sold at 60, 10, and 5% off. What was the price received?

7. Paid $2.80 for a book on which the bookseller allowed me a discount of 20% from the retail price. What was the retail price?

APPLICATIONS OF PERCENTAGE.

345. The principal applications of Percentage, in which *time* is not an element, are *Profit* and *Loss, Commission* and *Brokerage, Insurance, Taxes, Stocks,* etc.

Those in which *time* is an element are *Interest, Discount, Exchange, Equation of Accounts,* etc.

Some one of the *four formulas* of Percentage already considered is applicable in the solution of the problems arising in any of the above applications.

PROFIT and LOSS.

346. **Profit** and **Loss** are commercial terms used to express the gain or loss in business transactions, and are usually estimated at a *rate per cent.* on the *cost,* or the money *invested.*

347. The corresponding terms of Profit and Loss and of Percentage are as follows:

1. The **Cost**, or capital invested, is the *Base.*

2. The **Rate** % of profit or loss is the *Rate.*

3. The **Profit** or **Loss** is the *Percentage.*

4. The **Selling Price**, or cost *plus* the profit, is the *Amount.*

5. The **Selling Price**, or cost *minus* the loss, is the *Difference.*

MENTAL EXERCISES.

348. 1. Bought flour at \$8 a barrel, and sold it at a gain of $12\frac{1}{2}\%$. What was the *gain*, and what the *selling* price?

ANALYSIS.—Since the *gain* was $12\frac{1}{2}\%$, it was $\frac{12\frac{1}{2}}{100}$ or $\frac{1}{8}$ of the cost, $\frac{1}{8}$ of \$8 is \$1. The *selling price* was $\frac{9}{8}$ of the cost, or the cost \$8, plus the gain \$1, equal to \$9. Hence, etc.

Find the *gain* or *loss*, and *selling* price of

2. Tea bought at \$.50, and sold at a gain of 10%; 25%.
3. Cloth bought at \$3, and sold at a gain of $16\frac{2}{3}\%$; 20%.
4. If butter is bought at 30 cents a pound, for how much must it be sold to gain 5%? $12\frac{1}{2}\%$? 20%?
5. How must coffee that costs 20 cents a pound be sold to give a profit of 10%? $12\frac{1}{2}\%$? 15%? 5%? 30%?

349. 1. A grocer buys sugar at 8 cents a pound, and sells it at 10 cents. What per cent. does he gain?

ANALYSIS.—Since the gain on 8 cts. is 2 cts., or $\frac{1}{4}$ the cost, the gain *per cent.* is $\frac{1}{4}$ of 100%, or 25%. Hence, etc.

Find the *rate* of profit or loss on

2. Coffee, bought at 25 cts. a pound, and sold at 30 cents.
3. Apples bought at \$3 a barrel, and sold at \$4.
4. Cloth bought at \$4 a yard, and sold at \$3.50.
5. Sold a horse for $\frac{3}{4}$ of what he cost. What was the loss per cent?
6. Sold damaged cloth at $\frac{1}{5}$ less than cost. What was the loss %?
7. What per cent. is lost in buying oats at \$.50 a bushel and selling the same at \$.45?
8. What per cent. is gained or lost by selling an article for $\frac{3}{4}$ of its cost? For $\frac{2}{3}$? For $\frac{4}{5}$? For double the cost? For $\frac{1}{2}$ the cost?
9. Sold flour so as to gain $\frac{2}{5}$ as much as it cost. What per cent. was gained?

350. 1. A merchant sold goods at 20% more than cost, and gained 25 cents a yard. What was the cost?

ANALYSIS.—Since the gain 20% is $\frac{20}{100}$ or $\frac{1}{5}$ of 100%, 25 cts. is $\frac{1}{5}$ of the cost; and $\frac{5}{5}$, or the cost, is 5 times 25 cents, or $1.25. Hence, etc.

Find the *cost* of

2. Wheat, sold for 20 cts. a bushel above cost, or a gain of 10%.
3. A book, sold at a gain of 25 cts., or 5% above cost.
4. Oil, sold at a loss of $8\frac{1}{3}$%, or 12 cts. a gallon below cost.
5. Lumber, sold at a profit of $3 per M., or 25%.
6. Shirts, sold at an advance of 30 cts., or a gain of 15%.
7. A cow, sold for $9 less than cost, or a loss of $16\frac{2}{3}$%.
8. A broker lost $1600 by an investment in stocks, which was 10% of his capital. What sum had he invested?

351. 1. A horse was sold for $250, at a profit of 25%. What did he cost?

ANALYSIS.—The profit, 25% is $\frac{25}{100}$ or $\frac{1}{4}$ the cost. Then $250, the selling price, is $\frac{5}{4}$ of the cost. $\frac{1}{5}$ of $250 is $50, or $\frac{1}{4}$ of the cost; and $\frac{4}{4}$ is 4 times $50, or $200, the cost. Hence, etc.

Find the *cost* of

2. Silk, selling at $4 a yard, and yielding a profit of $33\frac{1}{3}$%.
3. Fruit, sold for $40, at a gain of 20%; at a loss of 25%.
4. A watch, sold for $120, at a loss of $16\frac{2}{3}$%.
5. A dealer sold two pianos, at $240 each, one for a profit of 20%, and the other at a loss of 20%. What was his entire gain or loss?
6. A tailor sold a suit of clothes for $42, and thereby made a profit of $16\frac{2}{3}$%. What % would he have gained or lost had he sold the suit for $30?
7. What per cent. is gained by buying an article for $\frac{3}{4}$ of the market price and selling at 25% above? By buying for $\frac{7}{8}$ of the market price and selling at $12\frac{1}{2}$% above?

WRITTEN EXERCISES.

352. ***To find the Profit or Loss and the Selling Price.***

1. Goods bought for \$850 were sold at a profit of 15%. What was the gain, and the selling price?

OPERATION.

\$850	Cost.
.15	Rate %.
\$127.50	Profit.
850.	Cost.
\$977.50	Selling Price.

EXPLANATION.—Since the gain was 15%, or $\frac{15}{100}$, equal to $\frac{3}{20}$ of the cost, the profit was .15, or $\frac{3}{20}$ of \$850, or \$127.50. \$850 + \$127.50 = \$977.50, the selling price. (**337**, 2.)

FORMULAS:

1. *Cost* × *Rate %* = *Profit or Loss.*
2. *Cost* + *Profit* } = *Selling Price.*
3. *Cost* − *Loss* } = *Selling Price.*

Find the profit or loss, and the selling price of

2. Goods, bought for \$750, and sold at a gain of 20%.
3. A piano, that cost \$450, and sold at a loss of 12%.
4. Coal, bought at \$4.20 a ton, and sold at a gain of 25%.
5. Coffee, bought at 30 cents, and sold at a loss of $2\frac{1}{2}$%.
6. Flour, that cost \$6.84 per bbl., and was sold at a gain of $16\frac{2}{3}$%.
7. At what price must cloth, that cost \$2.75 a yard, be marked to gain 24%? To lose 6%?
8. If a hhd. of sugar, weighing 8 cwt. 76 lb., cost \$61.32, and \$8.76 is paid for freight and cartage, at what price per pound must it be sold to gain $12\frac{1}{2}$%? 15%?
9. If 75 yards of cloth cost \$225, at what price must it be sold per yard to gain 10%? $18\frac{1}{2}$%? $31\frac{1}{4}$%?
10. Bought two houses for \$1500 each, and sold one at an advance of $33\frac{1}{3}$%, and the other at a loss of $33\frac{1}{3}$%. What did I gain or lose on both?

353. ***To find the Rate % of profit or loss.***

1. Sold a quantity of goods for \$620, that cost \$500. What was the gain per cent.?

OPERATION.

$\$620 - \$500 = \$120$ Whole gain.

$$\begin{array}{r|l} \text{Cost.} & \text{Profit.} \\ \$500 & \$120.00 \\ & .24 \quad \text{Rate \%.} \end{array}$$

EXPLANATION.—Since \$500 gain \$120, or $\frac{120}{500} = \frac{6}{25}$ of itself, the gain % is $\frac{6}{25}$ of 100%, which is 24%. Or, since the profit or loss is the *product* of the cost by the rate % (**352**, 1), the rate % is the *quotient* of the profit or loss divided by the cost, or $\$120 \div \$500 = .24$, or 24%. (**339.**)

FORMULA : *Profit or Loss* ÷ *Cost* = *Rate %.*

What is the rate per cent. of profit or loss on

2. Cloth, bought at \$.75 a yard, and sold at \$.87½?
3. Cloth, bought at \$.87½, and sold at \$.75?
4. Apples, bought at \$3.75 per bbl., and sold at \$5?
5. Sugar, bought at 7½ cents, and sold at 9 cents?
6. Tea, that cost \$1.20, and was sold at \$1.08?
7. Wheat, bought for \$1.32, and sold for \$1.54?
8. Two reams of paper, bought at \$3 a ream, and sold at 25 cents a quire?
9. 25 bbl. of flour, bought at \$6.80, and sold at a gain of \$51?
10. If $\frac{4}{5}$ of a certain investment is sold for what $\frac{1}{2}$ of it cost, what is the loss per cent.?
11. A merchant bought some goods for \$1500, which was \$500 less than their real value, and sold them at 25% above their real value. What per cent. profit did he make?
12. If $\frac{3}{4}$ of a hhd. of sugar is sold for what the whole cost, what is the gain per cent. on the part sold?
13. A man sold a house for \$350, and gained \$58⅓. If he had received \$70 more, what per cent. would he have gained?

354. ***To find the Cost.***

1. Sold a piece of property at an advance of $24.90, which was 20% of its cost. What was its cost?

EXPLANATION. Since the gain was 20% or $\frac{20}{100} = \frac{1}{5}$, $24.90 is $\frac{1}{5}$ the cost, etc. Or,

Since the profit or loss is the *product* of the cost by the rate %, the cost is the *quotient* of the profit or loss divided by the rate %; $24.90 ÷ .20 = $124.50. (**341.**)

OPERATION.

Rate %.	Profit.
.20)	$24.90
Cost,	$124.50

FORMULA: 1. *Profit or Loss ÷ Rate % = Cost.*

What is the cost of

2. Cloth, sold at an advance of $.70½ a yd., at 15% profit?
3. Tea, sold at $.12½ per lb. less than cost, at a loss of 12½%?
4. Corn, sold at $.17 per bu. less than cost, at a loss of 25%?
5. Pork, sold at $.50 per cwt. less than cost, at a loss of 10%?
6. A man sold 108 A. of land, at a loss of $486, which was 20% of the cost. What was the buying and selling price per acre?

7. By selling iron rails at $49.50 a ton, a profit of 12½% is made. What was the cost?

EXPLANATION. Since the gain is 12½%, or $\frac{1}{8}$ of the cost, the selling price is $\frac{9}{8}$ of the cost, or 8 times $\frac{1}{9}$ of $49.50, etc. Or,

Since 12½% of the cost is gained, the selling price is 112½%, or 1.12½ times the cost. Hence, $49.50 ÷ 1.125 = $44, the cost. (**343, 1.**)

OPERATION.

100% + 12½% = 112½%.

1+Rate %.	Selling Price.	
1.125)	$49.50	
	$44.00	Cost.

FORMULAS:

$$2.\ \textit{Selling Price} \div \begin{Bmatrix} 1.\ (1 + \textit{Rate \% of Gain}) \\ 2.\ (1 - \textit{Rate \% of Loss}) \end{Bmatrix} = \textit{Cost.}$$

8. What is the cost of silk, sold for $4.13 a yard, at a gain of 18%?

Find the cost of

9. Carpeting, sold at \$1.96 a yard, at a gain of $16\frac{2}{3}\%$.

10. A horse, sold for \$240, at a loss of 15%.

11. Coal, sold for \$6.50 a ton, at a gain of 30%.

12. A grocer sells molasses at \$.93, and makes 24% profit. What was its cost?

13. A merchant sold goods at a profit of 8%, and cleared \$1500. What did they cost?

14. By selling tea at an advance of $13\frac{1}{3}\%$, a profit of 10¢ a pound is made. What was the cost per pound?

15. A merchant sells muslins for $2\frac{1}{2}$ cents above cost, and makes a profit of 20%. What was the cost per yard?

16. How must goods be marked that cost \$3.50 a yard, so that $12\frac{1}{2}\%$ may be deducted from the marked price, and still a profit made of 15%?

First find the *selling price* at 15% profit; then regard the selling price as *cost*, and find the *asking price*.

$\$3.50 \times (1 + .15) = \$4.025$, the selling price. (**337**.)

$\$4.025 \div (1 - .12\frac{1}{2}) = \$4.60$, the marking price. (**354**, 2.)

Find the marking price when

17. The cost is \$1.12, so as to abate 5%, and still clear 25%.

18. The cost is 9 cents, so as to fall 10%, and make 50%.

19. The cost is \$60, so that 4% may be abated, and make 20%.

20. A case of goods that cost \$.80 a yard is somewhat damaged; how must they be marked, to abate 25% in the marked price, and still receive what they cost?

21. What must be the marked price of hats that cost \$3.29 each, that $12\frac{1}{2}\%$ may be deducted from the price, and still a profit of $12\frac{1}{2}\%$ be made?

22. Bought coal at \$3.75 a ton. How much must I ask a ton, that I may deduct 15% from my asking price, and still make a profit of $12\frac{1}{2}\%$ on the cost?

COMMISSION.

355. A **Commission Merchant, Agent,** or **Factor** is a person engaged chiefly in buying and selling merchandise, or other property, collecting, or investing money, or transacting other business for another.

The person for whom the business is transacted is called the **Principal**; the person who sends goods to be sold is called the **Consignor** or **Shipper**; the goods sent, a **Consignment**; and the person to whom they are sent, the **Consignee,** or **Correspondent.**

356. Commission is an allowance or compensation to a commission merchant, agent, or factor, for the transaction of business, and usually is a *percentage* of the money involved in the transaction.

357. The **Net Proceeds** of a sale, or consignment is the money due the *Consignor,* from the *Consignee,* after deducting the commission and all other charges.

358. A **Broker** is usually one who buys and sells stocks, bills of exchange, real estate, bonds, etc., for a commission, called *Brokerage.*

All problems of Commission and Brokerage may be solved by applying the formulas of Percentage already given.

359. The corresponding terms are as follows:

1. The **Amount of Sales, Money Invested** or **Collected,** is the *Base.*
2. The **Rate %** allowed for services is the *Rate.*
3. The **Commission** or **Brokerage** is the *Percentage.*
4. The **Amount of Sales,** or sum collected or invested, *plus* the commission, is the *Amount;* or *minus* the commission, is the *Difference.*

WRITTEN EXERCISES.

360. What is the *Commission* or *Brokerage*

1. On a sale of cotton for \$6450, at $1\frac{1}{2}\%$?

FORMULA: *Amt. of Sales* × *Rate %* = *Com.* (**337**, 1.)

2. On 225 bbl. of flour, at \$7.60 per barrel, at 2%?

3. On the purchase of 26 hhd. of sugar, each weighing 5 cwt. 45 lb., at $7\frac{1}{2}$ cents a pound, commission $\frac{7}{8}\%$?

4. On the purchase of a house for \$1500, at $3\frac{1}{2}\%$?

5. On a sale of \$5628.50 worth of goods, at $6\frac{1}{4}\%$?

6. On the purchase of 120 shares of stock, at \$$94\frac{1}{2}$, brokerage $\frac{1}{4}\%$?

7. My attorney collected 80% of a note for \$1200, and charged $5\frac{1}{2}\%$ commission. What amount should he pay me?

8. An agent sells a consignment of flour for \$7532.80, and then purchases 1840 bu. of wheat, at \$1.40 a bushel; his commission being $2\frac{1}{4}\%$, what sum must he remit to the consignor?

361. What is the *Rate* of commission or brokerage,

1. If \$35 is charged for selling a piece of property for \$700?

FORMULA: *Commission* ÷ *Sales* = *Rate %*. (**339.**)

2. If \$31.50 is charged for collecting a debt of \$1260?

3. If \$27 is charged for purchasing \$5400 of bank stock?

4. If \$162 is charged for selling \$3600 worth of furniture?

5. Paid a broker \$38.10 for buying 120 shares of railroad stock, at \$$95\frac{1}{4}$ a share. What was the rate of his brokerage?

6. An agent in Chicago remitted \$3795.66 on a sale of 540 bbl. of flour, at \$7.25 a barrel. What was his rate of commission?

7. A real estate broker charges \$182.34 for investing \$12156 in a factory. What was his rate of brokerage?

362. What is the *Amount of Sales*,

1. When a commission of \$360 is charged, at $2\frac{1}{2}\%$?

FORMULA: *Commission* $\div$ *Rate* % = *Amt. of Sales.* (**341.**)

2. When the brokerage charged is \$48, at $\frac{1}{4}\%$?
3. When a commission of \$24.52 is charged, at 2%?
4. When a commission of $\$57.82\frac{1}{2}$ is charged, at $4\frac{1}{2}\%$?
5. When the *net proceeds* are \$3870, commission $3\frac{1}{4}\%$?

FORMULA: *Net Pro.* $\div$ (1 − *Rate* %) = *Amt. Sales.* (**343,** 2.)

6. When the net proceeds are \$2444.55, brokerage $\frac{3}{4}\%$?
7. When the commission is \$135, and the rate $1\frac{1}{2}\%$?
8. When the net proceeds are \$2422.50, and the rate 5%?
9. When the net proceeds are \$5219.85, and the rate $2\frac{1}{4}\%$?
10. Paid an agent a commission of $\$133.12\frac{1}{2}$, at $2\frac{1}{2}\%$, to purchase wheat at $\$1.87\frac{1}{2}$ a bushel. How many bushels did he buy, and what was the amount of his bill?

363. Find the *Amount to be Invested*,

1. If \$582.40 is remitted, deducting 4% commission.

FORMULA: *Sum Rem.* $\div$ (1 + *Rate* %) = *Sum Inv.* (**343,** 1.)

2. If \$2846.25 is remitted, deducting $3\frac{1}{2}\%$ commission.
3. If \$4691.70 is received, and $\frac{1}{4}\%$ brokerage is retained.
4. If \$6500 is received, and $1\frac{1}{4}\%$ brokerage deducted.
5. What amount of wool, at \$.52 a pound, can be bought for \$3109.60 after deducting a commission of 4%?
6. A stock-broker received $\$45757.12\frac{1}{2}$, to invest in stocks, at $\$91\frac{2}{5}$ a share, after deducting $\frac{1}{8}\%$ brokerage. What amount of stock did he purchase?
7. Sent \$414 to an agent in Lowell, to be invested in prints, at $12\frac{1}{2}$ cents a yard, after taking out his commission of $3\frac{1}{2}\%$. How many yards can he purchase?

INSURANCE.

364. **Insurance** is indemnity secured against possible loss or damage. It is of two general kinds: Insurance on Property, and Insurance on Life.

365. **Property Insurance** includes *Fire Insurance,* or indemnity for loss of property by fire; *Marine Insurance,* for loss of vessel or cargo, whether at sea or on inland waters; and *Live Stock Insurance,* for loss of horses, cattle, etc.

366. The **Policy** is the contract or agreement between the insurer and the insured.

367. The **Premium** is the sum paid for insurance, and is a certain *percentage* of the amount insured.

The same elements are involved in Insurance as in the fundamental problems of Percentage.

368. The corresponding terms are as follows:

1. The **Amount Insured** is the *Base.*
2. The **Rate % of Premium** is the *Rate.*
3. The **Premium** is the *Percentage.*

WRITTEN EXERCISES.

369. What is the *Premium* for insuring

1. A house and furniture for \$3600, at 2%?

FORMULA: *Amt. Insured* × *Rate %* = *Premium.* (**337**, 1.)

2. A dwelling for \$2700, at $1\frac{1}{2}$%? For \$4100, at $\frac{3}{4}$%?
3. A cargo of 5840 bu. of wheat, valued at \$1.60 a bu., at $1\frac{1}{8}$% on $\frac{5}{8}$ of its value?

4. A drove of cattle for \$3500, at $\frac{1}{4}\%$? For \$6000, at $\frac{3}{8}\%$?

5. A mill worth \$18000, for $\frac{2}{3}$ of its value, at $1\frac{3}{4}\%$?

6. At $\frac{1}{4}$ of 1% a month, what will be the cost of insuring goods valued at \$7500, which remain in store 4 months?

7. What will be the cost of insuring my house for \$6500, at $\frac{3}{4}\%$; the furniture for \$2500, at $\frac{7}{8}\%$; and the barn and contents for \$3000, at 1%?

370. What is the *Rate* of *Insurance*,

1. If \$54 is paid for an insurance of \$3600?

FORMULA: *Premium* ÷ *Am't Ins.* = *Rate of Ins.* **(339.)**

2. If \$234 premium is paid for an insurance of \$5200?

3. If \$3 premium is paid for \$800 insurance?

4. If \$6.75 is paid for an insurance of \$900?

5. A vessel and cargo were valued at \$297000, and the premium paid for insurance on $\frac{5}{9}$ their value was \$2475. What was the rate of insurance?

6. Paid \$1443.75 for insuring 2500 bbl. of flour, worth $\$10\frac{1}{2}$ per bbl. What was the rate of insurance?

7. If it cost \$875 to insure a cargo of goods valued at \$50000, what is the rate of insurance?

371. What is the *Amount* of *Insurance*,

1. If \$175 premium is paid for insuring a hotel, at $1\frac{1}{4}\%$?

FORMULA: *Premium* ÷ *Rate of Ins.* = *Am't Ins.* **(341.)**

2. If \$187 is paid to insure a mill, at $1\frac{3}{8}\%$?

3. If the premium for insuring a house and furniture at $1\frac{1}{2}\%$ is \$79.14?

4. A drover paid a premium of \$73.50, at 40 cents per \$100, for insurance on $\frac{3}{4}$ the value of a herd of cattle. What was the value of the entire herd?

5. A speculator bought 1000 bbl. of flour, and had it insured for 80% of its cost, at $3\frac{1}{4}$%, paying a premium of \$214.50. At what price must he sell the flour to realize a profit of 20%?

6. A man paid \$175 for insuring his dwelling, at $\frac{3}{8}$%, and \$100 for insuring the furniture, at $1\frac{1}{4}$%. If both are destroyed by fire, how much is he entitled to receive?

7. Paid \$122.50 insurance on a shipment of beef, at $3\frac{1}{4}$%, to cover $\frac{3}{4}$ of its value. What was its total value?

8. If I pay \$100.25 to insure the transportation of goods, at $2\frac{1}{2}$%, what is the value put upon the goods?

TAXES

372. Taxes are sums of money assessed on persons, property, incomes, or products, for any public purpose.

373. A **Capitation** or **Poll-tax** is a tax assessed without regard to property, upon the person of every male citizen 21 years of age and upward not exempt by law.

374. A **Property Tax** is a tax upon property, and is assessed at a given rate per cent. of the valuation, or so many cents on the \$100, or mills on the dollar.

Property is of two kinds—*Real* and *Personal.*

375. Real Property, or **Real Estate,** is *immovable* property, as lands and houses.

376. Personal Property is *movable* property, as merchandise, furniture, tools, ships, cattle, money, stocks, mortgages, etc.

377. An **Assessor** is an officer appointed to estimate the value of property, and apportion the taxes.

378. The corresponding terms of Taxes and Percentage are as follows:

1. The **Valuation** of property is the *Base*.
2. The **Tax Rate** or **Tax** on $1 is the *Rate*.
3. The **Sum to be Raised** is the *Percentage*.
4. The **Sum Collected,** minus the Commission, is the *Difference*.

Written Exercises.

379. 1. The taxable property of a village, estimated at $575000, was assessed 7 mills on the dollar, for public improvements. What amount of tax was raised?

1. Formula: *Valuation* $\times$ *Tax Rate* $=$ *Sum Raised.* **(337.)**

2. What amount of tax must a man pay who is assessed $10500 for real estate, and $5000 for personal property, if he pays $1\frac{1}{5}\%$ City tax, $\frac{1}{2}\%$ State tax?

3. At what rate must property valued at $1250000 be assessed to raise a tax of $15000?

$$2.\ \text{Formula: } \frac{\textit{Sum to be Raised}}{\textit{Valuation}} = \textit{Rate of Tax.}\ \textbf{(339.)}$$

4. A tax of $7380 was levied upon the taxable property of a county, valued at $2460000. What was the rate, and what was the tax upon a farm assessed at $4000?

5. What is the valuation of a piece of property that pays a tax of $182, at the rate of $3\frac{1}{4}$ mills on the dollar?

$$3.\ \text{Formula: } \frac{\textit{Sum Raised}}{\textit{Rate of Tax}} = \textit{Valuation.}\quad \textbf{(341.)}$$

6. If a tax of $240 is assessed upon a cotton mill valued at $48000, what is the valuation of a piece of property that pays a tax of $35.50, at the same rate?

7. What sum must be assessed on a district, to build a school-house, at a cost of \$2730, and pay $2\frac{1}{2}\%$ for collection?

4. FORMULA: $\dfrac{\textit{Sum to be Raised}}{(1 - \textit{Rate of Collection})} = \textit{Sum to be Assessed.}$

8. A Town-house, costing \$12250, was built by a tax assessed upon the property of the town. The tax rate was 5 mills on the dollar, and the cost of collection 2%. What was the valuation?

RULE FOR GENERAL TAXES.—1. *From the sum to be raised, deduct the poll-tax, if any, and divide the remainder by the assessed value of the taxable property, real and personal; the quotient will be the rate of taxation.*

2. *Multiply the assessed value of each man's property by the rate, and to the product add his poll-tax, if any; the sum will be the whole tax.*

9. A tax of \$6971.60 is assessed upon a certain town, containing 430 taxable polls, at \$1.25 each, the real estate is valued at \$1354000, and the personal property at \$75800. What will be the rate of taxation, and what will James Ray's tax be, who pays for 4 polls, and whose taxable property is valued at \$5750?

$\$1.25 \times 430 = \$537.50$, total poll tax;
$\$6971.60 - \$537.50 = \$6434.10$, sum to be assessed on property;
$\$1354000 + \$75800 = \$1429800$, amount of taxable property;
$\$6434.10 \div \$1429800 = .004\frac{1}{2}$, rate of taxation;
$\$5750 \times .004\frac{1}{2} = \$25.87\frac{1}{2}$, Ray's property tax;
$\$25.87\frac{1}{2} + (\$1.25 \times 4) = \$30.87\frac{1}{2}$, Ray's whole tax.

10. In the above town, what was A's tax, whose property was assessed at \$3640, and who pays for 2 polls?

11. What was B's tax, whose property is assessed at \$7320, and who pays for 3 polls?

DUTIES and CUSTOMS.

380. Duties or **Customs** are taxes assessed by government upon imported goods, for revenue purposes, or for the protection of home industry.

Duties are of two kinds, *Ad Valorem* and *Specific.*

381. An **Ad Valorem Duty** is a certain per cent. assessed on the actual cost of the goods, in the country from which they were imported, as shown by the invoice, or fixed by appraisement.

382. A **Specific Duty** is a tax assessed upon the number, weight, or measure of the goods, per bale, ton, pound, hhd., gal., etc., without regard to their value.

Before estimating Specific Duties, certain allowances or deductions are made, called *Tare, Leakage, Breakage,* etc.

383. Tare is an allowance for the weight of the box, bag, or cask containing the merchandise.

384. Leakage is an allowance for the loss of liquids, ascertained by gauging the cask or barrel in which they are imported.

385. Breakage is an allowance for the loss of liquids imported in bottles.

386. An **Invoice** or **Manifest** is a written statement of a ship's cargo, showing the items, quantity, quality, and cost of goods, where shipped, and to whom consigned.

387. Gross Weight or **Value** is the weight or value of the merchandise before any allowances are made.

388. Net Weight or Value is the weight or value after all allowances are made.

The long ton of 2240 lb. is invariably used in the U. S. Custom Houses. In weights, less than $\frac{1}{2}$ lb. is not regarded, and more than $\frac{1}{2}$ lb. is taken as 1 pound.

389. The corresponding terms of *Ad Valorem* Duties and Percentage are the following :

1. The **Net Value** or **Quantity** is the *Base.*
2. The **Rate** or **Rate % Ad Valorem** is the *Rate.*
3. The **Duty** is the *Percentage.*

WRITTEN EXERCISES.

390. 1. Find the duty on 120 bags of coffee, gross weight 148 lb. each, allowing 3% tare, at $3\frac{1}{2}$ cents a pound.

OPERATION.

$$148 \text{ lb.} \times 120 = 17760 \text{ lb., gross;}$$
$$17760 \text{ lb.} \times .03 = 532.8 \text{ lb., tare;}$$
$$17760 \text{ lb.} - 533 \text{ lb.} = 17227 \text{ lb., net quantity;}$$
$$\$.035 \times 17227 = \$602.95\text{, duty.}$$

EXPLANATION. Since the tare is 3% of the gross weight, 17227 lb. is the *net* weight; and since the duty on 1 lb. is $3\frac{1}{2}$ cts., on 17227 lb. it is $602.95.

FORMULA: 1. *Rate* × *Net Quant.* = *Spec. Duty.* (**337**, 1.)

Find the *Duty*

2. On 50 hhd. of sugar, each weighing 480 lb., at $1\frac{1}{4}$ cts. a pound, tare 78 lb. per hhd.

3. On 360 doz. bottles of porter, duty $.50 a dozen, breakage 10%.

4. On 250 chests of tea, each 75 lb., invoiced at $.54 a pound, duty 30% ad valorem.

FORMULA: 2. *Net Invoice Cost* × *Rate %* = *Ad Val. Duty.* (337, 1.)

Find the *Duty*

5. On 16 tons of steel, invoiced at 18 cents per lb., at 25% ad valorem.

6. On 175 boxes of raisins, 18 lb. per box, at $1\frac{3}{4}$ cents per pound, tare 20%.

7. On 280 yd. of English Brussels carpet, 27 in. wide, invoiced at 8s. 6d. per yard, duty 28 cents per sq. yd., and 35% ad valorem.

8. On 40 cases of tobacco, each weighing 65 lb., and 20000 Havana cigars, weighing 200 lb.. invoiced at $45 per M., the duty on tobacco being $.30 per lb., and on cigars $$2\frac{1}{2}$ per lb. specific, and 40% ad valorem.

9. A. T. Stewart & Co. imported 10 cases of shawls, averaging 216 lb. a case, invoiced at 24884.10 francs, the duty being $.50 a pound, and 35% ad valorem. The invoice was paid with a bill of exchange, bought at 5.16 francs to the dollar. What was the duty, and what did the shawls cost, after paying other charges to the amount of $75.80?

10. Paid $22.40 duty on 100 bbl. of sugar, each weighing 220 lb., invoiced at 8 cts. a pound, tare 4%. What was the rate? **(339.)**

11. A merchant imported 80 pieces three-ply carpet, 75 sq. yd. in a piece, and paid $2591.84 duty, at 16 cts. per sq. yd., and 30% ad valorem. What was the invoice price per yard, in sterling money?

12. A wine merchant imported 6 casks of wine, and paid $432 duty, at $2 per gallon, leakage 10% allowed. How many gallons to each cask, had no leakage been allowed?

13. The duty on 300 drums of figs, containing 14 lb. each, invoiced at $5\frac{1}{4}$ cts. a lb., was $17.64; required, the rate.

14. The duty, at 19%, on an importation of satin, is $309.70; what is the invoice of the goods?

STOCKS AND INVESTMENTS.

391. Stock is the capital of an incorporated company. It is divided into equal parts, called shares, for distribution among several holders, and transfer from one to another.

It is sometimes called the *Capital Stock;* and the company a *Joint-stock Company.* Collectively, the shares of various companies in the market for sale and transfer are called *Stocks.*

The amount of each share varies in different companies. In railroad, bank, and insurance companies it is usually $100.

392. A Certificate of Stock is a written instrument, issued by a company, certifying the number of shares to which the holder is entitled.

These may be bought and sold like any other property.

393. The **Par Value** of stock is the original value, or amount specified in the certificate.

394. The **Market Value** of stock is the sum for which it can be sold at any time.

When stock can be sold for its original or face value, it is said to be *at par,* represented by 100%; when it will bring more than its face value, it is *above par,* or *at a premium;* when less, *below par,* or *at a discount.*

The market quotations are given in percentage; thus, 100 denotes *at par;* 110, 10% *above par;* 90, 10% *below par.*

395. A **Dividend** is a sum paid to stockholders out of the earnings of the company, and is always reckoned on the par value of the stock.

396. A **Stock Broker** is one who buys and sells stocks for a commission, called *Brokerage.*

The brokerage is usually computed at a certain rate % on the par value of the stock purchased or sold.

397. A **Bond** is an instrument in writing, given to secure the payment of a sum of money, at a specified time.

Bonds issued by the U. S. Government, or by States, cities, railroad companies, etc., and usually paying semi-annual interest, are dealt in as stocks, their value fluctuating according to the security afforded.

Bonds are either *Registered*, or *Coupon Bonds.*

U. S. bonds, and bonds issued by States, cities, corporations, etc., are quoted according to the rate of interest which they bear. Thus, U. S. 4's are bonds issued by the United States bearing 4% interest. Missouri 6's are bonds issued by the State of Missouri bearing 6% int. N. Y. 7's are bonds issued by the City of New York, bearing 7% int.

398. Coupons are certificates of interest attached to bonds, each to be cut off and presented for payment when the interest is due.

399. The corresponding terms used in Stocks and Percentage are as follows:

1. The **Par Value** is the *Base.*
2. The **Rate of Premium** or **Discount** is the *Rate.*
3. The **Premium** or **Discount** is the *Percentage.*
4. The **Market Value** is the *Amount* or *Difference.*

WRITTEN EXERCISES.

400. ***To find the cost of stock, when the market value is at a premium or discount.***

1. Find the cost of 150 shares of Illinois R. R. stock, the market value being $105\frac{1}{2}$, brokerage $\frac{1}{8}\%$.

EXPLANATION. Since the market value is $105\frac{1}{2}\%$, and the brokerage $\frac{1}{8}\%$, the cost of 1 share is $105\frac{5}{8}\%$ of \$100, equal to $\$105\frac{5}{8}$; the cost of 150 shares is 150 times $\$105\frac{5}{8}$; $\$105\frac{5}{8} \times 150 = \$15843.75$, *cost.*

FORMULAS:

1. *Par Value* × *Rate %* = *Premium*, or *Discount.*
2. *Par Value* + *Premium* or − *Discount* = *Market Val.*
3. (*Market Value of* 1 *Share* + *Brok.*) × *No. Shares* = *Cost.*

Find the *Cost*,

2. Of 36 shares Lake Shore R. R. stock, at $103\frac{3}{8}$, brokerage $\frac{1}{8}\%$.

3. Of 45 shares canal stock, at $12\frac{1}{2}\%$ discount. At $4\frac{1}{8}\%$ premium.

4. Of four \$1000 U. S. $4\frac{1}{2}$'s of 1891, at $109\frac{7}{8}$, brok. $\frac{1}{8}\%$.

5. A broker bought 72 shares of bank stock, at $2\frac{3}{4}\%$ discount, and afterwards sold them at $101\frac{1}{4}$. Find the gain.

401. ***To find the number of shares that can be bought for a given sum of money.***

1. How many shares of railroad stock, at $103\frac{3}{4}$, can be bought for \$26000, brokerage being $\frac{1}{4}\%$?

EXPLANATION. Since the market value is $103\frac{3}{4}$, each share, with the brokerage, will cost \$104; hence, as many shares can be bought for \$26000 as \$104 is contained times in \$26000, or 250 shares.

FORMULA: 1. *Amt. of Inv.* $\div$ *Cost* 1 *Share* $=$ *No. of Shares.*

2. How many \$500 Mo. 6's, at $97\frac{1}{4}$, brokerage $\frac{1}{4}\%$, can be bought for \$5850?

3. How many \$500 5% bonds, at $16\frac{5}{8}$ premium, brokerage $\frac{3}{8}\%$, can be bought for \$17550?

4. How many shares of Western Union Telegraph Stock can I buy for \$2610, at $108\frac{1}{2}$, brokerage $\frac{1}{4}\%$?

5. Bought U. S. Express stock at $32\frac{1}{8}$, and sold it at $33\frac{3}{4}$, paying $\frac{1}{8}\%$ brokerage each way, and gained \$1650. How many shares did I buy?

OPERATION. $(33\frac{3}{4} - \frac{1}{8}) - (32\frac{1}{8} + \frac{1}{8}) = 1\frac{3}{8}\%$ gain;
\$1650 $\div$ \$$1\frac{3}{8}$ = 1200, No. of shares.

FORMULA: 2. $\dfrac{\textit{Whole Gain or Loss}}{\textit{Gain or Loss on 1 Share}} = \textit{No. of Shares.}$

6. How many shares of Ohio Central Stock, bought at $20\frac{1}{2}$, and sold at $21\frac{3}{4}$, brokerage each way $\frac{1}{8}\%$, will yield a gain of \$1000?

402. ***To find what Investment must be made to produce a given income.***

1. What sum must be invested in U. S. registered 4's, at $107\frac{1}{2}$, to yield an income of \$1000 ?

EXPLANATION. The income on 1 share of 4% stock is \$4, and \$1000 ÷ \$4 = 250; hence, 250 shares. $\$107\frac{1}{2} \times 250 = \$26875$ *amount invested.*

FORMULAS :

1. *Given Income ÷ Income of 1 Share = No. of Shares.*
2. *Cost of 1 Share × No. of Shares = Sum Invested.*

What sum must be invested

2. In stock at $97\frac{1}{2}$, that pays a semi-annual dividend of 5%, to yield an annual income of \$1500?
3. In Ill. Central R. R. bonds, bearing 6% int., at $104\frac{3}{4}$, brokerage $\frac{1}{4}\%$, to secure an income of \$2400 per annum?

403. ***To find what Income any investment will yield.***

1. What income will be realized, by investing \$12760 in 5% bonds, purchased at $87\frac{1}{2}$, brokerage $\frac{1}{2}\%$?

EXPLANATION.—The cost of 1 share is $\$87\frac{1}{2}$ plus $\$\frac{1}{2}$, or \$88; and \$12760 ÷ \$88 = 145; hence 145 shares. \$5, income on 1 share × 145 = \$725, *income.*

$$\text{FORMULA: } \frac{\textit{Investment}}{\textit{Cost of 1 Share}} \times \textit{Income on 1 Share} = \textit{Income.}$$

2. What annual income shall I receive from an investment of \$15860 in R. R. stock, at 99, brokerage $\frac{1}{8}\%$, if 4% semi-annual dividends are declared?
3. What income will \$5000 in U. S. 4's, at $103\frac{1}{2}$, yield, brokerage $\frac{1}{4}\%$?

404. ***To find the Rate % of income.***

1. What rate per cent. will be realized from bonds bought at 120, and paying 5% semi-annual dividends?

EXPLANATION.—Since the cost of 1 share is $120, and the *annual* income is $10, the rate per cent. is $\frac{10}{120}$ or $\frac{1}{12}$ of 100%, equal to $8\frac{1}{3}$%

FORMULA: $\frac{\textit{Annual Inc. per Share}}{\textit{Cost per Share}} = \textit{Rate \% of Inc.}$ **(339.)**

Find the rate % of income realized

2. From 6% bonds, bought at 75; at $97\frac{1}{2}$; at 105.

3. From R. R. stock paying 5% semi-annual dividends, bought at a discount of 15%.

4. Which is the more profitable to buy, 7's, at 105, or 5's, at 75? N. Y. 7's, at 105, or 6's, at 84?

5. What per cent. of income do U. S. 6's yield, if bought at $108\frac{1}{2}$?

405. ***To find the Price at which stock should be bought to realize a specified rate of income.***

1. At what price should stock paying 9% annual dividends, be bought, to yield an income of $7\frac{1}{2}$%?

EXPLANATION.—Since the annual dividend or income per share is $9, this must be $7\frac{1}{2}$% of the price; $9 ÷ .07$\frac{1}{2}$, equals 120; hence, $120 is the *buying price.*

FORMULA: *Dividend ÷ Rate of Income = Buying Price.* **(341.)**

What must be paid

2. For 6% stock to yield an income of 8%? Of 10%?

3. For 8% stock to realize 6%? 7%? 10%? 12%?

4. At what price must 5's be bought, to pay as well as 6's bought at par?

5. What must I pay the government for $4\frac{1}{2}$'s of '91, to realize 7% on my investment?

Review

406. Problems in Percentage which do not involve the element of *time*.

1. Find 40% of 6 hr. 28 min. 15 sec.

2. What is 104% of 75 A. 80 P.? Of 1 T. 5 cwt. 25 lb.?

3. Bought 120 yd. of cloth, and found that it lacked 10 yd. of full measure. What per cent. did I lose?

4. A man saves $562\frac{1}{2}$ a year, which is $37\frac{1}{2}\%$ of his salary. What is his salary?

5. A drover having 45 head of cattle, sold 36 head for what the whole cost him. What was his gain per cent.?

6. A farmer sold 75 A. 80 P. of land, which was 40% of his whole farm. What was the value of his whole farm, at $32.60 an acre?

7. Bought 120 tons of coal for $408. If sold at an advance of 35%, what was the profit per ton?

8. If the insurance on $5600 worth of goods from New York to New Orleans is $70, what is the rate of insurance?

9. For what amount must merchandise valued at $8000 be insured, at $3\frac{1}{2}\%$, so as to receive this sum and the premium in case of total loss?

10. What is $\frac{3}{4}$ of a farm worth, if $2587.50 is 15% more than $\frac{9}{20}$ of its value?

11. What are the net proceeds of a sale of $16400, commission $2\frac{1}{2}\%$, and other charges $500?

12. If a lot of damaged goods are sold for $563.20, at a loss of 12%, what did they cost?

13. 1980 is 10% of 120% less than what number?

14. Silicate slates, marked 25 cents, are bought at 20, 10, and 10% off, and sold at an advance of 10 and 5% on the marked price. What is the gain per cent.?

15. What per cent. of the long ton is the short ton?

16. Sold part of a consignment of flour, at $10\frac{1}{2}$ per bbl., realizing a profit of 25%, but was obliged to sell the remainder at $8.40 per bbl. What was the gain or loss on this?

17. A lawyer receiving 5% for collection, received from one client $75, from another $84.50, and from another $200; what were the respective amounts collected?

18. Sold a piece of property at a loss of 16%, or $640 less than it cost. What would it have brought, had it been sold at a profit of 16%?

19. If I receive $558 as my share of a 9% dividend, how many shares of stock, at $50 each, do I own?

20. A speculator bought 25 shares of bank stock ($100), at a discount of $3\frac{1}{4}$%, and sold it at $1\frac{1}{2}$% premium. What was his gain?

21. A tax collector pays over $126740, and retains $1901.10. What rate per cent. is allowed him for collection?

22. Sent $5128.05 to a broker in Cincinnati, with directions to purchase pork at $12\frac{1}{2}$ per bbl., to insure it for 60 da., at 15 cents a $100, to pay storage, at 5 cts. a bbl., for 10 days, and to deduct his commission of 2% on the money expended. How many barrels of pork did he buy?

23. If from the retail price of a book, 20 and 10% are deducted, and the book then sells for $1.25, what is the retail price, and the rate of discount?

24. Received as net proceeds of a consignment $1520, after paying a commission of $2\frac{1}{2}$% for selling. What was the amount of sales?

25. Imported 28 bags of canary seed, gross weight 4284 lb., tare 84 lb. What was the duty paid, at $1 per bushel of 60 lb.?

26. Mr. Brown's tax is $621.50; he pays for 3 polls, at $2.25 each; the rate is 4 mills on the dollar. What is the valuation of his property?

27. Sold 56 acres of land for \$2047.50, which was 10% less than the asking price, but a gain of $62\frac{1}{2}$%. What was the asking price, the cost per acre, and the profit?

28. An agent bought 3000 lb. of wool at \$.46 a pound, and paid \$$7\frac{1}{2}$ for insurance, \$6.25 for cartage, his commission being $2\frac{3}{4}$%. What was the amount of his bill to his principal?

29. If the net earnings of a company with a capital of \$250000 are \$17000, and \$7000 are reserved for expenses, what rate of dividend can be declared, and what is the dividend on 35 shares, par value \$100?

30. If stock paying 10% dividends is bought at $112\frac{1}{8}$, brokerage $\frac{1}{8}$%, what per cent. of income will it yield?

31. A cotton mill valued at \$150000 is insured as follows: in Co. A for $\frac{1}{3}$ its value, at $\frac{3}{5}$%; in Co. B for $\frac{1}{4}$ its value, at $\frac{3}{4}$%; in Co. C for $\frac{1}{5}$ its value, at $\frac{2}{3}$%; in Co. D for $\frac{1}{8}$ its value, at $\frac{1}{2}$%. What is the total annual premium, and in case of loss by fire to the amount of \$25000, what is due from each company?

32. For what amount must a policy be made to insure \$12500 and the premium, at $3\frac{1}{2}$%, and what is the premium?

33. What must be paid for stocks yielding 8% dividends, to realize an annual income of 10%?

34. The Continental Ins. Co. took a risk at $2\frac{1}{4}$%, and reinsured $\frac{3}{5}$ of it in the Royal Ins. Co., at $2\frac{1}{2}$%. The premium received was \$72 more than was paid; what was the amount of the risk?

35. A tax of \$22768, besides $3\frac{1}{4}$% paid for collection, is to be raised in a certain town. There are 760 polls, assessed at \$$1\frac{1}{4}$ each; the real estate is assessed at \$700000, and the personal property at \$62000. Find the tax rate, and a person's tax whose real estate is valued at \$12500, personal property at \$2500, and who pays for 2 polls?

36. A man owning 250 shares of bank stock received a semi-annual dividend of \$1125. What was the rate?

37. S. G. & Co. of New York received per steamer Gallia, goods invoiced as follows:

480 yd.	Broadcloth, weight 265 lb.,	cost	14s.	sterl.	per yd.
1860 "	Brus'ls Tapestry, $\frac{3}{4}$ yd. wide,	"	7s. 6d.	"	"
640 "	Thread Lace,	"	8s.	"	"

The duty on broadcloth was \$.50 per lb. and 35%; on tapestry, 28 cents per square yard and 30%; and on lace, 25%. What amount of duty was paid, allowing the pound sterling to be \$4.866$\frac{1}{2}$?

38. Bought stock at 3$\frac{1}{2}$% discount, and sold the same at a premium of 2$\frac{1}{4}$%, gaining \$258.75. What was the par value of the stock?

39. If by selling broadcloth \$.75 per yard above cost, I gain 12$\frac{1}{2}$%, how much must I advance the price to realize a profit of 20%?

40. A store rents for \$2000, and the tax on the same is 2$\frac{1}{2}$%, on a valuation of \$22000. The owner sells it for \$35000, and invests the money in stock at 91, that pays a dividend of 7%. Is his yearly income increased or diminished, and how much?

41. If $\frac{5}{8}$ of a barrel of pork is sold for what the whole barrel cost, what per cent. is gained on the part sold?

42. A grocer imported 65 hhd. of sugar, each weighing 4 cwt. 2 qr. 16 lb., invoiced at 6 cts. per lb., tare allowed, 36 lb. per hhd., duty 2$\frac{1}{2}$ cts. per lb. and 20% additional; also, 35 hhd. of molasses, 84 gal. each, invoiced at 20 cts. per gal., 4% being allowed for leakage, duty 4$\frac{1}{2}$ cts. per gal. and 25% additional. What was the entire amount of duty?

43. What rate of income will be realized from money invested in stock paying a semi-annual dividend of 5%, purchased at 87$\frac{1}{2}$, brokerage $\frac{1}{4}$%?

INTEREST.

INDUCTIVE EXERCISES.

407. 1. If money is loaned at 6% a year, what decimal part of the money borrowed equals the sum paid for its use? If loaned at 7%? 8%? 5½%? 10%?

2. At 6%, how many dollars must be paid for the use of $100 for 1 year? For 2 yr.? 3 yr.? 5 yr.?

3. At 5%, how many dollars must be paid for the use of $200, for 1 yr.? $200, for 3 yr.? $400, for 2 yr.?

4. If $6 are paid for the use of $100 for 1 yr., what is the rate per cent.? If $18 are paid for the use of $100 for 3 yr.?

5. If I borrow $200, and agree to pay 6% a year for its use, how much shall I owe the lender at the end of 2 yr.?

DEFINITIONS.

408. Interest is money paid for the use of money.

409. The **Principal** is the money, for the use of which interest is paid.

410. The **Rate of Interest** is the per cent. of the principal paid for its use *one year*.

411. The **Amount** is the sum of the principal and interest.

412. Legal Interest is at the rate fixed by law.

413. Usury is interest at a higher than the legal rate.

When the rate of interest is not specified in notes, accounts, contracts, etc., the legal rate is always understood.

414. In some States, a higher than the legal rate is allowed, if agreed to in writing, as shown in the following

TABLE OF LEGAL RATES.

States.	*Rate %.*		*States.*	*Rate %.*		*States.*	*Rate %.*		*States.*	*Rate %.*	
Ala.	8		Ill.	6	8	Mo.	6	10	S. C.	7	Any.
Ark.	6		Ind.	6	10	Montana	10		Tenn.	6	10
Arizona	10	Any.	Iowa	6	10	N. H.	6		Texas	8	10
Cal.	10		Kan.	7	12	N. J.	7		Utah	10	Any.
Conn.	6		Ky.	6	10	N. Y.	6		Vt.	6	
Colo.	10	Any.	La.	5	8	N. C.	6	8	Va.	6	
Dakota.	7	Any.	Maine.	6	Any.	Neb.	10	12	W. Va.	6	
Del.	6		Md.	6		Nev.	10	Any.	W. T.	10	Any.
D. C.	6	10	Mass.	6	Any.	Ohio	6	8	Wis.	7	10
Fla.	8	Any.	Mich.	7	10	Oregon	10	12	Wy.	12	Any.
Ga.	7	12	Minn.	7	12	Penn.	6				
Idaho	10		Miss.	6	10	R. I.	6	Any.			

The legal rate in England and France is 5%; in Canada and Nova Scotia, 6%.

415. Corresponding terms of Interest and Percentage:

1. The **Principal** is the *Base.*
2. The **Rate % per Annum** is the *Rate.*
3. The **Interest** is the *Percentage.*

The sum of the principal and the interest is the *Amount.*

4. **Time** is an *additional* element introduced, to be considered always in connection with the *rate %* per annum.

416. PRINCIPLE. *Interest is the product of three factors, the principal, rate % per annum, and time expressed in years.*

If any three elements are given, the fourth may be found.

Of the various methods of computing interest, three of the most simple and practical are presented in this work.

1. As the legal rate of interest in the majority of the States is 6%, the six per cent. method is selected, as likely to be largely used.

2. The method by cancellation being *uniformly* applicable to *any rate* and *time,* and also simple, brief, and easily understood, is especially commended to the attention of the learner.

3. The method by Tables, which is in very general use among business men.

WRITTEN EXERCISES.

417. ***To find the Interest and the Amount.***

1. Find the interest and the amount of \$247.50, for 3 yr. 4 mo., at 8%.

EXPLANATION. Since the int. of any sum at 8%, for 1 yr., is .08 of the principal, the product of \$247.50 by .08, which is \$19.80, is the int. for 1 yr.; and the int. for 3 yr. 4 mo., or $3\frac{1}{3}$ yr., is $3\frac{1}{3}$ times \$19.80, or \$66, the required *interest;* and \$247.50 + \$66 = \$313.50, the *amount.* Hence,

OPERATION.

\$247.50	Principal.
.08	Rate.
\$19.8000	Interest for 1 year.
$3\frac{1}{3}$	Time in years.
\$66.000	Int. for 3 yr. 4 mo.
\$247.50	Principal.
\$313.50	Amount.

FORMULAS: $\begin{cases} 1.\ \textit{Principal} \times \textit{Rate}\ \% \times \textit{Time} = \textit{Interest.} \\ 2.\ \textit{Principal} + \textit{Interest} = \textit{Amount.} \end{cases}$

In computing interest, the partial results need not be carried to more than *four* decimal places.

Find the *interest* of

2. \$75, for 4 yr., at 6%; for 3 yr. 6 mo., at 7%; at 8%.
3. \$135.70, for 2 yr. 3 mo., at 5%; for 1 yr. 9 mo., at $6\frac{1}{4}$%.
4. \$762.40, for 3 yr. 3 mo., at 8%; for 2 yr. 4 mo., at 10%.
5. \$2684.80, for 6 yr. 6 mo., at 7%; for 9 mo., at 8%.
6. \$1500, for 4 yr. 8 mo., at $4\frac{1}{2}$%; for 5 yr. 2 mo., at 12%.

Find the *amount* of

7. \$87.60, for 10 mo., at 8%; at 7%; 6%; 12%.
8. \$564.80, for 2 yr. 4 mo., at $6\frac{1}{4}$%; for 3 yr. 9 mo., at 7%.
9. A owes a note for \$1200 and interest, at 6% for 2 yr. 4 mo. If he pays \$853.50 on it, how much is still unpaid?
10. If a man borrows \$1575, at 6%, and loans it immediately at $7\frac{1}{2}$%, what does he gain by the operation in 9 mo.?

11. Find the interest of \$216.50 for 1 yr. 5 mo. 18 da., at 7%.

EXPLANATION.—The int. for 1 yr. is .07 of the principal, or \$15.155; the int. for 1 mo. is $\frac{1}{12}$ the int. for 1 yr., which is \$1.2629; and for 1 yr. 5 mo. 18 da., or 17.6 mo., it is 17.6 times \$1.2629, which is \$22.23, the *interest* required.

OPERATION.

\$216.50	
.07	
12) \$15.1550	Int. for 1 yr.
\$1.2629	Int. for 1 mo.
17.6	Time in mo.
\$22.2270	Int. for 1 yr. 5 mo. 18 da.

Since 30 days are reckoned as a month, 3 days is $\frac{1}{10}$ (.1) of a month; hence, any number of days is readily reduced to *tenths* of a month by dividing by 3; thus, 18 da. equals .6 of a month; 19 da., .6$\frac{1}{3}$ of a month, etc.

Find the interest of

12. \$540, for 9 mo. 15 da., at 6%; 7%; 8%; 12%.

13. \$636.40, for 1 yr. 1 mo. 12 da., at 5%; for 10 mo. 20 da., at 6%.

14. \$1000, for 2 yr. 2 mo. 9 da., at 4½%; at 6½%.

15. \$960, at 6%, for 11 mo. 4 da.; for 5 mo. 27 da., at 7%.

16. \$2175, at 7%, for 1 yr. 15 da.; for 3 mo. 12 da., at 8%.

17. \$428.80, at 8%, for 7 mo. 14 da.; for 2 yr. 24 da.

18. \$1500, at 10%, for 1 yr. 11 mo.; for 10 mo. 10 da., at 12%.

19. Find the interest and the amount of \$374.60 from Sept. 4, 1879, to June 20, 1881, at 8%.

20. If a man pays \$375 a year rent for a dwelling, would he gain or lose, and how much, in 4 yr. 6 mo., by borrowing \$4500 at 6%, to purchase the house?

21. What is the int. of \$1630 from April 1, 1878, to Oct. 10, 1882, at 6%?

22. Bought a piece of property for \$6500, and agreed to pay for it in 8 mo. 15 da., with interest at 7%. What amount was due at the expiration of the time?

418. SIX PER CENT. METHOD.

The interest of $1 at 6%

For 12 mo., or	1 year,	is 6 cents,	or .06	of the principal.		
" 2 "	" $\frac{1}{6}$ year,	" 1 cent,	" .01	"	"	
" 1 "	" $\frac{1}{12}$ year,	" 5 mills,	" .005	"	"	
" 6 da.,	" $\frac{1}{5}$ month,	" 1 mill,	" .001	"	"	
" 1 da.,	" $\frac{1}{30}$ month,	" $\frac{1}{6}$ mill,	" $.000\frac{1}{6}$	"	"	

Hence, the following

419. PRINCIPLES.—I. *The interest of any sum of money at 6%, is one half as many hundredths of the principal as there are months in the given time.*

II. *The interest of any sum of money at 6%, is one sixth as many thousandths of the principal as there are days in the given time.*

Thus, the interest on any sum for 1 yr. 4 mo. (16 mo.) is $\frac{1}{2}$ of .16, or .08 of the principal; for 1 yr. 5 mo. (17 mo.) it is .085; for 2 mo. 12 da. (72 da.) it is $\frac{1}{6}$ of .072, or .012 of the principal; for 26 da., $.004\frac{1}{3}$; etc.

WRITTEN EXERCISES.

420. 1. What is the interest of $255 for 1 yr. 5 mo. 15 da., at 6%?

EXPLANATION.—The int. of any sum for 1 yr. 5 mo. 15 da., at 6%, is $.087\frac{1}{2}$ of the prin.; $\$255 \times .087\frac{1}{2} =$ $22.31, the required int.

OPERATION.

$$\begin{aligned} 1 \text{ yr. } 5 \text{ mo.} &= 17 \text{ mo.} \\ \tfrac{1}{2} \text{ of } .17 &= .085 \\ \tfrac{1}{6} \text{ of } .015 &= \underline{.002\tfrac{1}{2}} \\ \text{Int.} &= .087\tfrac{1}{2} \text{ of Prin.} \end{aligned}$$

$$\begin{array}{r} \$255 \\ \underline{.087\frac{1}{2}} \\ \$22.3125 \end{array}$$

RULE. *Multiply the principal by the decimal expressing one half the number of months as hundredths, and one sixth the number of days as thousandths; the product will be the interest at 6%.*

To find the interest at any other rate per cent., divide the interest at 6% by 6, and multiply the quotient by the given rate.

Find the *interest*, at 6%, of

2. \$840, for 8 mo. 15 da.
3. \$216.40, for 5 mo. 24 da.
4. \$148, for 2 yr. 3 mo.
5. \$1026, for 1 yr. 1 mo. 20 da.
6. \$98.60, for 11 mo. 28 da.
7. \$1000, for 3 yr. 1 mo. 6 da.
8. \$1684.75, for 93 days.
9. \$2500, for 63 days.
10. \$348.84, for 33 days.
11. \$1208.50, for 1 mo. 13 da.

Find the *interest*, at 5%, 7%, and 8%, of

12. \$750, for 1 yr. 1 mo. 3 da.
13. \$910, for 2 mo. 11 da.
14. \$256.40, for 90 days.
15. \$4562.50, for 2 mo. 21 da.
16. \$5000, for 3 mo. 10 da.
17. \$1476.90, for 127 days.

18. What is the amount of \$3725 from Oct. 10, 1880, to May 15, 1882, at $4\frac{1}{2}\%$?

METHOD BY CANCELLATION.

421. This method will commend itself to the judgment of the practical teacher, on account of its simplicity and its application to *any rate and time.*

422. The following *Formulas* cover all cases of simple interest that can arise:

1. When the time is expressed in years,

$$\textit{Principal} \times \textit{Rate}\ \% \times \textit{Time} = \textit{Interest}$$ (already illustrated).

2. When the time is expressed in months,

$$\frac{\textit{Principal} \times \textit{Rate}\ \% \times \textit{Time}}{12} = \textit{Interest.}$$

3. When the time is expressed in days,

$$\frac{\textit{Principal} \times \textit{Rate}\ \% \times \textit{Time}}{360} = \textit{Interest.}$$

WRITTEN EXERCISES.

423. 1. What is the interest of \$92.40 for 1 yr. 4 mo., at 6%?

OPERATION.

	\$92.40	Prin.
	$.\cancel{0}\cancel{6}$	Rate.
$\cancel{1}\cancel{2}$	$\cancel{1}\cancel{6}$ 8	Time.
	\$7.392	Int.

EXPLANATION. Since the interest on any sum for 1 yr., at 6%, is .06 of the principal, \$92.40 × .06 gives the int. for 1 yr., or 12 mo. This product divided by 12 gives the int. for 1 mo., which multiplied by 16, the number of months in the given time, gives \$7.39, the required *interest*.

Or, $\frac{16}{12}$ of 1 year's interest equals the required interest.

2. Find the interest of \$150 for 36 days, at 7%.

OPERATION.

	$\$15\cancel{0}$
	.07
$\cancel{3}\cancel{6}\cancel{0}$	$\cancel{3}\cancel{6}$
	\$1.05

EXPLANATION. Since the int. on any sum for 1 yr., at 7%, is .07 of the principal, \$150 × .07 gives the int. for 1 yr., or 360 da. This divided by 360 gives the int. for 1 da., which multiplied by 36 gives \$1.05, the required interest.

Or, $\frac{36}{360}$ of the int. for 1 yr. gives the required int.

RULE. 1. *On the right of a vertical line, place the principal, rate, and time. If the time is in years, place 1 on the left; if in months, place 12 on the left; if in days, 360, or its factors, 12 and 30. For "Exact Interest"* (**424**) *use 365.*

2. *Cancel equal factors, if any, on opposite sides of the line, and the product of the remaining factors on the right, divided by the factor on the left, if any, will give the required interest.*

PROOF. *Place the answer obtained on the left, in the statement as first written, and if the work all cancels, it is correct: but if anything remains on either side of the line, the answer is wrong.*

Some examples do not admit of cancellation, but by preserving the *method* the operations will be correctly indicated.

3. Find the interest of $\$40\frac{2}{3}$ for 7 mo. 10 da., at $\frac{3}{4}\%$ a mo.

EXPLANATION. $\$40\frac{2}{3}$ is $\$\frac{122}{3}$; 7 mo. 10 da. is $7\frac{1}{3}$, or $\frac{22}{3}$ mo., and $\frac{3}{4}\%$ a mo. is 9% a year. Or, the time may be reduced to days, 220 da., and 360 placed on the left.

The pupil is already familiar with the arrangement of placing numerators, or dividends, on the right, and denominators, or divisors, on the left of the vertical line. (**181.**)

OPERATION.

$$\begin{array}{r|l} \cancel{3} & \cancel{122}\ {}^{61} \\ {}^{4}\ \cancel{12} & .\cancel{09} \\ 3 & \cancel{22}\ {}^{11} \\ \hline 3 & 6.71 \\ \hline & \$2.236 \end{array}$$

4. Find the interest of \$218 for 9 mo. 18 da., at 7%, and for 3 mo. 12 da., at $7\frac{1}{2}\%$.

EXPLANATION. 9 mo. 18 da. is $9\frac{3}{5}$, or 9.6 mo., or 288 days; 3 mo. 12 da. is $3\frac{2}{5}$, 3.4 mo., or 102 days. If the *rate* is a mixed number, it may also be expressed in the form of a common or a decimal fraction. Thus, $7\frac{1}{2}\%$ is $\frac{15}{2}\%$, or .075. Either form may be used.

OPERATION.

$$\begin{array}{r|l} & \$218 \\ & .07 \\ \cancel{12} & \cancel{9.6}\ {}^{8} \\ \hline & \$12.208 \end{array} \qquad \begin{array}{r|l} & \$109 \\ & \$\cancel{218} \\ \cancel{2} & \cancel{15} \\ {}^{4}\ \cancel{360} & \cancel{102}\ {}^{17} \\ \hline 4 & 18.53 \\ \hline & \$4.63 \end{array}$$

5. Find the interest of \$750 for 14 da., at $1\frac{1}{2}\%$ a mo.; for 4 mo. 24 da., at $12\frac{1}{2}\%$; for 7 mo. 6 da., at $\frac{2}{3}\%$ a month.

OPERATION.

$$\begin{array}{r|l} & \$75\cancel{0} \\ & .\cancel{18} \\ \cancel{360} & \cancel{14}\ {}^{7} \\ \hline & \$5.25 \end{array} \qquad \begin{array}{r|l} & \$750 \\ & .125 \\ \cancel{12} & \cancel{4.8}\ {}^{.4} \\ \hline & \$37.50 \end{array} \qquad \begin{array}{r|l} & \$750 \\ & .08 \\ \cancel{12} & \cancel{7.2}\ {}^{.6} \\ \hline & \$36 \end{array}$$

The explanations already given of similar examples are sufficient to make the above operations fully understood.

Find the interest, by either method, of

6. \$420, for 2 yr. 4 mo., at 7%; at 8%; at 10%.
7. \$64.60, for 10 mo. 21 da., at 6%; at $5\frac{1}{2}\%$; at 4%.
8. \$1000, for 9 mo. 12 da., at 2% a mo.; at $\frac{3}{4}\%$ a mo.
9. \$175, for 60 days, at 7%; at 10%; at $\frac{1}{2}\%$ a mo.
10. \$364.50, for 19 da., at 2% a mo.; at $\frac{7}{8}\%$ a mo.

11. \$2364.80, for 90 da., at $6\frac{1}{4}\%$; for 63 da., at $5\frac{1}{2}\%$.

12. \$540, for 6 yr. 5 mo., at 7%; for 3 yr. 6 mo., at 8%.

13. \$2500, for 84 da., at $1\frac{1}{4}\%$ a mo.; for 65 da., at $1\frac{1}{2}\%$ a month; for 63 da., at 6%.

14. What is the amount of \$4500, for 20 da., at 8%?

15. What is the amount of \$360.40, for 1 yr. 24 da., at 1% a mo.? At $\frac{3}{4}\%$ a mo.?

16. What is the amount of \$5000, for 63 da., at 8%?

17. If \$2150 is placed at interest May 10, 1877, what amount will be due Jan. 1, 1881, at 6%?

18. If a man borrows \$6500 in New York, and loans it in Michigan, at legal interest, what is his gain in 1 yr. 5 mo.?

19. If I borrow \$4550 in Connecticut, and loan it in Texas, what do I gain in 3 yr. 9 mo.?

20. A note for \$900, on int., 6%, after 3 mo., was given May 10, 1879, and paid Sept. 4, 1881. What was the amt. due?

21. What is the interest on a note for \$500, dated June 10, 1880, and paid Jan. 3, 1881, at 1% a month?

22. What amount is due on a note for \$125 that has run 60 days, with 7% interest?

424. Exact Interest is obtained by reckoning 365 days to the year, instead of 360 da., the latter being 5 da., $\frac{5}{365}$ or $\frac{1}{73}$ *less* than a *common* year; 6 da., $\frac{6}{366}$ or $\frac{1}{61}$ *less* than a *Leap Year.*

For example, the *exact* interest of any sum for 45 da. is $\frac{45}{365}$ of the interest for 1 year; but this is $\frac{1}{73}$ less than that obtained by the ordinary methods. Hence,

RULE. *Find the interest for years and aliquot parts of a year by the common method, and for days take as many 365ths of 1 year's interest as there are days.* Or,

When the time in days is less than 1 year, find the interest by the ordinary method; then diminish this result by $\frac{1}{73}$ for a common year, or by $\frac{1}{61}$ for a leap year.

WRITTEN EXERCISES.

425. 1. What is the exact interest of \$960, for 63 days, at 8% ?

OPERATION.

$$\begin{array}{r|l} & \overset{\$192}{\cancel{\$960}} \\ 73 & .08 \\ \cancel{\$365} & 63 \\ \hline 73 & 967.68 \\ & 13.256 \end{array} \qquad \text{Or} \qquad \begin{array}{r|l} & \overset{\$8}{\cancel{\$960}} \\ & .08 \\ \cancel{\$60} & \cancel{\$3} \quad 21 \\ \hline & \$13.44 \end{array}$$

\$13.44 by common method;

$\$13.44 - (\frac{13.44}{73}) = \$13.256$, *exact int.*

Find the *exact* interest, by either method,

2. Of \$450, for 75 da., at 6%.
3. Of \$1250, for 104 da., at 7%.
4. Of \$650.50, for 93 da., at 8%.
5. Of \$250, for 33 da., at 6%.
6. Of \$384.80, for 28 da., at $5\frac{1}{2}$%.
7. Of \$816.40, for 126 da., at $6\frac{1}{4}$%.
8. Find the interest on a \$1000 U. S. Bond, at 5%, from May 1 to Oct. 20. From April 10 to Aug. 16.
9. Find the difference between the *exact* interest of \$845, at 8%, for 1 yr. 2 mo. 24 da., and the interest computed by the ordinary methods.
10. Find the interest on \$2500, at $5\frac{1}{2}$%, from Oct. 10, 1880 to May 9, 1881.
11. What is the difference between common and exact interest of \$975, at 6%, from April 4, 1881 to Aug. 10, 1882 ?
12. What is the exact amount of \$584.50, at 10%, from Jan. 1, 1879 to March 12, 1880?
13. What is the interest, at 6%, on three \$500 U. S. Bonds from March 1 to Sept. 10 ?
14. What is the exact amount of \$1200, at 8%, from June 5 to Sept. 15 ?

PROBLEMS IN INTEREST.

WRITTEN EXERCISES.

426. ***To find the Principal, when the interest or the amount, and the rate and the time are given.***

1. What principal will gain $105 in 2 yr. 4 mo., at 6%?

EXPLANATION. — The int. of any sum of money for a given time, at a given rate, is the int. of $1 taken as many times as there are dollars in the principal; and since the int. of $1 for $2\frac{1}{3}$ yr. is $.14, the principal is as many times $1 as $.14 is contained times in $105, which is 750 times; 750 times $1 equals $750, the required *principal.*

OPERATION.

Int. on $1, for $2\frac{1}{3}$ yr., at 6% = $.14.

$105 ÷ $.14 = 750 times.

$1 × 750 = $750, Prin.

2. What sum will *amount* to $855 in 2 yr. 4 mo., at 6%?

Since the am't is the principal plus the int., the am't must contain $1, plus the int. for the given time and rate, as many times as there are dollars in the principal. Hence,

OPERATION.

Am't of $1 for $2\frac{1}{3}$ yr., at 6% = $1.14.

$855 ÷ $1.14 = 750 times.

$1 × 750 = $750, Prin.

$$\text{FORMULAS:}\left\{\begin{array}{ll} 1. & \dfrac{\textit{Interest}}{\$1 \times \textit{Rate}\,\% \times \textit{Time}} \\[2ex] 2. & \dfrac{\textit{Amount}}{\$1 + \$1 \times \textit{Rate}\,\% \times \textit{Time}} \end{array}\right\} = \textit{Principal.}$$

What *Principal*

3. Will gain $218.40 in 4 yr. 8 mo., at 6%? At 8%?
4. Will gain $357 in 3 yr. 6 mo. 15 da., at 8%? At 10%?
5. Will gain $1290.78 in 2 yr. 4 mo. 5 da., at 7%? At 4%?
6. Will amount to $228.60 in 2 yr. 4 mo. 18 da., at 6%?
7. Will amount to $3160.87 in 7 mo. 7 da., at 7%?

427. ***To find the Rate when the principal, interest, and time are given.***

1. At what rate will \$640 gain \$192 int. in 3 yr. 9 mo.?

OPERATION.

Int. of \$640, at 1% for $3\frac{3}{4}$ yr. = \$24.

\$192 ÷ \$24 = 8 times.

1% × 8 = 8%, Rate.

EXPLANATION.— The int. is as many times 1% of the given principal as there are units in the rate.

Since the int. of \$640 for $3\frac{3}{4}$ yr. at 1% is \$24, the rate is as many times 1% of the principal as \$24 is contained times in \$192, which is 8 times; 8 times 1% equals 8%, the required *rate.* Hence,

FORMULA: *Interest* ÷ (*Principal* × 1% × *Time*) = *Rate %.*

At what *Rate per cent.*

2. Will \$325.41 gain \$54.235 in 3 yr. 4 mo.?
3. Will \$4180 gain \$189.377 in 7 mo. 23 da.?
4. Will \$3325 gain \$119.70 in 10 mo. 24 da.?
5. Will \$844.75 amount to \$910.22 in 93 da.?
6. At what rate per *month* will \$1200 gain \$24 in 60 da.?
7. A man invests \$9640 so that it yields him an annual income of \$1156.80. What is the rate of interest?
8. At what rate per annum will any sum double itself in 5 yr.? In $8\frac{1}{3}$ yr.? In 12 yr. 6 mo.?

At 1% any sum will double itself in 100 yr. To double itself in 5 yr., the rate must be as many times 1% as 100 yr. is times greater than 5 yr. 100 yr. ÷ 5 yr. = 20. Hence, the required rate is 20%.

9. What is the rate of interest when \$5000 yields an income of \$125 a month?
10. A note for \$320 with interest from May 10, 1879 to Sept. 4, 1881 amounted to \$365.48. What was the rate of interest?
11. At what rate will \$560 in 2 yr. produce the same interest as \$672, at 5%, in 3 yr.?

428. ***To find the Time, when the principal, interest, and rate are given.***

1. In what time will \$360 gain \$88.20, at 7%?

OPERATION.

$$\$360 \times .07 = \$25.20, \text{ Int. for 1 year.}$$
$$\$88.20 \div \$25.20 = 3.5 \text{ times.}$$
$$1 \text{ yr.} \times 3.5 = 3 \text{ yr. } 6 \text{ mo., Time.}$$

EXPLANATION. Since the int. is the product of three factors, viz., the principal, rate, and time, the time must be the quotient of the interest divided by the product of the other two factors, viz., the principal and the rate %. Since the product of these two factors is \$25.20, or the int. on the principal for 1 yr., the time is as many years as \$25.20 is contained times in \$88.20, which is 3.5 times; 3.5 times 1 yr. equals $3\frac{1}{2}$ yr., or 3 yr. 6 mo., the required *time*. Hence,

FORMULA: *Interest* ÷ (*Prin.* × *Rate %*) = *Time.*

In what *Time*

2. Will \$415.50 gain \$55.40, at 10%?
3. Will \$3703.92 gain \$418.23, at 8%?
4. Will \$530 amount to \$602.875, at 6%?
5. Will \$512.60 amount to \$538.32, at 7%?
6. In what time will any sum of money double itself, at simple interest, at 5%? $6\frac{1}{4}$%? 8%? 7%? 10%?

At 5%, it will require as many years as 5% is contained times in 100%, or 20 yr.

7. In what time will the interest of \$500, at 6%, equal the principal? Equal *one-half* the principal? *Twice* the principal?
8. In what time will \$1203.03 amount to \$1367.84, at 6%?
9. A certain sum of money, at 5%, in 3 yr. 6 mo., gained \$147 interest. In what time will the same sum of money gain \$115.20, at 6%?
10. How long must I keep on deposit \$932.50, at 8%, to pay a debt of \$1066.16?

429. Bankers, merchants, and business men, very generally, compute interest, both *simple* and *compound*, by means of tables. The following table shows the simple interest of $1, or the number of hundredths to be taken of any principal, at 5, 6, 7, and 8%, for years, months, and days:

Yr.	5%.	6%.	7%.	8%.	Yr.	5%.	6%.	7%.	8%.
1	.05	.06	.07	.08	4	.20	.24	.28	.32
2	.10	.12	.14	.16	5	.25	.30	.35	.40
3	.15	.18	.21	.24	6	.30	.36	.42	.48
Mo.					**Mo.**				
1	.00416	.005	.00583	.00666	7	.02916	.035	.04083	.04666
2	.00833	.01	.01166	.01333	8	.03333	.04	.04666	.05333
3	.01250	.015	.01750	.02000	9	.03750	.045	.05250	.06000
4	.01666	.02	.02333	.02666	10	.04166	.05	.05833	.06666
5	.02083	.025	.02916	.03333	11	.04583	.055	.06416	.07333
6	.02500	.03	.03500	.04000					
Da.					**Da.**				
1	.00013	.00016	.00019	.00022	16	.00222	.00266	.00311	.00355
2	.00027	.00033	.00038	.00044	17	.00236	.00283	.00330	.00377
3	.00041	.00050	.00058	.00066	18	.00250	.00300	.00350	.00400
4	.00055	.00066	.00077	.00088	19	.00263	.00316	.00369	.00422
5	.00069	.00083	.00097	.00111	20	.00277	.00333	.00388	.00444
6	.00083	.00100	.00116	.00133	21	.00291	.00350	.00408	.00466
7	.00097	.00116	.00136	.00155	22	.00305	.00366	.00427	.00488
8	.00111	.00133	.00155	.00177	23	.00319	.00383	.00447	.00511
9	.00125	.00150	.00175	.00200	24	.00333	.00400	.00466	.00533
10	.00138	.00166	.00194	.00222	25	.00347	.00416	.00486	.00555
11	.00152	.00183	.00213	.00244	26	.00361	.00433	.00505	.00577
12	.00166	.00200	.00233	.00266	27	.00375	.00450	.00525	.00600
13	.00180	.00216	.00252	.00288	28	.00388	.00466	.00544	.00622
14	.00194	.00233	.00272	.00311	29	.00402	.00483	.00563	.00644
15	.00208	.00250	.00291	.00333					

1. Find, by the table, the interest of \$325.60, for 2 yr. 5 mo. 12 da., at 6%.

The int. of \$1 for the given time $= \begin{cases} \$.12 & \text{Int. for 2 yr.} \\ .025 & \text{“ “ 5 mo.} \\ .002 & \text{“ “ 12 da.} \end{cases}$

The int. of \$1 for 2 yr. 5 mo. 12 da. is \$.147. Hence,

The interest of \$325.60 for the same time is .147 of \$325.60; \$325.60 × .147 = \$47.86, the required interest.

In like manner, find, at 7%, the interest of

2. \$456, for 1 yr. 9 mo. 20 da.
3. \$1364, for 3 yr. 1 mo. 9 da.
4. \$184.75, for 11 mo. 27 da.
5. \$1500, for 4 yr. 28 da.
6. \$962½, for 6 yr. 11 mo.
7. \$537.50, for 2 yr. 10 mo.

430. Compound Interest is interest on the interest added to the principal when it becomes due, the amount forming a new principal for the next period.

The int. may be compounded, or added to the principal, at the end of any period agreed upon, as annually, semi-annually, quarterly, etc.

WRITTEN EXERCISES.

431. 1. Find the amount and the compound interest of \$650, for 2 yr. 4 mo., at 6%.

OPERATION.

\$650	Prin. for 1st yr.
1.06	
\$689	Prin. for 2d yr.
1.06	
\$730.34	Prin. for 4 mo.
1.02	
\$744.95	Amt. for 2 yr. 4 mo.
650	
\$94.95	Comp. Int.

EXPLANATION. Since the *amt.* of any sum at 6% is 1.06 of the principal, the amt. of \$650 at the end of the 1st yr. is \$689, which is the principal for the 2d year. In the same manner we find \$730.34, the amt. at the end of the 2d yr., which forms the principal for 4 mo. Since 6% for 1 yr. is 2% for 4 mo., or ⅓ of a year, the principal, \$730.34, multiplied by 1.02 gives \$744.95, the amt. at the end of 2 yr. 4 mo. The amount, \$744.95, less the principal, \$650, gives \$94.95, the *compound interest* required.

RULE.—1. *Find the amount of the principal for the first period of time, at the end of which interest is due, and make it the principal for the second period.*

2. *Find the amount of this principal for the second period of time, making it the principal for the third period, and so continue for each successive period and fraction of a period, to the end of the given time.*

3. *Subtract the given principal from the last amount; the remainder will be the compound interest.*

When the time contains months and days, find the amount of the principal to the end of the last period; then compute the simple interest on that amount for the remaining time, and add it to the last amount for the total amount.

Find the amount and the compound interest of

2. \$475, for 2 yr., at 5%; \$640, for 2 yr. 6 mo. 15 da., at 6%.
3. \$1000, for 3 yr., at 7%; \$875.50, for 4 yr. 8 mo., at 5%.
4. \$2500, for 4 yr. 4 mo., at 4%; \$3600, for 1 yr. 9 mo., at $5\frac{1}{2}$%.
5. \$750, for 5 yr. 3 mo., at 6%; \$4500, for 3 yr. 4 mo., at 8%.
6. Find the amt. of a note for \$600, due in 2 yr. 6 mo., bearing compound interest, at 6%, payable semi-annually.

If the interest is compounded semi-annually, the rate % is *one half* of the yearly rate; if quarterly, *one fourth* the yearly rate; etc.

By reducing the time to periods, and the rate proportionally, 2 yr. 6 mo. in semi-annual payments at 6% is the same as 5 yr. at 3%.

If the time were 1 yr. 6 mo. 24 da. in quarterly payments at 8%, then it would be the same as 6 yr. at 2%, and 24 da. *simple* interest, at 8% must be computed on the last amount and added for the total amount.

7. To how much more would \$1460 amount at compound interest than at simple interest in 4 yr. 4 mo., at 6%?

8. What is the compound interest of \$540, for 1 yr. 6 mo. 24 da., at 8%, interest payable quarterly?

9. At 7% interest compounded semi-annually, what will \$360 amount to in 1 yr. 9 mo. 15 da.?

432. TABLE. Showing the amount of $1, at different rates, compound interest, from 1 to 15 years.

Yr.	2½ per ct.	3 per ct.	3½ per ct.	4 per ct.	5 per ct.	6 per ct.	7 per ct.	8 per ct.
1	1.0250	1.0300	1.0350	1.0400	1.0500	1.0600	1.0700	1.0800
2	1.0506	1.0609	1.0712	1.0816	1.1025	1.1236	1.1449	1.1664
3	1.0769	1.0927	1.1087	1.1249	1.1576	1.1910	1.2250	1.2597
4	1.1038	1.1255	1.1475	1.1699	1.2155	1.2625	1.3108	1.3605
5	1.1314	1.1593	1.1877	1.2167	1.2763	1.3382	1.4026	1.4693
6	1.1597	1.1941	1.2293	1.2653	1.3401	1.4185	1.5007	1 5869
7	1.1887	1.2299	1.2723	1.3159	1.4071	1.5036	1.6058	1.7138
8	1.2184	1.2668	1.3168	1.3686	1.4775	1.5938	1.7182	1.8509
9	1.2489	1.3048	1.3629	1.4233	1.5513	1.6895	1.8385	1.9990
10	1.2801	1.3439	1.4106	1.4802	1.6289	1.7908	1.9672	2.1589
11	1.3121	1.3842	1.4600	1.5395	1.7103	1.8983	2.1049	2.3316
12	1.3449	1.4258	1.5111	2.6010	1.7959	2.0122	2.2522	2.5182
13	1.3785	1.4685	1.5640	1.6651	1.8856	2.1329	2.4098	2.7196
14	1.4130	1.5126	1.6187	1.7317	1.9799	2.2609	2.5785	2.9372
15	1.4483	1.5580	1.6753	1.8009	2.0789	2.3966	2.7590	3.1722

1. Find, by the table, the amount of $784 for 5 yr. 6 mo., at 8%, interest payable semi-annually.

EXPLANATION.—5 yr. 6 mo. = 11 halves of a year; 8% per annum = 4% semi-annually. The am't of $1 for 11 yr. at 4% is 1.5395; $784 × 1.5395 = $1206.97, the *am't;* $1206.97 − $784 = $422.97, the *compound interest.*

In like manner find the am't and the comp. int. of

2. $396, for 5 yr., at 6%.
3. $1475, for 7 yr., at 5%.
4. $860.50, for 4½ yr., at 7%.
5. $1500, for 2½ yr., at 8%.
6. $1125, for 7 yr., at 7%.
7. $575, for 6 yr. 6 mo., at 8%.
8. $2500, for 5 yr. 6 mo., at 6%.
9. $1250, for 3½ yr., at 8%.

10. What is due on a note of $500, bearing semi-annual compound interest, at 7%, 3 yr. 9 mo. from date?

433. Annual Interest is *simple interest* on the principal and on each year's interest remaining unpaid.

Annual interest is not considered legal in some States. When allowed, notes or other contracts must contain the words, "with annual interest," or "interest payable annually," as *simple interest* is not due, and cannot be collected until the principal is due.

WRITTEN EXERCISES.

434. 1. Find the amount of $860, at 6%, for 3 yr. 6 mo., interest payable annually, but remaining unpaid.

OPERATION.

Int. of $860 for 3 yr. 6 mo.	=	$180.60
" $51.60 for 4 yr. 6 mo.	=	13.93
Total Int.,		$194.53

$860 + $194.53 = $1054.53, Am't.

EXPLANATION.—At 6% the simple int. of $860, due at the end of each year, is $51.60, and for 3 yr. 6 mo. it is $180.60. $51.60, the int. due at the end of the first year, draws interest for 2 yr. 6 mo.; that due at the end of the 2d year, for 1 yr. 6 mo.; and that due at the end of the 3d year, for 6 mo. Hence, there is simple interest due on $51.60 for 2 yr. 6 mo. + 1 yr. 6 mo. + 6 mo. = 4 yr. 6 mo.; and the int. of $51.60 for 4 yr. 6 mo., at 6%, is $13.92, and the total int. is $194.52; and $860 + $194.52 = $1054.52, the am't.

What is the *annual interest* and the *amount* of

2. $1000, for 4 yr. 6 mo., at 8%?
3. $765, for 5 yr., at 7%?
4. $2640, for 3 yr. 9 mo., at 6%?
5. $186.80, for 6 yr. 3 mo., at 5%?

6. What is the difference between the annual and the compound interest of $800, for 5 yr., at 6%?

7. How much interest is due at the end of 3 yr. 8 mo., on a note for $685, at 6%, interest payable annually, but remaining unpaid?

8. What is the amount due upon a note for $900 which has run 6 yr. 4 mo., at 7% interest, payable annually?

PARTIAL PAYMENTS.

DEFINITIONS.

435. A **Promissory Note** is a written promise by one party to pay a sum of money to another party, at a specified time, or on demand, for value received.

436. The **Face** of a note is the sum promised to be paid.

437. The **Maker** or **Drawer** of a note is the party who promises to pay, and who signs the note.

438. The **Payee** of a note is the party to whom, or to whose order, the money is to be paid.

439. The **Holder** of a note is the party who has legal possession of it.

A note to be valid must contain the words "value received."

If a note contains the words "with interest," or "with use," it draws interest from date. If these words are omitted, the note will not draw interest until after it is due. If no rate is mentioned, the legal rate prevails.

440. A **Negotiable Note** is a note made payable to *bearer*, or to the order of the payee.

Without the words "or order," or the words "or bearer," inserted the note is *not negotiable*, and is payable to the person only whose name is mentioned in it.

441. A **Bankable Note** is a note made payable at a bank. The following is a simple form :

250\frac{75}{100}$.   CHICAGO, April 1, 1881.

Sixty days after date, I promise to pay JOHN HAYES, *or order, two hundred fifty* $\frac{75}{100}$ *dollars, at the Citizens' Bank, value received.*

AMOS BROWN.

442. Partial Payments are part payments of notes, bonds, or other obligations.

443. Indorsements are statements of payments, written on the back of the note or obligation, stating the time and amount of the same.

The Rule prescribed by the Supreme Court of the United States, for partial payments, and adopted by most of the States, is based upon the following

444. PRINCIPLES. I. *That payments must be applied, first, to discharge accrued interest, and then the remainder, if any, toward the discharge of the principal.*

II. *Only unpaid principal can draw interest.*

UNITED STATES RULE.

1. *Compute the interest on the given principal from the date of the note to the time of the first payment. If this payment equals or exceeds the interest due, subtract the payment from the amount, and treat the remainder as a new principal.*

2. *If any payment is less than the accrued interest, compute the interest on the same principal, to a date when the sum of the payments equals or exceeds the interest then due, and subtract the sum of the payments from the amount, and regard the remainder as a new principal.*

3. *Proceed in the same manner with the remaining payments, until the date of settlement.*

The new method here presented of working examples in partial payments, usually so tedious and perplexing, is commended to the careful consideration of teachers, as not only brief and simple, but eminently practical for business calculation, being greatly superior in both these respects to the methods generally taught in arithmetical text-books.

WRITTEN EXERCISES.

445. (1.)

$2000.00. NEW YORK, Jan. 4, 1879.

On demand, I promise to pay A. S. BARNES & Co., *or order, two thousand dollars, with interest at six per cent., value received.*

JAMES FOSTER.

The following payments were indorsed on this note:

Feb. 19, 1880, $400; June 28, 1881, $1000; Nov. 13, 1881, $520. What was due Dec. 25, 1882?

First arrange a "Time Table," as follows:

In the first column place the date of the note, the dates of the several payments in their order, and the date of settlement. In the second column place the difference between each date and the succeeding one. This operation, and reducing results to lower terms, can usually be done mentally. In the third column place the payments in their order; and in the last column place the face of the note.

YR.	MO.	DA.	DIFFERENCE BETWEEN DATES.					PAYM'TS.	PRINCIPAL.
			yr.	mo.	da.	Days.	Months.		
1879	1	4							$2000
1880	2	19	1	1	15	= 405	= 13.5	$400	$1735
1881	6	28	1	4	9	= 489	= 16.3	$1000	$876.40
1881	11	13		4	15	= 135	= 4.5	$520	$376.12
1882	12	25	1	1	12	= 402	= 13.4		$401.32 *Ans.*

The *reduction* of the difference of time between the dates can be made as required for use, and need not appear in the table. The remainder after each payment is placed in the last column as a new principal. The design of the above is to represent the completed work.

The advantage of this simple and compact arrangement is that it shows at a glance the date of each payment and the time for which interest is to be computed; and, as the work progresses, it shows the amount due after each payment, used as a new principal, and the amount due at the time of settlement.

The time may be reduced to months and *tenths* of a month (**417**, Ex. 11), or to days, as shall best be adapted to the method used in computing the interest.

To illustrate the foregoing suggestions, the work is given below in full, following the United States Rule, and using the method by cancellation (**423**, Rule), leaving the cancelling to the ingenuity of the pupil. The amount due after each payment should be placed in the last column, and regarded as a new principal, and the previous work may be erased, so that when the work is complete, only the table will be left, the last amount in the last column being the required answer.

With the table before him, let the pupil compute the interest upon the principal from the date of the note to the time of the first payment, or for 1 yr. 1 mo. 15 da., equal to 13.5 months, etc.

FIRST STEP.

	\$2000	Prin.
	.06	Rate.
12	13.5	Time.
	\$135	Int.
	\$2000	
	\$2135	Amt.
	\$400	Payment.
	\$1735	1st Rem.

SECOND STEP.

	\$1735	1st new Prin.
	.06	Rate.
12	16.3	Time.
	\$141.40	Int.
	\$1735	
	\$1876.40	Amt.
	\$1000	Payment.
	\$876.40	2d Rem.

THIRD STEP.

	$876.40	2d new Prin.
	.06	Rate.
360	135	Time.
	$19.72	Int.
	$876.40	
	$896.12	Amt.
	$520	Payment.
	$376.12	3d Rem.

FOURTH STEP.

	$376.12	2d new Prin.
	.06	Rate.
12	13.4	Time.
	$25.20	Int.
	$376.12	
Ans.	$401.32	Last Amt.

(2.)

$675.00. SAN FRANCISCO, Aug. 1, 1878.

One day after date, I promise to pay T. H. ROSE, *or order, six hundred seventy-five dollars, with interest at* 8 *per cent, for value received.*

CHARLES H. ALLEN.

The following payments were indorsed on this note: Oct. 25, 1879, $40; Dec. 25, 1880, $122.60; June 14, 1881, $216.80.

What was due Jan. 2, 1882?

YR.	MO.	DA.	DIFFERENCE BETWEEN DATES.					PAYMENTS.	PRIN.
1878	8	1	yr.	mo.	da.	Days.	Months.		$675
1879	10	25	1	2	24	= 444	= 14.8	$40	$642
1880	12	25	1	2	0	= 420	= 14	$122.60	
1881	6	14		5	19	= 169	= $5.6\frac{1}{3}$	$216.80	$449.31
1882	1	2		6	18	= 198	= 6.6		$469.08 *Ans.*

In this example, the first payment is *less* than the interest due, and we compute the interest to the time of the second payment, and subtract the sum of the two payments.

3. A note for \$850, dated St. Louis, Feb. 1, 1878, had payments indorsed upon it as follows :

July 25, 1878, \$200. April 1, 1880, \$50.
June 13, 1879, \$100. Dec. 10, 1880, \$175.

What was due Aug. 12, 1881, interest 6%?

4. Take the same example, and find what was due May 6, 1881, interest 8%.

(5.)

\$784$\frac{60}{100}$. BOSTON, May 9, 1880.

Three months after date, we promise to pay JOHN ABBOTT, *or order, seven hundred eighty-four* $\frac{60}{100}$ *dollars, with 7 per cent. interest, value received.* NICHOLS & BROWN.

Indorsed as follows: Oct. 1, 1880, \$100 ; June 15, 1881, \$125 ; Nov. 2, 1881, \$300. Find the amount due April 4, 1882.

6. A mortgage for \$5400 was dated New Orleans, Jan. 1, 1879, and indorsed as follows : May 22, 1880, \$1200; Feb. 9, 1881, \$150; Oct. 28, 1881, \$1500. What was due March 1, 1882, interest 5% ?

7. What would be due upon the same, if settled Aug. 8, 1882, interest at 8%?

8. I held a bond against Ira Fox, dated May 1, 1879, for \$4000, on interest, at 6%. The following payments were indorsed on this bond :

May 21, 1880, \$800 ; June 10, 1881, \$1200 ; Aug. 10, 1882, \$1500. What was due May 1, 1883?

446. Merchants in computing interest on notes and accounts, running a year or less, or that require to be settled annually, semi-annually, or quarterly, on which partial payments have been made, often use the following method, called the

Mercantile Rule.

1. *Find the amount of the note or debt from the time interest commenced to the time of settlement.*

2. *Find the amount of each payment from the time it was made to the time of settlement.*

3. *From the amount of the note or debt subtract the sum of the amounts of the payments; the remainder will be the amount due.*

In the application of this rule, the time should be reduced to *days*, and the interest computed according to **424**.

1. Find the amount due on a note for \$940, at 6%, dated Jan. 1, 1881, on which were the following indorsements: March 10, \$250; May 25, \$175; Sept. 12, \$350. What was due Nov. 20, 1881?

OPERATION.

Amt. of \$940	from Jan. 1	to Nov. 20,	323 da.,			\$989.91
" " \$250	" Mar. 10	" "	255 "	\$260.48		
" " \$175	" May 25	" "	179 "	\$180.15		
" " \$350	" Sept. 12	" "	69 "	\$353.97		\$794.60
				Amt. due Nov. 20, 1881,		\$195.31

2. On a debt of \$1250, due June 1, 1881, payments were made as follows:

July 10, \$350; Oct. 26, \$228;
Sept. 9, \$155.50; Dec. 16, \$410.75.

What was due April 15, 1882, at 7%?

3. A note was given for \$3462, Jan. 4, 1880, at 7%. Indorsements: April 3, \$500; Aug. 12, \$650; Oct. 5, \$1000. What was due 1 year from date?

4. What would have been due upon the same note Nov. 10, interest at 8%?

DISCOUNT.

447. Discount is a deduction from the price of goods, or from the amount of a debt, a note, or other obligation, for payment before it is due.

448. The **Present Worth** of any debt or note is the amount of it less the discount.

449. The **True Present Worth** of a debt payable at a future time without interest is the sum which at legal interest would amount to the debt at the time it becomes due.

450. The **True Discount** is the difference between the amount of the debt and the true present worth.

451. The corresponding terms of True Discount and Percentage are as follows:

1. The **True Present Worth** is the *Base.*
2. The **Rate Per Cent.** is the *Rate.*
3. The **True Discount** is the *Percentage.*
4. The **Whole Debt** is the *Amount.*

WRITTEN EXERCISES.

452. ***To find the Present Worth and True Discount of a debt, payable at a future time.***

1. Find the present worth and the true discount of $353.60, payable in 1 yr. 6 mo., the rate being 7%.

EXPLANATION.—Since $1.105 is the amount of $1, at interest for 1 yr. 6 mo. at 7%, $353.60 is the amount of as many dollars for the same time and rate, as $1.105 is contained times in $353.60, or $320, the *present worth;* and $353.60−$320 = $33.60, the *discount.*

FORMULAS: 1. *Debt ÷ Amount of $1 = Present Worth.*
2. *Debt − Present Worth = Discount.*

Find the *Present Worth* and *True Discount*

2. Of \$475.50, at 7%, due in 2 yr. 9 mo.; at 6%, due in 9 mo.

3. Of \$535.60, at 6%, due in 6 mo.; at 8%, due in 10 mo.

4. Of \$860, at 6½%, due in 90 da.; at 10%, due in 6 mo. 21 da.

5. Of \$1575, at 5%, due in 1 yr. 3 mo. 15 da.; at 7%; at 8%.

6. What is the difference between the interest and true discount of \$1650, at 6%, due in 8 mo.?

7. What sum, due 73 da. hence, at 7%, should be discounted, so that the present payment may be \$900?

8. A merchant bought a bill of goods on 6 mo. credit amounting to \$1450. What will he gain by present payment of the bill, if allowed 5% off, money being worth ¾% a month?

9. Which is the better, to buy flour at \$8 a barrel, on 6 mo. credit, or \$7.50 cash, money being worth 8%?

10. What is the present worth of a debt of \$487.50 due in 8 mo. 24 da., at 10%?

11. Which is worth the most, \$640 in 12 mo., \$620 in 6 mo., or \$600 cash, money being worth 8%?

12. Bought a house for \$3786 ready money, and sold it for \$5250, payable in 1 yr. 6 mo. How much would be gained in ready money, discounting at the rate of 8%? How much, discounting at the rate of 6%?

13. A merchant bought goods to the amount of \$6300, ⅓ payable in 3 mo., ⅓ in 6 mo., and ⅓ in 9 mo. Money being worth 8%, what sum would have paid the bill at the time of purchase?

14. Jones holds two notes against Smith, one for \$356, due May 6, 1882, and the other for \$175.50, due Sept. 25, 1882; how much money will cancel both notes Oct. 11, 1881, discounting at the rate of 6%?

Bank Discount.

453. **Bank Discount** is a deduction usually made by banks for paying a note before it is due. This deduction is the interest on the face of the note for the time it has to run, including three additional days, called *Days of Grace.*

454. **Days of Grace** are three days allowed to be added to the time specified in the note, before it is legally due.

455. The **Proceeds** or **Avails** of a note is its present worth when discounted; that is, its face less the discount.

456. The **Term of Discount** is the time the note has to run after being discounted.

457. The **Maturity** of a note is the expiration of the time it has to run, including the days of grace.

When a note falls due on Sunday or a legal holiday, it must be paid the day previous.

458. A **Protest** is a statement, made by a *notary public,* giving legal notice to the maker and indorsers of a note, of its non-payment.

If a note is not protested on the third day of grace, the indorsers are released from all obligation to pay it.

459. The corresponding terms of Bank Discount and Percentage are as follows:

1. The **Face of the Note** is the *Base.*
2. The **Rate Per Cent.** is the *Rate.*
3. The **Bank Discount** is the *Percentage.*

WRITTEN EXERCISES.

460. ***To find the Bank Discount and Proceeds of a note.***

1. Find the bank discount and the proceeds of a note for \$684, due in 90 days, at 7%?

EXPLANATION.—The TERM OF DISCOUNT is 93 da., and the int. of \$684, for 93 da., at 7%, reckoning 365 da. to the year, is \$12.20, the BANK DISCOUNT. Reckoning 360 da. to the year,* the bank discount is \$12.37. The *face of the note*, minus the *bank discount*, \$12.20, or \$12.37, gives the *proceeds*, \$671.80, or \$671.63. Hence,

FORMULAS:
1. *Face* × *Rate %* × *Term of Discount* = *Bank Discount*. (**417,** 1.)
2. *Face* − *Bank Discount* = *Proceeds*. (**337,** 3.)

If the note is on interest, find the discount on the *amount* of the note at maturity.

Find the *Bank Discount* and the *Proceeds*, reckoning 360 da. to the year,

2. Of \$540, for 3 mo. 15 da., at 6%.
3. Of \$875, for 60 da., at 6%. Of \$620, for 6 mo., at 8%.
4. Of \$2144.50, for 3 mo. 10 da., at 6%. At 8%.
5. Of \$312.80, at 6%, for 30 da. For 2 mo. 15 da.
6. Of 2500, at 7%, for 3 mo. Of \$1150, at 6%, for 2 mo.
7. What is the difference between the *bank* and the *true* discount on a note of \$1250, at 6%, payable in 60 da.?
8. Find the bank discount and the proceeds of a note for \$1500, dated Sept. 10, 1880, payable in 6 mo., with interest, at 8%, and discounted Nov. 25, 1880, at the same rate.
9. What are the proceeds of a note for \$750, due in 90 da., bearing interest at 8%, and discounted at a bank at the same rate?
10. A note for 3 mo., dated Aug. 14, 1881, for \$862.50, was discounted, at 6%, Sept. 25. What were the proceeds?

* The year is considered as having only 360 days, unless otherwise stated

Find the *Date of Maturity*, the *Term of Discount*, and the *Proceeds* of the following notes, reckoning 365 days to the year:

11.

456\frac{50}{100}$. COLUMBIA, MO., July 1, 1881.

Four months after date I promise to pay to the order of WM. A. CAUTHORN, *four hundred fifty-six* $\frac{50}{100}$ *dollars, with interest, value received.* JAMES B. EADS.

Discounted Aug. 9, at 6%.

12.

$735^{00}. PHILADELPHIA, Oct. 4, 1880.

Sixty days after date I promise to pay to GEO. H. STUART, *or order, Seven hundred thirty-five dollars, at the Commercial Bank, value received, without defalcation.*

JOHN B. JONES.

Discounted Oct. 25, at 6%.

13.

$1000. ST. LOUIS, May 14, 1882.

Six months after date we promise to pay J. M. BAKER, *or order, One thousand dollars, value received.*

ELLIS, MCKNIGHT & CO.

Discounted at the Marine Bank, Sept. 1, at 1% a month.

14.

296\frac{75}{100}$. CHARLESTON, S. C., March 8, 1881.

Three months after date I promise to pay to the order of WM. THOMPSON, *two hundred ninety-six* $\frac{75}{100}$ *dollars, with 7% interest, value received.* AMOS KENDALL.

Discounted at the Planters' Bank, April 15, at same rate.

15.

$1260. New York, Jan. 1, 1880.

Thirty days after date we promise to pay to Henry Curtis, *or order, twelve hundred sixty dollars, at the Chemical Bank, value received.* A. D. Randolph & Co.

Discounted Jan. 1, at 6%.

16. I hold a note for $1176.40, dated Oct. 18, 1880, and due in 3 mo., bearing interest at 8%. If discounted at a bank Nov. 10, 1880, at $1\frac{1}{2}$% a month, what will be the proceeds?

17. A note for $5000, dated June 11, 1881, and payable in 6 mo., with interest at 10%, was discounted Aug. 1, at 2% a month. What were the discount and the proceeds?

461. ***To find the Face of a note, the proceeds, time, and rate being given.***

1. For what sum must a note be drawn, payable in 60 da., at 8%, that when discounted the proceeds may be $824?

Explanation.—Since the bank discount of $1 for 63 da., at 8%, is $.014, the *proceeds* of $1 is $1 – $.014, or $.986; hence, $824 is the proceeds of as many dollars as $.986 is contained times in $824, or $835.70, the *face* of the note. Hence,

Formula: $\dfrac{\textit{Proceeds of Note}}{\textit{Proceeds of \$1}} = \textit{Face of Note.}$

Required the *Face* of a note, that the proceeds may be

2. $810.60, for 60 da., at 7%; $601.09, for 6 mo., at 6%.

3. $1478.82, for 30 da., at 6%; $2072.60, for 40 da., at 8%.

4. $496, for 6 mo., at 2% a mo.; $575, for 60 da., at $1\frac{1}{2}$% a month.

5. Bought goods for $1621.20 cash, but gave a note, payable in 60 da., at 7% bank discount. What was its face?

6. The avails of a 4 mo. note, discounted at 8%, were \$875.40. What was the face of the note?

7. I wish to raise \$1275.25 at a bank, by a note for 15 da., to be discounted at 8%. For what sum must I draw the note?

8. For what sum must a note be drawn, dated Aug. 10, 1881, payable in 90 da., that when discounted at 7%, Sept. 15, the proceeds may be \$640?

9. For what sum must a produce dealer make his note at 4 mo., that when discounted by a bank at 6%, the proceeds will pay for 1500 bu. of wheat, at \$1.62½ a bushel?

10. For what sum must a note be drawn, dated Aug. 20, 1881, payable in 5 mo., that when discounted at a bank Oct. 3, at 6%, the proceeds will pay for 328 tons of coal at \$4.50 a ton?

EXCHANGE.

DEFINITIONS.

462. Exchange is a method of paying debts to persons at a distance by means of written orders, called *Bills of Exchange* or *Drafts.*

In this way the inconvenience and risk of remitting money to persons at a distance are obviated.

Exchange is of two kinds—*Domestic* or *Inland,* and *Foreign.*

463. Domestic or Inland Exchange is that which is made between different places in the same country.

464. Foreign Exchange is that which is made between different countries.

465. A **Bill of Exchange** is a written order for the payment of money to a certain person at a specified time. In domestic exchange it is called a **Draft.**

These bills are usually drawn in duplicate or triplicate, each copy being valid until the amount is paid. These copies may be sent by different conveyances, so as to avoid miscarriage; or one may be retained by the buyer of the bill for the sake of security. Together these copies constitute a **Set of Exchange.**

466. A **Sight Bill** is one payable *at sight*, or on presentation.

467. A **Time Bill** is one that is payable at a specified time, either after date, or after sight.

468. The **Drawer of a Bill** is the party who draws it; the **Drawee** is the party upon whom it is drawn; the **Payee** is the party in whose favor it is drawn, or to whom the money is directed to be paid.

The payee may by *indorsement* transfer the payment to any other person. A *Special Indorsement* is an order to pay the bill to some particular person called the **Indorsee.**

469. The **Acceptance** of a bill is the consent of the *drawee* to pay it at maturity.

This he gives by writing across the face of the bill "*accepted,*" with the date of his signature. The drawee then becomes the *acceptor*, and the bill an *acceptance*. Days of grace are usually allowed on bills of exchange, as on promissory notes.

470. The **Par of Exchange** is the established value of the monetary unit of one country as compared with that of another.

The *Intrinsic Par of Exchange* is the standard value of the monetary unit, depending upon its comparative weight and purity; the *Commercial Par of Exchange* is the market value at any particular time, dependent on the *Course of Exchange*.

471. The **Rate** or **Course of Exchange** is the current price of Bills of Exchange upon different countries.

The rate of exchange depends on the course of trade and the commercial credit of the places between which the exchange is effected. Exchange is *at par, above par,* or *below par,* according to the fluctuations of trade between the two countries or places. If, for example, as between London and New York, the balance of indebtedness is in favor of the former place, the exchange at the latter will be at a *premium,* or above par; if the reverse, at a *discount,* or below par.

DOMESTIC OR INLAND EXCHANGE.

472. FORM OF AN INLAND BILL OR DRAFT.

600\frac{00}{}$. NEW YORK, June 10, 1881.

At sight, pay to the order of LEVI WOODMAN *Six hundred dollars, value received, and charge to the account of*

JOHN H. BOWEN.

TO BROWN BROS.,
Chicago, Ill.

In time drafts, instead of the words "at sight," "— days after sight," or "— days after date" are used. *After sight* is equivalent to *after acceptance.*

For inland bills or drafts the rate oi exchange is expressed by the rate of premium or discount at the time, depending on the course of trade.

WRITTEN EXERCISES.

473. ***To find the Cost of a sight or time draft.***

1. What is the cost of a sight draft on Chicago for $1680, at $1\frac{1}{8}\%$ premium?

EXPLANATION.—Since the rate of premium is $1\frac{1}{8}\%$, the course of exchange is $1.01\frac{1}{8}$, and the cost of exchange for $1 is 1.01\frac{1}{8}$; hence, the cost of $1680 is 1680 times 1.01\frac{1}{8}$, or $1698.90.

2. What is the cost of a draft on Portland for $1360, payable in 60 da. after sight, at 6% interest, exchange at $\frac{7}{8}$% premium?

EXPLANATION.—Since the premium is $\frac{7}{8}$%, the rate of exchange is $1.00\frac{7}{8}$, and the *bank discount* for 63 da. is .0105; hence, the cost of exchange for $1 is $1.00875 minus $.0105, or $.99825; and the cost of $1360 will be 1360 times $.99825, or $1357.62. Hence,

FORMULA: *Face* × *Cost of $1 Exchange* = *Cost of Draft.*

Find the *cost* of sight drafts

3. For $650, premium 2%.
4. For $1500, premium $\frac{3}{4}$%.
5. For $275.50, premium $1\frac{1}{2}$%.
6. For $3240, discount $\frac{1}{2}$%.
7. For $500, discount $1\frac{1}{4}$%.
8. For $395.75, discount $\frac{4}{5}$%.

Find the *cost* of drafts

9. For $380, premium $\frac{1}{2}$%, time 60 da., interest 4%.
10. For $2500, premium $\frac{3}{4}$%, time 70 da., interest 5%.
11. For $1462, discount $1\frac{1}{4}$%, time 30 da., interest 6%.

12. Find the cost of a sight draft on St. Louis for $2160, at $1\frac{3}{8}$% premium.

13. Find the cost of a draft in New York, on San Francisco, at 90 da., for $4684, the course of exchange being $1.01\frac{5}{8}$.

14. What will be the cost of a draft in Chicago, on New Orleans, at 30 da. sight, for $1920, at $\frac{3}{8}$% discount, interest 7%?

474. ***To find the Face of a sight or time draft.***

1. How large a draft can be bought for $3000, exchange at $1\frac{1}{2}$% discount?

EXPLANATION.—Since the rate of discount is $1\frac{1}{2}$%, the cost of exchange for $1 is $.985; hence, the face of a draft that will cost $3000, will be as many dollars as $.985 is contained times in $3000, or $3045. 69.

2. Find the face of a draft on St. Paul at 90 da., purchased for \$1250, exchange being $101\frac{1}{2}$, interest 6%.

EXPLANATION.—Since the course of exchange is $101\frac{1}{2}$, the cost of exchange for \$1 is \$1.015, if payable *at sight;* but since it is not to be paid until 93 da., bank discount is allowed for that time at 6%, that is, \$.0155 for every dollar; hence, the cost of exchange for \$1 is \$1.015 minus \$.0155, or \$.9995; and \$1250 will purchase a draft for as many dollars as \$.9995 is contained times in \$1250, or $\$1250.62\frac{1}{2}$.

FORMULA: $\dfrac{\textit{Cost of draft}}{\textit{Cost of \$1 exchange}} = \textit{Face.}$

Find the *Face* of a draft which costs

3. \$950, premium $1\frac{3}{4}\%$.
4. \$756.40, premium $2\frac{1}{8}\%$.
5. \$375.50, premium $\frac{3}{5}\%$.
6. \$485, discount $1\frac{1}{8}\%$.
7. \$1500, discount $\frac{3}{4}\%$.
8. \$2540, discount $1\frac{3}{5}\%$.

9. What is the face of a 6 months' draft costing \$600, premium $1\frac{1}{2}\%$, interest 5%?

10. How large a draft can be bought for \$3195.20, payable in 60 da., interest 8%, exchange $101\frac{1}{4}$?

11. An agent in Boston sold a consignment of goods for \$4130, commission on the sale $2\frac{1}{2}\%$. He remitted the proceeds by draft on New York, exchange $\frac{1}{2}\%$ premium. What amount did he remit?

FOREIGN EXCHANGE.

475. FORM OF A BILL OR SET OF EXCHANGE.

£600. NEW YORK, Feb. 1, 1881.

At sight of this FIRST *of* EXCHANGE (*Second and Third of the same tenor and date unpaid*), *pay to the order of* WM. BAYSTOCK, *six hundred pounds sterling, for value received, and charge the same to the account of*

TO COHEN & CO., BROWN BROS. & CO.

London, Eng.

The foregoing is the form of the *first* bill; in the second and third bills, the only change required is the substitution of the word *second* or *third* for *first*.

Bills of Exchange are usually made payable either 3 days after sight, or 60 days after sight. Hence, sixty-day bills are quoted at a lower rate, on account of the discount.

476. Exchange with Europe is effected mainly through the great financial centres, London, Paris, Antwerp, Berlin, Hamburg, Frankfort, and Amsterdam.

In exchange on Paris, Antwerp, and Switzerland, the unit is the *franc;* on Amsterdam, the unit is the *guilder;* and on Hamburg, Frankfort, Bremen, and Berlin, the unit is *four marks.*

477. Sterling Exchange consists of bills on any part of Great Britain.

478. Quotations of Exchange give the market value of the foreign monetary unit in U. S. money, or the U. S. unit in foreign money; thus,

Sterling Exchange,	£1	= $4.86		(market value).
Paris	"	$1	= 5.21 francs	"
Antwerp	"	$1	= 5.23 "	"
Berlin	"	4 marks	= $.94½	"
Bremen	"	"	= $.95	"
Amsterdam	"	1 guilder	= $.40½	"

479. Money of Account is the money of any particular country, consisting of the various denominations in which accounts are kept.

The Act of Congress of March 3, 1873, provides that, "The value of foreign coin, as expressed in the money of account of the United States, shall be that of the pure metal of each coin of standard value; and the values of the standard coins in circulation, of the various nations of the world, shall be estimated annually by the Director of the Mint, and be proclaimed on the first day of January, by the Secretary of the Treasury.

480. *The following Table, published by the Secretary of the Treasury, Jan. 1, 1881, shows the values in United States money of the pure gold or silver representing respectively the monetary units and standard coins of foreign countries.*

COUNTRY.	MONETARY UNIT.	STANDARD.	VALUE IN U. S. MONEY.
Austria	Florin	Silver	.40, 7
Belgium	Franc	Gold and silver	.19, 3
Bolivia	Boliviano	Silver	.82, 3
Brazil	Milreis	Gold	.54, 6
Brit. Pos. in N. A.	Dollar	Gold	$1.00
Chili	Peso	Gold and silver	.91, 2
Cuba	Peso	Gold and silver	.93, 2
Denmark	Crown	Gold	.26, 8
Ecuador	Peso	Silver	.82, 3
Egypt	Piaster	Gold	.04, 9
France	Franc	Gold and silver	.19, 3
Great Britain	Pound	Gold	4.86, 6½
Greece	Drachma	Gold and silver	.19, 3
German Empire	Mark	Gold	.23, 8
India	Rupee	Silver	.39
Italy	Lira	Gold and silver	.19, 3
Japan	Yen	Silver	.88, 8
Liberia	Dollar	Gold	1.00
Mexico	Dollar	Silver	.89, 4
Netherlands	Florin	Gold and silver	.40, 2
Norway	Crown	Gold	.26, 8
Peru	Sol	Silver	.82, 3
Portugal	Milreis	Gold	1.08
Russia	Rouble	Silver	.65, 8
Sandwich Islands	Dollar	Gold	1.00
Spain	Peseta	Gold and silver	.19, 3
Sweden	Crown	Gold	.26, 8
Switzerland	Franc	Gold and silver	.19, 3
Tripoli	Mahbub	Silver	.74, 3
Turkey	Piaster	Gold	.04, 4
U. S. of Colombia	Peso	Silver	.82, 3

WRITTEN EXERCISES.

481. ***To find the Cost of a Foreign Bill of Exchange.***

1. Find the cost of a bill of exchange on London for £427 10s. 6d., at 3 da. sight, sterling exchange being 4.87½.

EXPLANATION.—£427 10s. 6d. is £427.525; and since £1 is worth \$4.875, £427.525 is worth 427.525 times \$4.875, or \$2084.18.

2. What is the cost of a bill on Paris for 675 francs, at 5.18 francs to the dollar?

EXPLANATION.—Since 5.18 francs cost \$1, 675 francs will cost as many dollars as 5.18 francs is contained times in 675 francs, or \$130.30.

FORMULAS: 1. *Face × Value of Monetary Unit in U. S. Money* / 2. *Face ÷ Value of \$1 in Foreign Money* } = *Cost.*

3. Find the cost of a bill on Berlin for 2150 marks, at 96½.

EXPLANATION.—Since 4 marks cost \$.96½, 1 mark will cost \$.24⅛; and 2150 marks will cost \$518.69.

Find the cost of a bill

4. On London, for £520 12s. at 4.85½, brokerage ½%.
5. On Paris, for 3640 francs, at 5.22½, brokerage ¼%.
6. On Geneva, Switzerland, for 8750 francs, at 5.18¼.
7. On Amsterdam, for 6500 guilders, at 41¾, brokerage ⅛%.
8. On Berlin, for 3200 marks, at 98½.
9. What is the cost of a bill of exchange on Glasgow for £623 4s. 9d., the rate being \$4.88¼?
10. Find the cost of a bill on Hamburg for 2800 marks, at 95½, brokerage ⅛%.
11. What will be the cost of a bill on Amsterdam for 3164 guilders, quoted at 41⅛, adding brokerage at ⅛%?

482. ***To find the Face of a Bill of Exchange.***

1. What is the face of a bill on London that can be bought for \$11652, exchange selling at 4.85½?

EXPLANATION.—Since £1 cost \$4.85½, \$11652 will buy as many pounds as \$4.855 is contained times in \$11652, or £2400.

2. How large a bill on Antwerp can I buy for \$1650, when exchange is quoted at 5.20?

EXPLANATION.—Since \$1 will buy 5.20 francs, \$1650 will buy 1650 times 5.20 francs, or 8580 francs.

FORMULAS: $\left\{\begin{array}{l} \text{1. } \textit{Cost} \div \textit{Value of Monetary Unit in U. S. Money} \\ \text{2. } \textit{Cost} \times \textit{Value of \$1 in Foreign Money} \end{array}\right\} = \textit{Face.}$

3. What is the face of a bill on Dublin, bought for \$5000, exchange being 4.87?

4. Find the face of a bill on Frankfort, bought for \$2040, exchange at 96.

EXPLANATION.—Since 4 marks can be bought for \$.96, 1 mark can be bought for \$.24; and as many marks can be bought for \$2040 as \$.24 is contained times in \$2040, or 8500 marks.

5. What will be the face of a bill on Sheffield, England, that can be bought for \$6400, rate of exchange 4.87½?

6. A clothing merchant in New York gave \$4350 for a bill on Paris, at 5.18. What was its face?

7. Find the face of a bill on Amsterdam, that cost \$1043.225, exchange at 41⅞.

8. A. T. Stewart & Co. instructed their agent at Berlin to draw on them for a bill of goods of 45000 marks, exchange at 97⅛, brokerage ¼%. What did they pay in U. S. money for the goods?

9. A jeweler paid \$1600 for a bill on Geneva, Switzerland, exchange at 5.20¾, brokerage ¼%. What was its face?

EQUATION OF PAYMENTS.

483. Equation of Payments is the process of finding the average time or date for paying the whole amount of several debts due at different times. It is sometimes called *Average of Accounts.*

484. The **Equated Time** is the date at which the several sums, due at different times, may equitably be paid.

485. The **Term of Credit** is the time the debt has to run before it becomes due.

486. The **Average Term of Credit** is the time at the end of which several sums of money due at different times may equitably be paid in one amount.

In finding the equated time of a series of debits and credits, the date which is assumed as a standard from which to reckon the terms of credit, is sometimes called the *Focal Date.*

487. An **Account Sales** is a statement in writing, made by a commission merchant or agent to his principal, of merchandise consigned and sold, including the price, charges, and net proceeds.

The greater the term of credit, the greater the interest allowed to the debtor on the amount he owes; but, if he pays it before it is due, he loses a certain amount of interest; and if he retains it after it is due, he receives more interest than he is entitled to. By the equation of payments, neither party loses or gains any interest, on the following

488. PRINCIPLE. *The rate and interest being the same, the greater the principal the less the time, and the less the principal the greater the time.*

WRITTEN EXERCISES.

489. ***To find the Equated Time and the Average of terms of credit, beginning at the same date.***

1. I owe \$200, payable in cash, \$400 due in 3 mo., \$500 due in 5 mo., and \$700 due in 8 mo. In what time should the whole be paid?

OPERATION.

$$\begin{array}{rcrcr} 200 & \times & 0 & = & 0 \\ 400 & \times & 3 & = & 1200 \\ 500 & \times & 5 & = & 2500 \\ \underline{700} & \times & 8 & = & \underline{5600} \\ 1800 & & &) & \underline{9300} \\ & & & & 5\frac{1}{6}\text{ mo.} \end{array}$$

EXPLANATION.—On the first item no interest is due; the interest of \$400 for 3 mo. is the same as the int. of \$1 for 1200 mo.; and the int. of \$500 for 5 mo. is the same as the int. of \$1 for 2500 mo.; and of \$700 the same as \$1 for 5600 mo. Therefore, the whole amount of int. is that of \$1 for 9300 mo. But the interest is on \$1800; hence, the time is $\frac{1}{1800}$ of 9300 mo., or $5\frac{1}{6}$ mo., which is the *average term of credit*, or time required.

2. What is the average term of credit of \$500 due in 1 mo., \$800 due in 3 mo., and \$900 due in 6 mo.?

3. Bought a bill of goods Dec. 1, 1880, amounting to \$1200, of which \$250 was payable in cash, \$450 in 3 mo., and the balance in 6 mo. What is the equated time of payment?

The *equated time* is found by adding the average term of credit to the date at which the credits commence.

4. A person owes \$2400, of which $\frac{1}{3}$ is due in 4 mo., $\frac{1}{4}$ in 6 mo., and the remainder in 8 mo. What is the average term of credit?

5. Find the average term of credit and the equated time of payment from January 15, 1880, of \$1500 of which 20% is due in 30 da., 25% in 60 da., 30% in 90 da., and the remainder in 120 days.

6. A grocer owes $1800, to be paid $\frac{1}{3}$ in 5 mo., $\frac{1}{6}$ in 10 mo., $\frac{1}{9}$ in 18 mo., and the remainder in 20 mo. What is the average term of credit?

7. Bought a bill of goods amounting to $1200, on 6 mo. credit. Paid cash on account $100; at the end of 3 mo. paid $300 more; and 2 mo. afterward paid $400, giving a note for the balance. For what time was the note drawn?

OPERATION.

$$
\begin{array}{rcl}
100 \times 6 & = & 600 \\
300 \times 3 & = & 900 \\
\underline{400} \times 1 & = & \underline{400} \\
800 & & 1900
\end{array}
$$

$$1900 \div 400 = 4\tfrac{3}{4}$$

$$(6 \text{ mo.} - 5 \text{ mo.}) + 4\tfrac{3}{4} \text{ mo.} = 5\tfrac{3}{4} \text{ mo.}$$

EXPLANATION.—The int. of $100 paid in cash, or 6 mo. before it was due, is the same as the int. of $1 for 600 mo.; that of $300 for 3 mo. is the same as $1 for 900 mo.; and $400 for 1 mo. as $1 for 400 mo. Hence, the debtor is entitled to a credit of $1 for 1900 mo.; or of $400, the unpaid balance, for $\frac{1}{400}$ of 1900 mo., which is $4\frac{3}{4}$ mo. Therefore, the note was drawn for 1 mo. plus $4\frac{3}{4}$ mo., or $5\frac{3}{4}$ mo. from the time of the last payment

8. On a bill of goods bought March 1, amounting to $1500, on 8 mo. credit, the following payments were made: May 1, $350; Aug. 1, $500; Sept. 1, $150. What is the equated time for the payment of the balance?

9. A person owes $350, due in 3 mo., and $750, due in 6 mo.; but at the end of 2 mo. he pays $200, and 3 mo. afterward, $500. When is the remainder due?

10. Asa May has given three notes; one for $300, due May 1; one for $350, due June 15; and one for $550, due Aug. 1. Desiring to exchange them for two notes of $600 each, he makes one payable June 15; when should the other fall due?

11. Bought goods to the amount of $10000, of which $2000 was to be paid in 1 mo.; $2000 in 2 mo.; $4000 in 3 mo., and the balance in 6 mo. If a note is given for the whole amount, how long should it run?

490. ***To find the Equated Time and the Average of terms of credit beginning at different dates.***

1. Bought goods as follows: May 1, 1880, \$300 on 3 mo. credit; June 10, \$200 on 4 mo.; Aug. 1, \$500 on 6 mo.; and Sept. 15, \$600 on 4 mo. What is the average term of credit, and the equated time of payment?

OPERATION.

Aug. 1	(May 1 + 3 mo.)	\$300, cash.	
Oct. 10	(June 10 + 4 mo.)	$200 \times 70 =$	14000
Feb. 1	(Aug. 1 + 6 mo.)	$500 \times 184 =$	92000
Jan. 15	(Sept. 15 + 4 mo.)	$600 \times 167 =$	100200
		1600	) 206200
			$128\frac{7}{8}$, or

129 da., the average term of credit from Aug. 1; and the equated time of payment, *Dec. 8*.

Explanation.—The term of credit of each item, reckoned from Aug. 1, the earliest date at which any of the sums become due, is 70 da., 184 da., and 167 da., respectively. The average term from that date, found as in the preceding case (**489**), is 129 da.; and the equated time is, therefore, 129 da. from Aug. 1, or Dec. 8.

Proof.—Take the *latest date*, Jan. 15; calculate the average term from that; and *subtract* the number of days thus found from the date assumed.

If the earliest date is not the *first day* of the month, assume that as the standard date, for the sake of convenience.

2. Bought mdse. as follows: Aug. 15, 1880, on 3 mo., \$600; Sept. 10, 1880, on 4 mo., \$750; Nov. 5, 1880, on 6 mo. \$900. Find the equated time of payment.

3. Find the equated time for the payment of three notes, as follows: \$350, dated July 12, 1881, for 90 da.; \$720, dated Sept. 10, 1881, for 60 da.; and \$1200, dated Nov. 5, 1881, for 120 da.

4. I owe \$2150, due Nov. 16; I pay \$500, Oct. 4; when is the remainder due?

5. Bought of A. T. Stewart & Co., the following bills of goods on 5 mo. credit: Feb. 10, 1880, $900; March 15, 1880, $2000; May 10, 1880, $750; June 12, 1880, $2000. Find the present worth of a note drawn July 1 in payment of the whole, discounted at 6%.

6. Four notes, made by Ira Day, and payable as follows: $560, due Sept. 10, 1880; $800, due Oct. 15,1880; $1100, due Dec. 1, 1880; $900, due Feb. 1, 1881, were exchanged for a single note. When will it fall due?

7. CHARLES ADAMS *to* SAMUEL LEWIS, *Dr.*

May 1, 1882.	To	Mdse.	. . . .	$720.
June 15, "	"	"	. . . .	450.
Aug. 1, "	"	"	. . . .	1000.
Sept. 5, "	"	"	. . . .	900.
Oct. 20, "	"	"	. . . .	590.

What is the equated time of payment, allowing 60 da. credit on each item?

8. Bought goods at different dates, as follows:

Aug. 15,	amounting to	$475,	on 6 mo.	credit.
Sept. 10,	" "	600,	" 5 mo.	"
Oct. 5,	" "	750,	" 4 mo.	"
Nov. 1,	" "	450,	" 3 mo.	"

What sum will equitably discharge the whole debt November 10, allowing true discount at 7%?

9. Purchased Mdse. of J. V. Farwell & Co., as follows:

Jan. 1,	a bill amounting to	$375.50,	on 4 mo.	credit.
" 20,	" "	168.75,	" 5 mo.	"
Feb. 4,	" "	386.25,	" 4 mo.	"
March 11,	" "	144.60,	" 5 mo.	"
April 7,	" "	386.90,	" 3 mo.	"

What is the present worth of a note made May 1, in payment of the whole, discounted at 6%?

AVERAGING ACCOUNTS.

491. An **Account** is a record of the amount and date of debit and credit transactions.

The term *debit* implies that something is owed; *credit*, something received. Thus, the purchase of goods on time is a *debit* transaction; the payment of money on account, a *credit* transaction.

492. Averaging Accounts is finding the average or equated time for paying the balance, or calculating the cash balance at any particular time.

493. ***To find the Equated Time for paying the balance of an account.***

1. What is the equated time for the payment of the balance of the following account?

Dr. WILLIAM MARSTON. *Cr.*

1880.			1880.		
Feb. 15	To mdse. @ 3 mo.	$400	June 4	By cash . .	$400
Mar. 3	" " @ 4 mo.	500	" 10	" " . .	200
" 24	" " @ 5 mo.	900	July 5	" " . .	600
Apr. 15	" " @ 3 mo.	1000			

OPERATION.

Take the earliest date at which any item on either side matures, as the *standard*.

May 15.	$400 \times 0 =$	0	
July 3.	$500 \times 49 =$	24500	
July 15.	$1000 \times 61 =$	61000	
Aug. 24.	$900 \times 101 =$	90900	
	2800	176400	
	1200	43800	
	1600	) 132600	

June 4.	$400 \times 20 =$	8000
June 10.	$200 \times 26 =$	5200
July 5.	$600 \times 51 =$	30600
	1200	43800

Hence, the equated time is 83 da. *after* May 15, or *Aug. 6.*

PROOF. (by reversing the standard).

Aug. 24.	900 × 0	= 0	July 5.	600 × 50	= 30000
July 15.	1000 × 40	= 40000	June 10.	200 × 75	= 15000
July 3.	500 × 52	= 26000	June 4.	400 × 81	= 32400
May 15.	400 × 101	= 40400		1200	77400
	2800	106400			
	1200	77400			
	1600	) 29000			

Hence, the equated time is 18 da. *before* Aug. 24, or *Aug. 6.*

EXPLANATION.—Since $400 is due May 15, if the settlement were made at that date, no interest would be due on that item; but the next item of $500, due July 3, would be paid 49 da. before being due; and therefore the debtor would be entitled to the int. for that time. For the same reason he would be entitled to the int. of $1000 for 61 da., and of 900 for 101 da., making in all the int. of $1 for 176400 da. For a similar reason the creditor is entitled to an allowance of int. equal to that of $1 for 43800 da. Hence, the balance of interest in favor of the debtor is that of $1 for 132600 da., which is the same as the int. of $1600, the balance, for 83 da.; and, consequently, the equated time is 83 da. after the assumed date, May 15, or Aug. 6.

The reasoning in the Proof is based on precisely the same principle.

This is sometimes called the method of *products;* but instead of this we may find the *interest* at any rate per cent (12 is the most convenient), on each debit and credit item, and divide the *balance of interest* by the *interest on the balance of the account for one day.* The quotient will give the number of days to be added to, or subtracted from, the standard date to give the equated time.

In all cases where payment is made by note, add the three days of grace to the time the note has to run in reckoning the terms of credit pertaining to it.

2. Find the *equated time* for the following:

Dr. WILLIAM RALSTON. *Cr.*

1881.			1881.		
May 1	To Mdse. @ 2 mo. .	$800	June 15	By Cash. .	$500
May 25	" " .	500	July 1	" "	400
June 20	" " @ 3 mo. .	750	Aug. 15	" "	1000
July 18	" " @ 2 mo. .	2000			

3. Find the equated time of the following:

Dr. SAMUEL SLOAN. *Cr.*

1880.			1880.		
Aug. 1	To Mdse. @ 6 mo. .	$1500	Sept. 25	By Cash. .	$900
Sept. 5	" " .	1200	Nov. 1	" "	700
Oct. 10	" " @ 3 mo. .	800	Nov. 15	" "	450
Nov. 3	" "	950	Dec. 1	Note, 60 da.	550
			1881.		
Dec. 1	" " @ 4 mo. .	2000	Feb. 10	By Cash. .	1500

494. ***To find the Cash Balance at any given time.***

1. What is the cash balance of the following account Jan. 2, 1881, interest at 6%?

Dr. ALEXANDER MATTHEWS. *Cr*

1880.			1880.		
May 1	To Mdse. @ 6 mo. .	$700	Aug. 15	By Cash. .	$400
June 5	" " @ 4 mo. .	500	Sept. 1	" "	300
July 10	" " @ 5 mo. .	1000	Oct. 10	" "	600

OPERATION.

The equated time is 176 da. from Aug. 15, or Feb. 7, 1881 (**493**), the difference between which date and Jan. 2, 1881, is 36 da. Hence, the true present worth of $900, the balance, for that time, at 6%, which is $894.63, is the cash balance at that date.

The cash balance may also be found by finding the interest due on each item at the time specified, and deducting the balance of interest from the same due on the account, if in favor of the debtor, or adding it, if in favor of the creditor.

2. Of the following account, what is the cash balance Feb. 1, 1882, allowing interest at 6%?

Dr. James Harrison. *Cr.*

1881.			1881.		
July 10	To Mdse. @ 90 da.	$875	Oct. 1	By Cash.	$750
Sept. 5	" " @ 60 da.	925	Nov. 15	" "	500
Oct. 8	" " @ 30 da.	463	Dec. 1	" "	350
Nov. 15	" " @ 90 da.	517	Dec. 30	" "	600
			1882.		
Dec. 1	" " @ 60 da.	1550	Jan. 20	" "	400

3. Allowing 60 da. credit on each debit item of the following account, what is the cash balance Sept. 15, 1880, the rate of interest being 7%?

Dr. Amos Brown *in acct. with* Ira Smith. *Cr.*

1880.			1880.		
Jan. 10	To Mdse. .	$540	April 1	By note @ 60 da.	$400
May. 15	" " .	490	May 30	" Mdse. @ 30 da.	300
June 12	" " .	670	Sept. 1	" Cash. . . .	500
Aug. 1	" " .	900			

4. Edgar Foreman gives his note at 3 mo. for the balance of the following account July 1. What is the face of the note, allowing interest at 6%?

Dr. Edgar Foreman. *Cr.*

1882.			1882.		
Jan. 5	To Mdse. @ 3 mo.	$950	Mar. 1	By Cash, . . .	$500
Feb. 10	" " @ 60 da.	435	Apr. 10	" Note, 60 da.	350
Apr. 12	" " @ 3 mo.	598	" 25	" Cash, . . .	250
May 3	" " @ 60 da.	1150	May 28	" " . . .	450
June 5	" " @ 90 da.	725	July 1	" " . . .	500

5. Sold to John Rose, on 4 mo. credit, Mdse. as follows: Feb. 12, a bill amounting to $300, and March 20, a bill of $400. July 1, he paid $200; from what date must I reckon interest on the balance?

RATIO

DEFINITIONS

495. Ratio is the relation of one number to another of the same denomination, expressed by the quotient obtained by dividing the first by the second.

Thus, the ratio of 6 to 2 is 3, of $27 to $3 is 9, and of 3 to 9 is $\frac{1}{3}$. A ratio is an abstract number; for, when the dividend and divisor are of the same denomination, the quotient is abstract (**95**, I). Ratio can exist only between numbers, or things of the same kind; a mile or a yard cannot be compared with a day or an hour; but a mile can be compared with a yard, when reduced to the same denomination, or 3 miles with 2 miles; and so a day may be compared with an hour, or 5 days with 6 days.

In order to form a ratio, length must be compared with length, time with time, weight with weight, and so on.

496. The **Special Sign of Ratio** is the *Colon.*

Thus, 8 : 4 is read the *ratio of 8 to 4,* or 8 is to 4, or 8 divided by 4. Any one of the signs of division may be used instead of the colon.

For example, the ratio of 8 to 4 may be expressed thus, 8 ÷ 4, or $\frac{8}{4}$. The colon is an abbreviation of the ordinary sign of division.

497. The **Terms** of a ratio are the two numbers compared.

Thus, in the expression 81 : 27, the terms of the ratio are 81 and 27.

498. The **Antecedent** of a ratio is the first term, and the **Consequent** is the second term. The former is a dividend, the latter a divisor.

The Antecedent and Consequent together form a *Couplet*. Ratios are of two kinds: *Simple Ratios* and *Compound Ratios.*

499. A **Simple Ratio** is the ratio of two numbers.

Thus, 5 : 6, $\$8\frac{1}{2}$: \$12, and 4 qt. : 12 qt., are simple ratios.

500. A **Compound Ratio** is the ratio of the products of the corresponding terms of two or more ratios.

Thus, the ratio *compounded* of 8 : 4 and 9 : 12 is $8 \times 9 : 4 \times 12$, or 72 : 48. When the multiplication is performed, the result is a simple ratio.

501. The **Reciprocal of a Ratio** is the quotient found by dividing the consequent by the antecedent.

Thus, the reciprocal of the ratio 8 : 4 is $\frac{4}{8}$ or $\frac{1}{2}$.

To find the ratio of two *fractions*, reduce them to fractions having a common denominator, and then compare their numerators.

$$\text{FORMULAS}: \begin{cases} 1.\ \textit{Ratio} = \textit{Antecedent} \div \textit{Consequent}. \\ 2.\ \textit{Consequent} = \textit{Antecedent} \div \textit{Ratio}. \\ 3.\ \textit{Antecedent} = \textit{Consequent} \times \textit{Ratio}. \end{cases}$$

PRINCIPLES.

502. Since the antecedent is a dividend, and the consequent a divisor, it follows, that

I. *Multiplying the antecedent, or Dividing the consequent,* } *Multiplies the ratio.* **(105,** I.)

II. *Dividing the antecedent, or Multiplying the consequent,* } *Divides the ratio.* **(105,** II.)

III. *Multiplying or dividing both terms of a ratio by the same number* } *Does not change the value of the ratio.* **(105,** III.)

GENERAL STATEMENT.

503. *A change in the first term of a ratio produces a* LIKE *change in the value of the ratio; but a change in the second term produces an* OPPOSITE *change in the value of the ratio.*

EXERCISES.

504. 1. Express the ratio of 28 to 14; of 14 to 28; of \$12 to \$6; of \$8 to 50 cents; of 3 wk. to 7 da.

2. Find the ratio of 3 yd. 1 ft. to 2 ft. 6 in.

3. Can you find the ratio of \$18 to 3 ft.? Why not?

4. Reduce each of the following ratios to its lowest terms: $\frac{12}{15}$, 18 : 6, \$12 ÷ \$8, 36 yd. : 4 ft., 1 bu. : 16 qt.

5. Find the ratio of $\frac{3}{4}$ to $\frac{1}{2}$; of $\frac{4}{5}$ to $\frac{3}{8}$; of $1\frac{1}{2}$ to $\frac{2}{3}$.

6. Reduce each of the following ratios to another whose terms shall be integral: $\frac{5\frac{1}{2}}{3}$; $6\frac{3}{4} : 8\frac{5}{6}$; $37\frac{1}{2} : 18\frac{3}{4}$.

7. Find the ratio which is compounded of 3 : 5 and 7 : 9.

8. What is the difference between 5 : 3, and the *reciprocal* of 3 : 5? Between 9 : 4 and 4 : 9?

9. The first term of a ratio is $\frac{13}{28}$, and the second term $3\frac{1}{4}$; what is the value of the ratio?

10. The first term is 75, and the ratio 7; find the second term.

11. The second term is \$6, and the ratio $3\frac{1}{2}$; find the first term.

12. Find the number which has to 27 the ratio of 5 to 3.

13. Which is greater, 8 : 5 or 9 : 6? 5 : 8 or 15 : 24? $\frac{4}{8}$ or $\frac{5}{10}$? $\frac{6}{11}$ or $\frac{2}{5}$? $\frac{1}{8}$ or $\frac{2}{15}$?

14. Name three ratios which are equal to 10 : 2; three which are equal to $\frac{2}{3}$.

PROPORTION

INDUCTIVE EXERCISES.

505. 1. Compare $6:2$ with $24:8$.

Which of the following statements are correct?

2. $27:9 = 81:27$; $3:12 = 12:36$; $2:3 = 5:6$; $3:4 = 6:8$; $28:4 = 16:2$.

3. The ratio of 5 to 3 is equal to the ratio of 10 to what number?

4. $27 : 9 = 9 : ?$
5. $3 : 2 = ? : 4$.
6. $8 : ? = 12 : 6$.
7. $? : 5 = 8 : 4$.
8. $\$30 : \$6 = ? : 8$.
9. $4 \div \frac{1}{2} = 6 \div ?$
10. $\frac{3}{4} \div \frac{1}{2} = ? \div \frac{1}{3}$.
11. $\frac{7}{8} \div ? = \frac{2}{3} \div \frac{5}{7}$.
12. $? \div \frac{5}{6} = \frac{1}{8} \div \frac{1}{9}$.
13. $\frac{5}{16} \div \frac{3}{8} = ? \div \frac{2}{5}$.

DEFINITIONS.

506. A **Proportion** is an equation in which each member is a ratio, both terms of which are expressed.

The equality of the ratios may be indicated either by the sign $=$, or by the double colon $::$.

Thus, the equality of the ratios of 10 to 5 and 8 to 4, may be indicated in any of the following ways: $10:5 = 8:4$, $10:5 :: 8:4$, $\frac{10}{5} = \frac{8}{4}$, $10 \div 5 = 8 \div 4$.

This proportion in any of its forms, is read, *the ratio of 10 to 5 is equal to the ratio of 8 to 4*, or, *10 is to 5 as 8 is to 4*.

507. The **Terms** of a proportion are the numbers which are compared.

The first and second terms form the *first couplet ;* the third and fourth, the *second couplet.*

Since it requires two numbers to form a ratio, every proportion must contain at least four terms.

508. A **Proportional** is any term of a proportion.

509. The **Antecedents** in a proportion are the antecedents of the ratios, or the first and third terms.

510. The **Consequents** are the consequents of the ratios, or the second and fourth terms.

511. The **Extremes** in a proportion are the first and fourth terms.

512. The **Means** are the second and third terms.

If three numbers are such that the ratio of the first to the second is equal to that of the second to the third, the second is said to be the **Mean Proportional** between the first and second.

Thus, in the proportion $3:6 = 6:12$, 6 is the mean proportional between 3 and 12, and 12 is the third proportional to 3 and 6.

513. A **Simple Proportion** is a proportion in which each ratio is simple. Thus, $2:4 = 6\frac{1}{2}:13$ is a simple proportion.

514. A **Compound Proportion** is a proportion which has a compound ratio.

Thus, $\left.\begin{matrix}2:3\\5:6\end{matrix}\right\} = 20:36$, is a compound proportion. This proportion may be read, *2 × 5 is to 3 × 6 as 20 is to 36.*

515. The proportion $2:3 = 4:6$ may be written $\frac{2}{3} = \frac{4}{6}$ **(506)**. Reducing $\frac{2}{3}$ and $\frac{4}{6}$ to equivalent fractions having a common denominator, $\frac{2 \times 6}{18} = \frac{4 \times 3}{18}$. Since these fractions are equal and have a common denominator, their numerators are equal; that is, $2 \times 6 = 4 \times 3$.

Hence, the following

516. PRINCIPLES.—I. *The product of the extremes of a proportion is equal to the product of the means.*

II. *Either extreme is equal to the product of the means divided by the other extreme.*

III. *Either mean is equal to the product of the extremes divided by the other mean.*

These principles are applicable also to a compound proportion.

Thus the compound proportion $\left.\begin{matrix}2:3\\5:6\end{matrix}\right\} = 20:36$ may be reduced to the simple proportion $10:18 = 20:36$, and $10 \times 36 = 18 \times 20$.

EXERCISES.

517. Find the omitted term, which is represented by x, in each of the following proportions:

1. $12:1 = x:144$.
2. $20:x = 50:120$.
3. $x:4 = \$80:\$4$.
4. $\$180:\$x = \frac{1}{8}:\frac{3}{8}$.
5. $37\frac{1}{2}:4\frac{1}{4} = x:11\frac{1}{3}$.
6. $1.875:.625 = 12.5:x$.
7. $17\frac{1}{2}:2\frac{1}{2} = 875:x$.
8. $4\frac{1}{3}$ yd. : x yd. = \$9.50 : \$28.50.
9. $\left.\begin{matrix}2:3\\5:6\end{matrix}\right\} = 20:x$.
10. $\left.\begin{matrix}4:6\\5:7\end{matrix}\right\} = x:72$.
11. $\left.\begin{matrix}2:3\\10:x\end{matrix}\right\} = 10:18$.
12. $\left.\begin{matrix}4:6\\7:8\end{matrix}\right\} = \left\{\begin{matrix}7:10\\5:x\end{matrix}\right.$

518. The **Statement** of a problem in proportion consists in arranging the given and the required quantities in the form of a proportion.

It is usually convenient to make the required quantity the fourth term of the proportion, or the consequent of the second couplet; the given quantity of the same kind or denomination will be the antecedent of that couplet, or third term of the proportion.

WRITTEN EXERCISES.

519. 1. If 3 yd. of cloth cost \$15, what will 12 yd. cost at the same rate?

EXPLANATION.—Denote the cost of 12 yd. by $\$x$; then it is evident that $\$x$ is as many times greater than \$15 as 12 yd. is greater than 3 yd. Hence, by PRIN. II,

$$x = \frac{\$15 \times \cancel{12}^{4}}{\cancel{3}} = \$60.$$

STATEMENT.

$$3 \text{ yd.} : 12 \text{ yd.} = \$15 : \$x.$$

$$\overline{\$15 \times 12} \div 3 = \$60.$$

$$\begin{array}{r|l} \cancel{3} & \$15 \\ x & \cancel{12}^{\,4} \\ \hline \multicolumn{2}{c}{x = \$60} \end{array}$$

2. A man performs a journey in 20 hr. by traveling 2 mi. an hour; how many hours will he require to perform the same journey if he travels 6 mi. an hour?

EXPLANATION.— Denote the required number of hours by x; then it is evident that 20 hr. is as many times greater than x hr. as 6 mi. is greater than 2 mi., for the time required diminishes in the same ratio as the rate of travel increases. Hence,

STATEMENT.

$$6 \text{ mi.} : 2 \text{ mi.} = 20 \text{ hr.} : x \text{ hr.}$$

$$\begin{array}{r|l} {}^{3}\cancel{6} & 20 \\ x & \cancel{2} \\ \hline 3x & 20 \end{array}$$

$$x = 6\tfrac{2}{3} \text{ hr.}$$

$$x = \frac{20 \times \cancel{2}}{\cancel{6}^{\,3}} = 6\tfrac{2}{3} \text{ hr.} \quad (\text{PRIN. II.})$$

The ratio of the times is in the *inverse ratio* of the rates of travel; or, the time is *inversely proportional* to the rate of travel.

RULE.—*1. Write for the third term that number which is of the same kind as the number to be found.*

2. *Write the two remaining numbers for the first couplet, so that their ratio shall be equal to that of the third term to the number sought.*

3. *Divide the product of the means by the given extreme, and the result will be the fourth term, or the number sought.*

If the first ratio is compound, compare every two numbers of the same unit value, and arrange the terms of each couplet in respect to the third term of the proportion as if it were the first couplet of a simple proportion.

3. If 5 men earn \$60 in 8 days, how much will 7 men earn in 12 days at the same rate?

STATEMENT.

$$\left.\begin{matrix} 5 \text{ men} : 7 \text{ men} \\ 8 \text{ da.} : 12 \text{ da.} \end{matrix}\right\} = \$60 \quad \$x.$$

$$\begin{array}{r|l} \cancel{5} & \$\cancel{60}\ ^{\$6} \\ \cancel{8} & 7 \\ x & \cancel{12}\ ^{3} \\ \hline \end{array}$$

$$x = \$126$$

EXPLANATION.—The work of 5 men for 8 da. is equivalent to the work of 1 man for 5×8, or 40 da., and the work of 7 men for 12 da. is equivalent to the work of 1 man for 7×12, or 84 days. Hence, the question may be stated as follows: If a man can earn \$60 in 40 da., how much can he earn in 84 da.? By the rule, $40 : 84 = \$60 : \$x$; whence, by PRIN. II,

$$x = \frac{\overset{\$6}{\$\cancel{60}} \times 7 \times \overset{3}{\cancel{12}}}{\cancel{5} \times \cancel{8}} = \$126.$$

Such questions as this are usually regarded as belonging to *Compound Proportion*, but a simple reduction brings them under the preceding rule.

4. If 16 horses eat 96 bu. of oats in 42 da., in how many days will 7 horses eat 66 bushels?

5. If 900 lb. of merchandise can be carried 26 mi. for 60 cents, how many miles can 3 tons be carried for \$8?

CAUSE AND EFFECT.

520. The relations of the terms of a proportion may also be regarded as a comparison of two *causes* and their corresponding *effects*, expressed as follows:

1st Cause : 2d Cause = 1st Effect : 2d Effect.

WRITTEN EXERCISES.

521. 1. If 12 men earn \$72 in 1 wk., how much will 18 men earn, at the same rate, and in the same time?

STATEMENT.

1st Cause. 2d Cause. 1st Eff. 2d Eff.

$$12 \text{ men} : 18 \text{ men} = \$72 : \$x.$$

$$\begin{array}{r|l} \$x & \cancel{\$72}^{\$6} \\ \cancel{12} & 18 \\ \hline \multicolumn{2}{c}{x = \$108} \end{array}$$

EXPLANATION.—Denote the required term by $\$x$. Then the first cause is 12 men and the second cause is 18 men; and since like causes can be compared, they may form the first couplet. The first effect is \$72 earned, and the second effect is $\$x$ earned; and since like effects have the same ratio as their causes these form the second couplet of the proportion, and the whole may be read, If 12 men in 1 wk. earn \$72, 18 men in the same time will earn $\$x$, or how many dollars? Hence, 12 men : 18 men = \$72 : $\$x$; $x = \frac{\$72 \times 18}{12} = \$108$.

2. If 6 men in 4 da., working 10 hr. a day, can reap 16 acres, in how many days can 10 men, working 12 hr. a day, reap 24 acres?

STATEMENT.

1st Cause. 2d Cause. 1st Effect. 2d Effect.

$$\left\{\begin{array}{c} 6 \text{ men} \\ 4 \text{ da.} \\ 10 \text{ hr.} \end{array}\right\} : \left\{\begin{array}{c} 10 \text{ men} \\ x \text{ da.} \\ 12 \text{ hr.} \end{array}\right\} = 16 \text{ acres} : 24 \text{ acres}.$$

$$\begin{array}{r|l} \cancel{10} & \cancel{6}^{3} \\ x & \cancel{4} \\ \cancel{12} & \cancel{10} \\ \cancel{16} & \cancel{24} \\ \hline \multicolumn{2}{c}{x = 3 \text{ days.}} \end{array}$$

EXPLANATION.—The first cause is the labor of 6 men, for 4 da. of 10 hr. each; the second cause is the labor of 10 men, for x days of 12 hr. each; the first effect is the reaping of 16 acres; and the second effect is the reaping of 24 acres. The question will then read, If 6 men in 4 da., working 10 hr. a day, reap 16 acres, 10 men in x, or how many days, working 12 hr. a day, will reap 24 acres? Hence,

$$x = \frac{\overset{3}{\cancel{6}} \times \cancel{4} \times \cancel{10} \times \cancel{24}}{\cancel{10} \times \cancel{12} \times \cancel{16}} = 3 \text{ da.}$$

Solve by either method the following.

3. If 10 bu. of wheat produce 3 bbl. of flour, how many bushels will be required to produce 120 bbl. of flour?

4. If 6 cloaks can be made from 27 yd. of cloth, how many yards will be required for 32 cloaks?

5. If 9 bales of cotton can be carried 100 mi. for \$6, how far can 17 bales be carried for the same money?

6. If $2\frac{3}{4}$ yd. of broadcloth are worth \$23.10, what is the value of $16\frac{7}{8}$ yd., at the same rate?

7. If $\frac{3}{4}$ of an acre of land cost \$60, what will $87\frac{1}{2}$ acres cost, at the same rate?

8. How many yards of carpeting $\frac{3}{4}$ of a yard wide, will cover a floor 30 ft. long and 24 ft. wide?

9. If I lend a man \$300 for 6 mo., for what time should I have the use of \$450 to equalize the favor?

10. If \$700 gain \$84, what will \$400 gain in the same time?

11. If I borrow a sum of money 6 mo. when interest is 8%, for what time should I lend the same sum, when interest is 6%, to requite the favor?

12. How many yards of cambric, 1 yd. wide, will be required to line 16 yd. of silk, 24 in. wide?

13. If a pole 26 ft. high cast a shadow of 8 ft., what length of shadow will a church spire $346\frac{3}{4}$ ft. high cast at the same time?

14. If 15 cwt. of sugar cost \$81, what will be the cost of 2 T. 17 cwt. 50 lb., at the same rate?

15. If 24 men can dig a trench in 12 days, how long would the same work employ 9 men?

16. What is the cost of 20 yd. of muslin, $1\frac{1}{8}$ yd. wide, if 16 yd. of the same quality, $1\frac{1}{4}$ yd. wide, cost $\$2\frac{1}{2}$?

17. If 72 yd. of carpeting, $\frac{3}{4}$ yd. wide, will cover a floor, how many yards, $\frac{7}{8}$ yd. wide, will cover the same floor?

18. If 25 men can do a piece of work in 24 da., working 8 hr. a day, how many hours a day will 30 men be required to work, in order to do the same work in 16 days?

19. If a pile of wood 36 ft. long, 4 ft. wide, and 5 ft. high, costs $58.50, what is the cost of a pile 60 ft. long, 4 ft. wide, and 6 ft. high, at the same price?

20. If 15 A. 100 P. of land produce 245 bu. of grain, how many bushels will 62 A. 80 P. of the same land produce?

21. If 10 men can perform a piece of work in 24 da., how many men can perform another piece of work, 7 times as great in one-fifth of the time?

22. If .85 of a gallon of wine cost $2.72, what is the cost of .25 of a gallon?

23. What is the cost of 38.25 A. of land, if $4\frac{1}{2}$ A. cost $25.50?

24. If 2.5 tons of freight can be conveyed $24\frac{3}{4}$ mi. for $5, how far can 22.5 cwt. be conveyed for the same money?

25. If a field of 10 acres be sufficient to pasture 20 head of cattle through the summer, how many acres will be sufficient for 35 head of cattle for the same time?

26. If 20 masons build a wall 50 ft long, 2 ft. thick, and 14 ft. high, in 12 da., of 7 hr. each, in how many days, of 10 hr. each, will 60 masons build a wall 500 ft. long, 4 ft. thick, and 16 ft. high?

27. If 150000 bricks are required to build a wall $1\frac{1}{2}$ ft. thick, 30 ft. high, and 216 ft. long, how many will be required to build a wall 2 ft. thick, 24 ft. high, and 324 ft long?

28. What is the weight of a block of stone 12 ft. 6 in. long, 6 ft. 6 in. broad, and 8 ft. 3 in. deep, if a block of the same stone 5 ft. long, 3 ft. 9 in. broad, and 2 ft. 6 in. deep, weighs 7500 lb.?

PARTNERSHIP.

522. Partnership is the association of two or more persons for the carrying on of some particular business, with joint capital.

The persons thus associated are called *Partners;* and the association is called a *Firm*, *Company*, or *House.*

523. The **Capital** consists of the money, or other property invested by the several partners, constituting what is called the *Investment,* or *Joint Stock.*

524. The **Resources, Assets,** or **Effects** of a firm, consist of the property which it owns, and the debts due to it.

525. The **Liabilities** of a firm are its debts.

The excess of resources over liabilities at any time constitutes its *Net Capital.*

Each partner's stock increased by gain, or diminished by loss, at the time of settlement, is called his *interest in the business* at that time.

The process of finding the share of the whole gain or loss to be apportioned to each member of a firm at the time of final settlement, is called *Partnership Average.*

When each partner's stock is employed for the same period of time, it is sometimes called *Simple Average;* and when employed for different periods of time, it is called *Compound Average.*

At the expiration of certain periods, the gains or losses are apportioned among the members of the firm according to the following

526. PRINCIPLE. *The greater the amount of capital invested by any partner, and the longer it remains invested, the greater his apportionment of gain or loss.*

WRITTEN EXERCISES.

527. ***To apportion the gain or loss when each partner's investment has been employed for the same period of time.***

1. Three persons engage in business with a capital of \$10000, of which A's share is \$3500; B's, \$4000; and C's, \$2500. They gain \$4000; what is each partner's share of the gain?

OPERATION I. (*By Fractions.*)

$$\$3500 + \$4000 + \$2500 = \$10000.$$

$$\tfrac{3500}{10000} = \tfrac{7}{20}\,;\ \$4000 \times \tfrac{7}{20} = \$1400, \text{ A's gain.}$$
$$\tfrac{4000}{10000} = \tfrac{2}{5}\,;\ \$4000 \times \tfrac{2}{5} = \$1600, \text{ B's gain.}$$
$$\tfrac{2500}{10000} = \tfrac{1}{4}\,;\ \$4000 \times \tfrac{1}{4} = \$1000, \text{ C's gain.}$$

EXPLANATION.—Since A's share of the capital, \$3500, is $\frac{7}{20}$ of the whole capital, his share of the gain is $\frac{7}{20}$, or \$1400; and for a similar reason, B's share is $\frac{2}{5}$, or \$1600; and C's, $\frac{1}{4}$, or \$1000.

OPERATION II. (*By Percentage.*)

\$4000 (gain) $= \frac{4000}{10000}$, or $\frac{2}{5}$, or 40%, of the whole capital; hence,

$$\left.\begin{array}{l}\text{A's share}\\ \text{B's}\ \ ``\\ \text{C's}\ \ ``\end{array}\right\} = 40\% \text{ of } \left\{\begin{array}{rl}\$3500 = & \$1400, \text{ A's gain.}\\ 4000 = & 1600, \text{ B's}\ \ ``\\ 2500 = & 1000, \text{ C's}\ \ ``\end{array}\right.$$

EXPLANATION.—Since \$4000, the whole gain, is 40% of \$10000, the whole capital, the shares are 40% respectively of the several shares of the capital.

OPERATION III. (*By Proportion.*)

$$\$10000 : \left\{\begin{array}{r}\$3500\\ 4000\\ 2500\end{array}\right\} = \$4000 : \left\{\begin{array}{l}\$1400, \text{ A's gain.}\\ 1600, \text{ B's}\ \ ``\\ 1000, \text{ C's}\ \ ``\end{array}\right.$$

EXPLANATION.—The ratio of the whole capital to each partner's share of the same is equal to the ratio of the whole gain or loss to each partner's share of the gain or loss.

2. At the expiration of a year from the commencement of their business, Adams, Morton & Co., after taking an account of stock, find the amount of merchandise, as per inventory, to be \$17450; cash on hand, \$10250; debts due the firm, \$11300; amount of firm's indebtedness, \$15500. Make out a statement, showing the resources and liabilities of the firm, with the net capital and gain; and find each partner's share of the latter, the respective shares of capital being as follows: J. Adams, \$8000; W. Morton, \$5000; and J. French, \$3000.

OPERATION.

Resources.

Mdse. as per inventory, ...	\$17450	
Cash in hand, ...	10250	
Debts due the company, ...	11300	
		\$39000

Liabilities.

Debts owed by the company, ...		15500
Net capital, ...		\$23500

Investments.

J. Adams, ...	\$8000	
W. Morton, ...	5000	
J. French, ...	3000	
Total investments, ...		\$16000
Net gain, ...		\$7500

$$\left.\begin{array}{lll}\text{J. Adams is entitled to} & \frac{8000}{16000}, \text{ or } \frac{1}{2} \\ \text{W. Morton "} & \text{"} \quad \frac{5000}{16000}, \text{ or } \frac{5}{16} \\ \text{J. French "} & \text{"} \quad \frac{3000}{16000}, \text{ or } \frac{3}{16}\end{array}\right\} \times \$7500 = \left\{\begin{array}{r}\$3750.00 \\ 2343.75 \\ 1406.25\end{array}\right.$$

3. A, B, and C enter into business with a capital of \$30000, of which A puts in \$14000; B, \$9500; and C, the remainder. They gain \$16000; what is each partner's share?

4. Simpson, Brown & Co. fail in business, their liabilities amounting to \$35000, and their resources to \$16500. They owe A \$9000, B \$7500, and C \$11000. What will each creditor receive?

5. Johnson Bros. & Co. find the condition of their business as follows: Mdse. as per inventory, \$49500; notes due the firm, \$32500; cash on hand, \$27000; due, as per ledger, \$31400; notes against the firm, \$29500; other indebtedness, \$15000. The investments are: W. Johnson, \$15000; B. Johnson, \$10000; C. Mott, \$7000. Make a statement, and apportion the gain or loss.

528. ***To apportion the gain or loss according to amount of capital invested, and time it is employed.***

1. A engaged in business with a capital of \$5000; 3 mo. afterward he took in B, with a capital of \$6000; and 4 mo. later he took in C, with \$10000. At the end of the year their profits were \$10000. What is the share of each?

OPERATION.

$$\$5000 \times 12 = \$60000; \quad \$6000 \times 9 = \$54000;$$
$$\$10000 \times 5 = \$50000.$$

$$\$60000 + \$54000 + \$50000 = \$164000.$$

$$\$164000 : \left\{\begin{matrix} \$60000 \\ 54000 \\ 50000 \end{matrix}\right\} = \$10000 : \left\{\begin{matrix} \$3658.54, \text{ A's gain.} \\ 3292.68, \text{ B's } `` \\ 3048.78, \text{ C's } `` \end{matrix}\right.$$

EXPLANATION.—The use of \$5000 for 12 mo. is the same as the use of \$60000 for 1 mo.; the use of \$6000 for 9 mo. is the same as that of \$54000 for 1 mo.; and the use of \$10000 for 5 mo. the same as that of \$50000 for 1 mo. Hence, the use of the whole capital during the year is the same as that of \$164000 for 1 mo. The ratio of the whole capital to each partner's share of the same is equal to the ratio of the whole gain or loss to each partner's share of the gain or loss.

2. Jan. 1, 1881, two men, A and B, commenced business with \$2400 capital, each furnishing \$1200; March 1, B put in \$1200 more; and April 1 they took in C, with \$3000. At the end of a year how should a gain of \$2000 be apportioned?

3. A and B are in business Jan. 1, with a capital of \$30000, of which A owns $\frac{5}{8}$ and B $\frac{3}{8}$. March 1, A puts in \$10000; and B draws out \$1500; and June 1, A draws out \$15000, and B puts in \$5000. At the end of the year, how shall a gain of \$10000 be apportioned?

INVOLUTION.

DEFINITIONS.

529. A **Power** of a number is either the number itself, or the product of factors, each of which is equal to that number.

Thus, 5 is the *first* power of 5; 5×5, or 25 is the *second* power of 5; $5 \times 5 \times 5$, or 125, is the *third* power of 5, and so on.

The second power of a number is sometimes called the *square* of that number, because the area of a square is equal to the second power of one of its *sides*.

The third power of a number is sometimes called the *cube* of that number, because the volume of a cube is equal to the third power of one of its *edges*.

530. The **Root** or **Base** of a number, regarded as a power, is either the number itself, or one of its equal factors; and *The Degree* of a power is equal to the number of times the root is used to produce the power.

Thus, $4 \times 4 \times 4$, or 64, is the third power of 4, and 4 is the root or base of that power.

531. An **Exponent** of a power is a number used to denote the degree of a power. It is written to the right of the base and a little above it.

Thus, 27^2, $(\frac{3}{14})^3$, $(4\frac{2}{5})^4$, indicate respectively, the square of 27, the cube of $\frac{3}{14}$, and the fourth power of $4\frac{2}{5}$.

532. **Involution** is the process of finding any power of a number, and a number is said to be *involved* or *raised* to a power, when any power of it is found.

533. A **Perfect Square** is a number which can be resolved into *two* equal factors; a **Perfect Cube** is a number which can be resolved into *three* equal factors; and so on.

Thus, 81 is a perfect square, and 27 is a perfect cube; for $81 = 9 \times 9$, and $27 = 3 \times 3 \times 3$.

Written Exercises.

534. ***To find any power of a number.***

1. Find the third power of 27.

Explanation.—The product of 27 multiplied by itself is 729; and 729 multiplied by 27 is 19683, which, being the product of 27 by 27 by 27, is the third power of 27.

OPERATION.

$$27 \times 27 = 27^2 = 729;$$
$$729 \times 27 = 27^3 = 19683.$$

Rule.—*Find the product of the given number used as many times as a factor as there are units in the exponent of the power.*

2. Find the fifth power of $\frac{2}{3}$.

OPERATION.

$$\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{2 \times 2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3 \times 3} = \frac{2^5}{3^5} = \frac{32}{243}.$$

Hence,

A fraction may be raised to any power by raising each of its terms to the required power.

Involution may sometimes be facilitated by means of the following

PRINCIPLE.—*If two or more powers of the same number be multiplied together the product will be a power, the degree of which is equal to the sum of the degrees of the given powers.*

Thus, $3^2 \times 3^3 \times 3^4 = 3^9$.

The seventh power of 2 may be found by multiplying 16, which is the fourth power of 2, by 8, which is the third power of 2.

3. Find the squares of the numbers from 1 to 12 inclusive.
4. Find the cubes of the numbers from 1 to 12 inclusive.
5. Find the squares of $\frac{3}{4}$, $\frac{4}{7}$, $\frac{7}{12}$, $2\frac{1}{2}$, and $8\frac{3}{4}$.
6. Find the cubes of $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$, $2\frac{1}{4}$, and $3\frac{1}{2}$.
7. Find the squares of .1, .02, 3.5, 1.85, .003.

If a number containing a decimal fraction be squared, will the number of decimal places be even or odd? Why?

8. Find the cubes of .2, .03, 2.5, 1.05, 1.001.
9. Find the square, cube, and fourth power of 1500.

If a number is terminated by ciphers, how many ciphers will there be in its square? In its cube? In its fourth power?

10. Multiply 15^3 by 15^2.

What power of a number is produced by multiplying its cube by its square?

11. Find the product of 8, 8^2, and 8^3.

What power of a number is produced by multiplying together its first power, square, and cube?

12. What is the square of 5^3? The cube of 5^2? The cube of 5^3?

What power of a number is the square of its cube? The cube of its square? The cube of its cube?

13. What is the difference between $(2 \times 3 \times 5)^2$ and $2^2 \times 3^2 \times 5^2$?

Is the square of the product of two or more factors equal to the product of the squares of these factors?

535. ***To find the square of a number in terms of its tens and units.***

1. Find the square of 87 in terms of its tens and units.

EXPLANATION.—The product of $80+7$ multiplied by 7 is $80 \times 7 + 7^2$, and the product of $80+7$ multiplied by 80 is $80^2 + 80 \times 7$; hence, $80^2 + 2 \times 80 \times 7 + 7^2$, which is the sum of these partial products, is the square of $80+7$. Hence,

OPERATION.

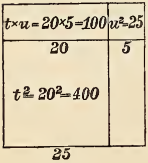
$$\begin{array}{rcl}
87 & = & 80 + 7 \\
87 & = & 80 + 7 \\
\hline
609 & = & 80 \times 7 + 7^2 \\
6960 & = & 80^2 + 80 \times 7 \\
\hline
7569 & = & 80^2 + 2 \times 80 \times 7 + 7^2
\end{array}$$

PRINCIPLE.—*The square of a number composed of tens and units consists of three parts; 1st, the square of the tens; 2d, twice the product of the tens by the units; 3d, the square of the units.*

Denote the tens and units of a number by t and u respectively; then,

FORMULA: $(t + u)^2 = t^2 + 2 \times t \times u + u^2$.

GEOMETRICAL ILLUSTRATION.



Let ABCD be a square, each side of which is 25, and let lines be drawn as indicated in the figure; then it is evident that the square ABCD, which is the square of 25, is composed of two squares, one of which is the square of the tens $(20)^2$, the other the square of the units $(5)^2$, and two rectangles, each of whose areas is 20×5.

1. Find the square of 42 by the preceding principle.

OPERATION.

$$\begin{array}{rl} t^2 = & 40^2 = 1600 \\ 2 \times t \times u = & 2 \times 40 \times 2 = 160 \\ u^2 = & 2^2 = \quad 4 \\ \hline t^2 + 2 \times t \times u + u^2 = & 40^2 + 2 \times 40 \times 2 + 2^2 = 1764 = 42^2. \end{array}$$

In like manner

2. Find the square of 37. Of 56. Of 48. Of 65.
3. Find the second power of 92. Of 125. Of 132.

536. ***To find the cube of a number in terms of its tens and units.***

Find the cube of 87 in terms of its tens and units.

OPERATION.

$$\begin{array}{lll} 87^2 & = 80^2 + 2 \times 80 \times 7 + 7^2 & \\ 87 & = 80 + 7 & \\ \hline 87^2 \times 7 = & & 80^2 \times 7 + 2 \times 80 \times 7^2 + 7^3 \\ 87^2 \times 80 & = 80^3 + 2 \times 80^2 \times 7 + 80 \times 7^2 & \\ \hline 87^3 & = 80^3 + 3 \times 80^2 \times 7 + 3 \times 80 \times 7^2 + 7^3 & \end{array}$$

EXPLANATION.—The square of 87 is $80^2 + 2 \times 80 \times 7 + 7^2$ (**535**), and the product of this square by $80 + 7$ is $80^3 + 3 \times 80^2 \times 7 + 3 \times 80 \times 7^2 + 7^3$, which is the cube of $80 + 7$. Hence,

537. PRINCIPLE. *The cube of a number composed of tens and units consists of four parts: 1st, the cube of the tens; 2d, three times the square of the tens multiplied by the units; 3d, three times the tens multiplied by the square of the units; 4th, the cube of the units. Hence,*

FORMULA: $(t + u)^3 = t^3 + 3 \times t^2 \times u + 3 \times t \times u^2 + u^3$.

In like manner

1. Find the cube of 64. Of 72.
2. Find the third power of 84. Of 125.

GEOMETRICAL ILLUSTRATION.

Fig. 1.

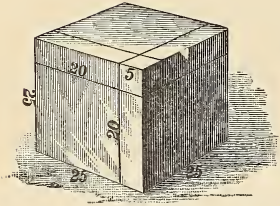
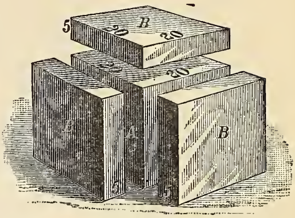
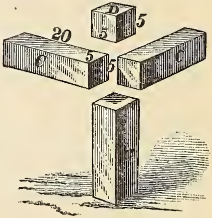


Fig. 2.



Let Fig. 1 represent a cube, the edge of which is 25, and suppose it to be divided into parts as indicated in the figure; then it is evident by Fig. 2 and Fig. 3 that these parts are (1) the cube A whose edge is 20 and volume 20^3, (2) the three equal rectangular solids B, whose combined volumes amount to $3 \times 20 \times 20 \times 5$, or $3 \times 20^2 \times 5$, (3) the three equal rectangular solids C, whose combined volumes amount to $3 \times 20 \times 5^2$, (4) the cube D whose edge is 5 and volume 5^3.

Fig. 3.



3. Find the cube of 37 by the preceding principle.

OPERATION.

$$
\begin{aligned}
30^3 &= 27000 \\
3 \times 30^2 \times 7 &= 18900 \\
3 \times 30 \times 7^2 &= 4410 \\
7^3 &= 343 \\
\hline
& 50653
\end{aligned}
$$

In like manner

4. Find the cube of 25. Of 39. Of 45. Of 51.
5. Find the third power of 96. Of 101. Of 126. Of 142.

EVOLUTION.

DEFINITIONS.

538. A **Root** of a number is either the number itself or one of its equal factors.

Thus, the square root of 49 is 7, for $7^2 = 49$, and the cube root of 64 is 4, for $4^3 = 64$.

The first root of a number is the number itself.

539. Evolution is the process of finding a root of a number. The root is then said to be *evolved* or *extracted.*

Evolution is the *converse* of Involution. In Involution, the root is given and the power is required; in Evolution the power is given and the root is required.

540. The **Radical Sign,** $\sqrt{\ }$, is the character used to indicate that a root is to be extracted.

541. The **Index** of a root is the number placed over the radical sign to show what root is to be extracted or expressed.

Thus, $\sqrt[3]{27}$, $\sqrt[4]{64}$, $\sqrt[5]{32}$ indicate respectively, the cube root of 27, the fourth root of 64, and the fifth root of 32.

The *square root* is usually indicated by the radical sign without the index.

Evolution is sometimes indicated by a fractional exponent. Thus, $9^{\frac{1}{2}}$ is equivalent to $\sqrt{9}$, $27^{\frac{1}{3}}$ is equivalent to $\sqrt[3]{27}$.

The numerator of the exponent indicates a power, and the denominator a root. Thus, $8^{\frac{2}{3}}$ is equivalent to $\sqrt[3]{8^2}$.

SQUARE ROOT.

542. PRINCIPLES. I. *The square of a number contains either twice as many figures as that number, or twice as many, less one.* Thus,

$$2^2 = 4, \qquad 10^2 = 100, \qquad .5^2 = .25,$$
$$9^2 = 81, \qquad 100^2 = 10000, \qquad .05^2 = .0025,$$
$$99^2 = 9801, \qquad 1000^2 = 1000000, \qquad 2.5^2 = 6.25.$$

Hence,

II. *If any perfect square be separated into periods of two figures each, beginning at the place of the decimal point, the number of periods will be equal to the number of figures in the square root of that number.*

If the given number is integral and not a perfect square, the number of periods will be equal to the number of figures in the entire part of the root.

WRITTEN EXERCISES.

543. ***To find the square root of a number.***

1. Find the square root of 625.

OPERATION.

$$\begin{array}{l} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad t \qquad\; u \\ t^2 + 2 \times t \times u + u^2 = 625\,(\,20 + 5 = 25 \\ t^2 \quad = \quad 20^2 \quad = 400 \\ \hline 2 \times t \times u + u^2 = \qquad\qquad\qquad\qquad 225 \\ 2 \times t \times u + u^2 = 2 \times 20 \times 5 + 5^2 = 225 \\ \hline \end{array}$$

EXPLANATION.—Since 625 has two periods, its square root has two figures. Since the square of tens contains no significant figure of a lower order than hundreds, 25 cannot be a part of the square of the tens of the root.

Hence, the tens of the root must be found from the first period 6. The greatest number of tens whose square is contained in 6, is 2. Subtracting the square of 2 tens from 625, the remainder is 225. This remainder consists of twice the product of the tens of the root by the units, and the square of the units (**535**). Hence, if 225 be divided by 40, which is twice the tens of the root, the quotient 5 will be either equal to the units' figure of the root or greater. Now if $2 \times 20 \times 5 + 5^2$, or $(40+5) \times 5$ be subtracted from 225, the remainder will be 0. Hence 25 is the required root.

In practice the work is usually abridged as follows:

```
       6,2 5 ( 2 5
       4
       ─────
4 5 )  2 2 5
       2 2 5
       ─────
```

In this example, 40 is the *partial* or *trial divisor,* and 45 is the *complete divisor.*

Geometrical Illustration.

Let Fig. 1 represent a square whose area is 625 square feet. The area of a square is equal to the square of one of its sides; hence, a side may be found by extracting the square root of the area. The square root of 625 lies between 20 and 30, for 20^2 is less than 625, and 30^2 is greater. The square root of 625 is therefore 2 tens plus an unknown number of units. If the square, whose side is 20 ft. and area 400 sq. feet, be taken from Fig. 1, the remainder, Fig. 2, will contain 225 sq. feet. This remainder consists of two equal rectangles, each of which is 20 ft. long, and a square whose side is equal to the width of the rectangles.

Fig. 1.

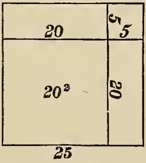
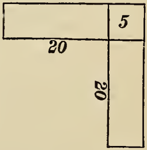


Fig. 2.



If the area and length of a rectangle are known, its width may be found by dividing the area by the length. Now the two rectangles contain the greater part of the 225 sq. feet; hence, 2×20, the length of the two rectangles taken together, may be used as a trial divisor, to find the width.

Dividing 225 by 40, the quotient is 5. But the two rectangles do not contain all the area of Fig. 2; hence, this quotient may be too great. To test this quotient, find the area of Fig. 2, considering 5 feet as the width of the rectangles. Thus, $2 \times 20 \times 5 + 5^2 = (40 + 5) \times 5 = 225$. This is the area which Fig. 2 is known to contain; hence, 5 feet is the true width. Therefore, a side of the given square is 25 feet.

2. Find the square root of 42876304.

OPERATION.

$$
\begin{array}{rl}
 & 42,87,63,04\ (6000+500+40+8=6548 \\
6000^2= & \underline{36\ 00\ 00\ 00} \\
6000\times 2=12000\ ) & 6\ 87\ 63\ 04 \\
(12000+500)\times 500= & \underline{6\ 25\ 00\ 00} \\
6500\times 2=13000\ ) & 62\ 63\ 04 \\
(13000+40)\times 40= & \underline{52\ 16\ 00} \\
6540\times 2=13080\ ) & 10\ 47\ 04 \\
(13080+8)\times 8= & \underline{10\ 47\ 04}
\end{array}
$$

The partial divisor at each step is found by doubling that part of the root already found.

ABRIDGED OPERATION.

$$
\begin{array}{rl}
 & 42,87,63,04\ (6548 \\
 & \underline{36} \\
125\ ) & 687 \\
 & \underline{625} \\
1304\ ) & 6263 \\
 & \underline{5216} \\
13088\ ) & 104704 \\
 & \underline{103704}
\end{array}
$$

3. Find the square root of 232.5625.

OPERATION.

$$
\begin{array}{rl}
 & 2,32.56,25\ (15.25 \\
 & \underline{1} \\
25\ ) & 132 \\
 & \underline{125} \\
302\ ) & 756 \\
 & \underline{604} \\
3045\ ) & 15225 \\
 & \underline{15225}
\end{array}
$$

EXPLANATION.—Begin at the decimal point, and proceed both toward the left and the right to separate into periods; then proceed as in the extraction of the square root of integers. The number of decimal places in the root will be equal to the number of decimal places in the root will be equal to the number of decimal periods in the given number.

4. Find the square root of $\frac{625}{6561}$.

EXPLANATION.—Since a fraction may be raised to any power by raising each of its terms to the required power (**533**), it follows that the square root of a fraction is found by extracting the square root of each of its terms.

OPERATION.

$$\sqrt{\frac{625}{6561}} = \frac{\sqrt{625}}{\sqrt{6561}} = \frac{25}{81}$$

5. Find the square root of $\frac{1024}{2704}$.
6. Find the square root of $\frac{9216}{10816}$.
7. Find the square root of 12321.

RULES.

I. To find the square root of an integer, or of a number containing a decimal.

1. *Separate the given number into periods of two figures each, beginning with the place of the decimal point.*

2. *Find the greatest number whose square is contained in the period at the left. This will be the first figure of the root. Subtract the square of this figure from that period, and to the remainder annex the second period to form a dividend.*

3. *Divide the dividend by twice the first figure of the root, regarded as tens; annex the quotient to the first figure of the root, and also to the divisor; then multiply the divisor thus completed by the second figure of the root, and subtract the product from the dividend.*

4. *If there are more periods to be brought down, proceed as before, using the part of the root already found in the same way as the first figure in the previous process.*

II. To find the square root of a common fraction when both terms are perfect squares.

Extract the square root of the numerator and of the denominator for the terms of the required root.

When the denominator is not a perfect square, reduce the fraction to a decimal, and extract the square root.

Find the square root of

8. 225.	12. 21025.	16. 295.06624.
9. 289.	13. 173056.	17. .14356521.
10. 576.	14. 998001.	18. 2.36144689.
11. 1444.	15. 978121.	19. 2950.6624.

Find the square root of

20. .16.	23. 12.96.	26. $\frac{676}{1024}$.	28. $\frac{3\frac{1}{2}}{4\frac{1}{2}}$.	30. $8\frac{17}{64}$.
21. 1.6.	24. 1.0404.	27. $\frac{5.04}{.021}$.		31. $(\frac{4}{5})^3$.
22. .016.	25. .00081.		29. $17\frac{3}{4}$.	32. $.3^4$.

33. A square field contains 1960000 sq. ft. What is the length of one side?

34. Find the side of a square field equal in area to a rectangular field 2800 yards long and 700 yards wide.

35. A piece of zinc contains 3600 square inches, and its length is 4 times its breadth. What are its dimensions?

36. A square field contains 1 A. 22 P. $7\frac{9}{16}$ sq. yd. Find the length of one side.

37. How much more will it cost to enclose with a fence, 160 A. of land, at $\$4\frac{1}{2}$ a rod, if in the form of a rectangle 320 rd. long and 80 rd. wide, than if in square form?

CUBE ROOT

544. Principles. I. *The cube of a number has three times as many figures as that number, or three times as many, less one, or two.* Thus,

$3^3 = 27$,	$10^3 = 1000$,	$.5^3 = .125$,
$9^3 = 729$,	$100^3 = 1000000$,	$.05^3 = .000125$,
$99^3 = 970299$,	$1000^3 = 1000000000$,	$2.5^3 = 15.625$.

Hence,

II. *If any perfect cube be separated into periods of three figures each, beginning at the place of the decimal point, the number of periods will be equal to the number of figures in the cube root of that number.*

If the number is integral and not a perfect cube, the number of periods will be equal to the number of figures in the *entire* part of the root.

WRITTEN EXERCISES.

545. *To find the cube root of a number.*

1. Find the cube root of 658503.

OPERATION.

$$
\begin{array}{lr|l}
t^3+3\times t^2\times u+3\times t\times u^2+u^3 = & & \quad 658{,}503\,(\overset{t}{80}+\overset{u}{7}=87 \\
t^3 = 80^3 \qquad\qquad\qquad\qquad = & & \quad 512\ 000 \\
\hline
\quad 3\times t^2\times u+3\times t\times u^2+u^3 = & & \quad 146\ 503 \\
\quad 3\times t^2 \quad = 3\times 80^2 \qquad = & 19200 & \\
\quad 3\times t\times u = 3\times 80\times 7 \quad = & 1680 & \\
\qquad\quad u^2 = 7^2 \qquad\qquad = & 49 & \\
\hline
\quad 3\times t^2 \quad +3\times t\times u+u^2 = & 20929 & \\
\end{array}
$$

$$3\times t^2\times u+3\times t\times u^2+u^3 = 20929\times 7 = 146\ 503$$

EXPLANATION.—Since 658503 has two periods, its cube root has two figures. Since the cube of tens contains no significant figure of a lower order than thousands, 503 cannot be a part of the cube of the tens of the root. Hence, the tens of the root must be found from the first period 658. The greatest number of tens whose cube is contained in 658 is 8. Subtracting the cube of 8 tens from 658503, the remainder is 146503. This remainder is composed of three times the square of the tens of the root multiplied by the units, three times the tens multiplied by the square of the units, and the cube of the units (**537**). Hence, if 146503 be divided by three times the square of the tens of the root, that is, by 19200, the quotient 7 will be either equal to the units' figure of the root, or greater. Subtracting $3\times 80^2\times 7+3\times 80\times 7^2+7^3$, or $(3\times 80^2+3\times 80\times 7+7^2)\times 7$ from 146503, nothing remains. Hence, 87 is the required root.

The preceding work may be abridged as follows·

		658,503	(87
	$8^3 =$	512	
$3 \times 80^2 =$	19200	146 503	
$3 \times 80 \times 7 =$	1680		
$7^2 =$	49		
	20929	146 503	

In this example, 19200 is the *partial* or *trial divisor*, and 20929 is the *complete divisor.*

Geometrical Illustration.

Fig. 1.

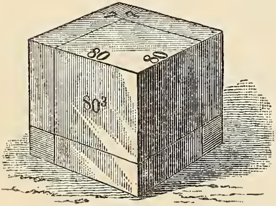
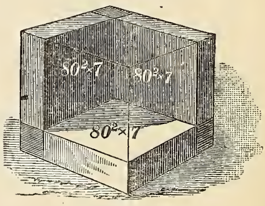


Fig. 2.



Let Fig. 1 represent a cube whose volume is 658503 cubic feet. The volume of a cube is equal to the cube of one of its edges; hence, an edge may be found by extracting the cube root of its volume. The cube root of 658503 lies between 80 and 90; it is therefore 8 tens plus an unknown number of units. If the cube whose edge is 80 feet and volume 512000 cubic feet be taken from Fig. 1, the remainder, Fig. 2, will contain 146503 cubic feet. This remainder consists of solids like those marked B, C, and D, in Fig. 2 and Fig. 3, Art. **537.**

If the volume and the area of the base of a rectangular solid are known, the height, or thickness, may be found by dividing the volume by the area of the base. Now the three equal rectangular solids, each of which is 20 feet square, contain the greater part of the 146503 cubic feet; hence, 3×80^2, or 19200, may be used as a trial divisor, to find the thickness. Dividing 146503 by 19200, the quotient is 7. This quotient may be too great. To test it, find the volume of Fig. 2, considering 7 feet as the thickness. Thus, $3 \times 80^2 \times 7 + 3 \times 80 \times 7^2 + 7^3 = 146503$. Hence, 7 feet is the true thickness.

2. Find the cube root of 95443993.

OPERATION.

		95,443,993 (457
	$4^3 =$	64
1st Partial Divisor $= 3 \times 40^2 =$	4800	31443
$3 \times 40 \times 5 =$	600	
$5^2 =$	25	
1st Complete Divisor $=$	5425	27125
2d Partial Divisor $= 3 \times 450^2 =$	607500	4318993
$3 \times 450 \times 7 =$	9450	
$7^2 =$	49	
2d Complete Divisor $=$	616999	4318993

EXPLANATION.—The partial divisor at each step is found by squaring that part of the root already found and multiplying by 3.

31443 ÷ 4800 gives 6 as the nearest quotient; but 6 will be found to be too large; try 5.

3. Find the cube root of 48228.544.

EXPLANATION.—Begin at the decimal point, and proceed both toward the right and left to separate into periods; then proceed as in the extraction of the cube root of integers. The number of decimal places in the root is equal to the number of decimal periods in the given number.

OPERATION.

		48,228.544 (36.4
	$3^3 =$	27
$3 \times 30^2 =$	2700	21228
$3 \times 30 \times 6 =$	540	
$6^2 =$	36	
	3276	19656
$3 \times 360^2 =$	388800	1572544
$3 \times 360 \times 4 =$	4320	
$4^2 =$	16	
	393136	1572544

4. Find the cube root of $\frac{125}{343}$.

EXPLANATION.—The cube root of a fraction is equal to the cube root of the numerator divided by the cube root of the denominator.

OPERATION.

$$\sqrt[3]{\frac{125}{343}} = \frac{\sqrt[3]{125}}{\sqrt[3]{343}} = \frac{5}{7}.$$

5. Find the cube root of 85184. Of 9.261.

6. Find the cube root of 857375. Of 405.224.

7. Find the cube root of $\frac{4913}{9261}$. Of $\frac{1124864}{1331000}$.

RULES.

I. To find the cube root of an integer, or of a number containing a decimal.

1. *Separate the given number into periods of three figures each, beginning with the place of the decimal point.*

2. *Find the greatest number the cube of which is contained in the period at the left. This will be the first figure of the root. Subtract the cube of this figure from that period, and to the remainder annex the second period to form a dividend*

3. *Divide this dividend by the trial divisor, which is three times the square of the first figure of the root regarded as tens, and write the quotient as the second figure of the root. To the trial divisor add three times the product of the second figure of the root by the first, considered as tens, and the square of the second figure. The result will be the complete divisor. Multiply the complete divisor by the second figure of the root, and subtract the product from the dividend.*

4. *If there are more periods to be brought down, proceed as before, using the part of the root already found in the same way as the first figure in the previous process.*

II. To find the cube root of a common fraction, when both terms are perfect cubes.

Extract the cube root of the numerator and of the denominator for the terms of the required root.

When the denominator of the given fraction is not a perfect cube, reduce the fraction to a decimal and extract the cube root.

Find the cube root of

8. 110592.	12. 17173512.	16. 32.461759.
9. 300763.	13. 259694072.	17. .000912673.
10. 2406104.	14. 926859375.	18. .001906624.
11. 69426531.	15. 219365327791.	19. .000024389.

Find the cube root of

20. 3.	23. $\frac{8}{27}$.	26. $405\frac{28}{125}$.	29. $\frac{23}{729}$.
21. .3.	24. $\frac{250}{863}$.	27. $7\frac{1}{5}$.	30. $38\frac{22}{31}$.
22. .03.	25. 44.6.	28. $\frac{125 \cdot 728}{16384}$.	31. 32.65^2.

Find the second member in the following equations:

32. $\sqrt[3]{2.5^5} - \sqrt[3]{1.44} = ?$ $\sqrt[3]{\frac{1331}{3375}} \times \sqrt[3]{\frac{256}{1024}} = ?$
$27^{\frac{1}{3}} + (103.823)^{\frac{1}{3}} \times (.125)^{\frac{1}{3}} = ?$

33. $\sqrt{16^6} \div \sqrt[3]{125} = ?$ $\sqrt[3]{(4096)^{\frac{1}{2}}} \times \sqrt[3]{\frac{3375}{8000}} \div \sqrt[3]{.5^3} = ?$

34. What is the length of an edge of a cubical box that contains 50653 cubic feet?

The volume of a cube is equal to the cube of one of its edges; hence, an edge is equal to the cube root of the volume.

35. Find the entire surface of a cube whose volume is 37 cu. ft. and 64 cu. in.

36. The edge of a cube is 2 ft. long; find the length of an edge of another cube containing 3 times as much volume.

37. A cubical box contains 9261 cubic inches. How many square inches in one of its faces?

38. Find an edge of a cube of marble which contains 32768 cubic inches.

39. If 108 cords of wood be piled in the form of a regular cube, what will be the length of one of its edges?

40. What is the area of the bottom of a cubical cistern whose capacity is 110592 cubic inches?

ROOTS OF HIGHER DEGREES.

546. Any root, the index of which contains the factors 2 and 3, and no other prime factors, may be extracted by means of the square and cube roots.

If any power of a number be raised to any required power, the result will be that power of the given number denoted by the product of the exponents. Thus, $(2^3)^3 = 2^3 \times 2^3 = 2^6$ (**533,** PRIN.).

Hence, if two or more roots of a number be extracted successively, the result will be that root of the given number denoted by the product of the indices.

WRITTEN EXERCISES.

547. 1. Find the 6th root of 2985984.

EXPLANATION.—The index of the required root is 6, or 2×3; hence, find the square root of the given number, and then the cube root of the result; or, find the cube root of the number, and the square root of the result.

OPERATION.

$$\sqrt{2985984} = 1728$$
$$\sqrt[3]{1728} = 12$$
Or,
$$\sqrt[3]{2985984} = 144$$
$$\sqrt{144} = 12$$

RULE. *Extract, in succession, the roots indicated by the prime factors of the given index; the final result will be the root sought.*

2. Find the 4th root of 1296.
3. Find the 6th root of 2176782336.
4. Find the 8th root of 10995.11627776.
5. Find the 9th root of $512 \times 512 \times 512$.

TEST EXAMPLES.

548. The following examples cover nearly all the practical and important subjects taught in arithmetic, and are designed to test the pupil's knowledge of the principles and processes presented in the preceding pages of this work, and his ability to solve readily the more difficult problems.

The pupil should be required to state briefly, and with clearness and accuracy, so far as the nature of the question will permit, what is given and what is required in each question, the relation of what is given to what is sought, and the steps and processes necessary to obtain the answer.

1. What is the cost of digging a cellar 45 ft. long, 28 ft. wide, and 8 ft. 6 in. deep, at \$.42 a cubic yard?

2. If a town 5 miles square is divided into 150 farms of equal size, what is the area of each farm?

3. How many bushels of wheat will fill a bin 20 ft. long, 12 ft. wide, and 5 ft. deep?

4. Bought R. R. stock at 7% discount, the par value of which was \$5200, and sold the same at $103\frac{1}{2}$. What was the whole gain?

5. A man sold 320 A. of land, which was 20% of 75% of his whole farm. How many acres had he left?

6. A flour merchant had 1000 bbl. of flour insured for 80% of their cost, at $3\frac{1}{4}$%, paying \$214.50 premium. At what price per barrel must he sell the flour, to gain 20% on the entire cost?

7. A cistern has 4 pipes; the first can fill it in 20 minutes, the second in 30 minutes, the third in 40 minutes, and the fourth can empty it in 1 hour. If these pipes are all opened at 2 o'clock P. M., the cistern being empty, at what time will it be filled?

8. How many barrels of water, of $31\frac{1}{2}$ gal. each, will fill a tank 13 ft. long, 10 ft. wide and 10 ft. deep?

9. What will be the expense of plastering a room 40 ft. long, $36\frac{1}{2}$ ft. wide, and $22\frac{1}{4}$ ft. high, at $.36 a sq. yard, allowing 1375 sq. ft. for doors, windows, etc.?

10. How many shares of bank stock, bought at $110\frac{1}{2}$, and sold at $116\frac{3}{4}$, brokerage $\frac{1}{8}\%$ each way, will gain $1200?

11. What decimal part of 1 acre is a piece of land 121 yd. long, and 75 ft. wide?

12. A merchant marked a piece of goods 25% above cost, but anxious to effect a sale, sold it at a discount of 20% from the marked price, supposing he should still gain 5%. Did he gain or lose?

13. Three hundred seventy-five thousandths of the lumber in a yard was destroyed by fire. If the entire lot was worth $10000, what was the loss of a firm that owned .12 of it?

14. Reduce $\left(\frac{1\frac{3}{4}}{4\frac{1}{2}} \div \frac{2\frac{1}{3}}{2\frac{1}{4}}\right) \times \frac{4}{5}$ of $\frac{1}{2}$, to a decimal.

15. Bought a quantity of coffee at 20 cts. a pound. Allowing a loss of 5% in weighing it out, and 10% of the sales for bad debts, at what price per pound must it be sold, to gain 14% on the cost?

16. What will 20 yards of cloth cost in U. S. money, if $4\frac{1}{2}$ yd. cost £6 8s. 10d. sterling?

17. Reduce $.8\dot{3}$, $.\dot{9}0\dot{9}$, $.47\dot{7}\dot{2}$, and $.\dot{1}4285\dot{7}$ to equivalent common fractions, and find their sum.

18. For what sum must a note be drawn at 60 da., to net $2500, if discounted at 6%?

19. At what price must books, bought at 10 and 5% off from $1 per copy, and on 4 mo. time, be sold for cash, to make a profit of 20%, money being worth 6%?

20. If 46 lb. of tea are worth 184 lb. of coffee, how many pounds of tea are worth 126 lb. of coffee?

21. What length must be cut from a plank $3\frac{1}{2}$ in. thick, and $6\frac{1}{4}$ in. wide, to contain $37\frac{1}{2}$ ft. board measure?

22. How must damaged flour, that cost $6.80 a barrel, be marked, so that 20% may be abated, and sell at cost?

23. If a road of uniform grade rises 462 ft. in $1\frac{1}{2}$ mi., how much does it rise in 100 ft.?

24. What must be the asking price for tea that cost $.56 a pound, in order to abate 10%, and still make a profit of 20%?

25. If a house worth $12500 rents for $937.50 per annum, what per cent of the value of the house is the rent?

26. What amount must be paid for insuring a vessel worth $24500, to save the premium in case of loss, the rate of insurance being $4\frac{1}{2}$%?

27. If it take $3\frac{1}{2}$ yd. of stuff, of a certain width, to line $2\frac{3}{4}$ yd. of another breadth, what quantity of the latter will be required to line $24\frac{1}{2}$ yd. of the former?

28. At 6 cts. a foot, board measure, what is the cost of 6 pieces of sawed timber, each measuring 24 ft. long, 1 ft. 8 in. wide, and 10 in. thick?

29. An agent in London bought a quantity of wine, amounting to £1650 15s. 6d. What was his commission in U. S. money, at $2\frac{1}{2}$%?

30. Three adjacent tracts of land, containing respectively, 362 A. 100 P., 464 A. 120 P., and 484 A. 56 P., are divided into 24 lots of equal size. How much does each lot contain?

31. How many rods of fencing will be required to enclose 44 A. 16 P. in a square form?

32. I hold a bond dated May 1, 1876, for \$1300, bearing interest at 6%, payable annually, but the interest remains unpaid. What amount is due May 1, 1882?

33. How many men can do a piece of work in 84 da., that 54 men can do in 126 days?

34. Find the side of a square field, the area of which is 48 A. 10 P. $22\frac{1}{2}$ sq. yd. 4 sq. ft.

35. If I buy goods for \$1500 cash, how large a note payable in 60 da., at 7% bank discount, must I make, that the proceeds shall pay for the goods?

36. What is the entire surface of a cube, the contents of which are 42875 cubic feet?

37. If goods are marked at two prices, one for cash and the other on 6 mo. credit, what ratio ought the prices to bear to each other, money being worth 6% per annum simple interest?

38. A bond for \$500, dated May 1, 1877, bearing interest at 6%, has the following indorsements upon it: May 1, 1878, \$100; May 1, 1879, \$50; June 10, 1880, \$20; Aug. 1, 1880, \$250. What amount is due on the bond May 1, 1881?

39. How many acres of land in a farm $\frac{1}{3}$ of a mile long, and $\frac{2}{7}$ of a mile wide?

40. If a rent of \$400, payable annually, remains unpaid, what will be due the landlord at the end of 4 years, allowing compound interest at 6%?

41. What is the face of a bill of exchange on London, in sterling, that costs \$880, premium 10%?

42. A can do a piece of work in 9 da., which B can do in 12 da., and C. in 18 da. In what time can the three do it, working together?

43. What will it cost to enclose a garden that is $12\frac{2}{3}$ rods long, and $9\frac{1}{4}$ rods wide, with a stone wall 4 ft. high, and $3\frac{1}{2}$ ft. thick, at \$.$62\frac{1}{2}$ a perch?

44. What sum invested in bank stock, at par, paying 8% annually, will yield a quarterly dividend of $450?

45. Which is the cheaper, a suit of clothes that costs $48 and will last 18 months, or a suit that costs $30 and will last 10 months; and how much will a man save in 15 years, who wears the cheaper suit, interest not being reckoned?

46. A note for $324.61 was paid at the end of 2 yr. 7 mo. 13 da., the amount being $384.131. What was the rate %?

47. A broker receives $5270 to be invested in stock, after retaining 3% commission on the money paid out. How much money did he pay out, and how much did he retain for his commission?

48. A man, engaging in trade, added 20% to his capital the first year; to his capital so increased he added 25% the second year; and the third year he added 28%. What per cent of the capital with which he commenced, was his capital at the end of the third year? If he commenced with $5000, how much had he at the close of the third year?

49. A traveler on arriving at Columbia, Mo., found his watch, which indicated the correct time of the place from which he started, 1 hr. 4 min. fast. How many degrees of longitude had he traveled over, and in what direction?

50. How large must a cubical bin be, to hold 500 bushels of wheat?

51. The taxable property in a town of 871 polls is $3407160. A tax of $33456.77 is to be raised. If a poll-tax of $1.25 is levied, what should be the rate of tax?

52. If a publisher sells to the jobber a book at 40% above the cost of manufacture, and the jobber sells to the retail merchant at 20% advance, who in turn makes 25% by selling it at $1, what is the cost of manufacture?

53. Bought a bill of exchange on Paris for 6500 francs, at 5.22, $\frac{1}{4}$% brokerage. What did it cost in U. S. money?

54. A mason wishes to make a cubical cistern that shall contain 250 barrels of water. What must be the length of its edges?

55. What must be paid for paving a court 50 ft. 6 in. by 36 ft., at $.30 a square yard?

56. What is the cost of 500 boards, one-half of them 12 ft. 6 in. long, and 10 in. wide, the other half 13 ft. long and 9 in. wide, at $3 per C., board measure?

57. Bought goods of A. S. Barnes & Co., amounting to $728.50, for which I pay $\frac{2}{5}$ cash, and give a 4 mo. note, with interest at 6% added, for the remainder. What is the face of the note?

58. Imported from Havre 160 baskets of Champagne, 12 bottles each, 5% breakage, duty 40%, freight and other charges $134.40, the whole cost being $1459.20. What did it cost per bottle in Havre, what in store, and what must it be sold for per bottle to clear 35%?

59. Wm. Gallup & Co. bought goods of Peake & Co. as follows: July 1, $150, at 3 mo.; July 20, $200, at 4 mo.; Aug. 16, $300, at 2 mo.; and Oct. 4, $250, at 4 mo. Find the equated time of payment, and what would be due on the account March 15 following, if nothing had been paid, at 6% interest?

60. In what time will $800, at 6% simple interest, yield an amount equal to itself?

61. If, on a note made for $700, bearing interest at 6%, and dated Jan. 1, 1881, $50 is paid on the first of every month, commencing Feb. 1, following the date, what is due Jan. 1, 1882?

62. A man hired a house for $500 a year, payable in quarterly payments at the end of the quarter, but he wishes to pay the whole at the commencement of the year, discount being at the rate of 6% per annum. What must he pay?

63. A starts on a journey at the rate of $3\frac{1}{2}$ miles an hour; B follows in $1\frac{1}{2}$ hours at the rate of 4 miles an hour. How far must B travel to overtake A?

64. A regiment of 745 men is to be clothed, each suit to contain $3\frac{1}{2}$ yd. of cloth, $1\frac{3}{8}$ yd. wide, and lined with serge, $\frac{7}{8}$ yd. wide. How many yards of serge will be required?

65.

$\$1824\frac{60}{100}$ INDIANAPOLIS, May 7, 1881.

For value received, eighteen months after date, I promise to pay to the order of M. R. BARNARD, Eighteen hundred twenty-four $\frac{60}{100}$ dollars, with interest at 10 per cent.

J. E. BOWEN.

Discounted June 24, 1882, at $1\frac{1}{4}\%$ a month.

Find the *date of maturity*, *term of discount*, and *proceeds*.

66. Bought 50 yards of broadcloth, at $2.25 a yard, and a certain number of yards of flannel. The cost of both was $169.20, and for every 5 yards of broadcloth there were 9 yards of flannel. How many yards of flannel were bought, and at what price per yard?

67. At $\$1.87\frac{1}{2}$ a bushel, what is the value of the wheat filling a bin $9\frac{1}{2}$ ft. long, 6 ft. wide, and 4 ft. 3 in. deep?

68. What will be the cost of raising the surface of a half acre of ground 9 inches, at $.75 per cubic yard?

69. HORACE P. GRIFFIN.

Dr. *Cr.*

1880.			1880.		
Jan. 4	To Mdse. @ 4 mo.	$375.25	July 4	By Cash. .	$300
Feb. 19	" " "	420.60	Aug. 14	" "	300
May 4	" " "	372.50	Sept. 4	Note, 2 mo.	300

Find the *cash balance* on this account on July 4, 1882, allowing interest at 7%.

70. Compare the amounts of a \$500 8% note, running 4 years, at simple interest, at annual interest unpaid, and at compound interest.

71. If a manufacturer sells to a jobber at 15% profit, and the jobber sells to the retailer at 20% profit, and the retailer sells to the consumer at a profit of 25%, what part of the price paid by the consumer is profit?

72. A man harvested 72 bushels of wheat, which was only $\frac{3}{5}$ more than he sowed. How much did he sow?

73. Smith owes Jones four several amounts as follows: \$264.50, due in 60 days; \$374.25, due in 75 da.; \$375.60, due in 80 da.; and \$120.50, due in 90 da. Find the average term of credit, and the equated time of payment from April 12.

74. A wheel 15 ft. in circumference revolves 600 times in going over a certain distance. What is the distance?

75. A mechanic worked 1 yr. 5 mo. for a monthly salary of \$75, payable at the end of the month, but remaining unpaid until the end of his service. What amount is he entitled to receive, allowing simple interest at 7% per annum?

76. A man bought 75 shares of bank stock at 98½. He held it for 3 yr. 6 mo., receiving a semi-annual dividend of 4% on the same, when he sold it at 105. Money being worth 6% simple interest, what was his gain?

77. James Webb purchased goods of Davis, Banks & Co. to the amount of \$4000; \$1000 payable April 2, \$1200 payable May 5, \$750 payable June 15, and the remainder June 30. At what date must a note for the full amount be drawn, payable in 3 months, to fall due at the average date?

78. Three men undertake a job for \$750. The first furnishes 5 men 9 da.; the second, 6 men 8 da.; and the third 12 men 3 da. What share of the money is each entitled to receive?

79. I sold $\frac{1}{2}$ of a lot of land for $\frac{3}{4}$ of the cost of the lot, then sold the remaining half for $\frac{1}{2}$ of what I received for the first part. What per cent did I gain on the whole debt?

80. I hold three notes against Amos Dix, as follows: One dated Oct. 10, for $200, at 2 mo.; another dated Nov 10, for $250, at 3 mo.; and the other dated Dec. 10, for $200, at 3 mo.; Jan. 10 he paid $300. From what date should interest be reckoned on the remainder?

81. A dealer paid $364 for 40 bbl. of flour, giving $10 for first quality, and $7 for second quality. How many barrels were there of each?

82. When cotton is quoted in New Orleans at 16 cts. a pound, what should it be worth in Liverpool, exchange being quoted at 4.87$\frac{1}{2}$, and freight 1$\frac{1}{4}$ cents a pound?

83. Jan. 1, 1881, three men form a partnership for 1 year, each furnishing $2000 capital. May 1, A puts in $1000 more; June 1, B puts in $1500, and C draws out $500; Oct. 1, A withdraws $500, and B and C put in $1000 each. Having gained $4000, at the close of the year the partnership is dissolved. What is each partner's share of the gain?

84. How many tons of water will a cistern contain that is 8 ft. 8 in. long, 6 ft. 4 in. wide, and 6 ft. 9 in. deep, allowing a cubic foot of water to weigh 1000 oz.?

85. R. B. Maddex held a bond against John Walker, dated May 1, 1877, for $1500, bearing interest at 6%. Payments were made on this bond as follows:

June 1, 1878, $597.50.	May 1, 1880, $100.
July 1, 1879, $265.	July 11, 1881, $400.
Nov. 10, 1879, $25.	

What was due Dec. 26, 1882?

86. What must be the length of an edge of a cubical cistern that will contain 4913 cubic feet?

87. What principal must be loaned May 1, at 8%, to be repaid by 4 installments of $2000 each, payable on the first day of the four succeeding months?

88. From a piece of ground 25 rd. long and 15 rd. wide, a lot 25 ft. by 100 ft. was sold for $300. What is the value of the remainder at the same rate?

89.

$756. St. Louis, Jan. 1, 1880.

One year after date, I promise to pay to the order of Chas. Tuthil, seven hundred fifty-six dollars, with six per cent interest, for value received.

Charles Gay.

Indorsed as follows: April 1, 1880, $12; July 1, 1880, $30; Nov. 1, 1880, $325. Find the amount due Jan. 1, 1881, first by the mercantile rule (**446**); next by accurate interest (**424**); and then by U. S. Rule (**444**), allowing 360 days to the year.

90. Divide $1260 among 3 persons, so that the second shall have $\frac{3}{4}$ as much as the first, and the third $\frac{1}{2}$ as much as the other two together.

91. If 10 men can build a wall in $7\frac{1}{2}$ da., and 4 of them are absent at first $2\frac{1}{2}$ days, how many more men must they bring with them to complete the work in the same time?

92. How many boards of the longest possible equal length will enclose a rectangular field 9893 ft. long, and 8047 ft. wide, with a straight fence 6 boards high?

93. Bought a bill of goods, for which I paid $\frac{2}{5}$ cash, and gave my note at 4 mo. with 6 per cent. interest added, for the remainder. The face of the note was $1689.12; what was the amount of the bill?

94. Which is the better investment, railroad stock paying an annual dividend of 6%, bought at a discount of 25%, or money loaned at 10%, payable annually? What per cent. better?

APPENDIX.

PROGRESSIONS.

DEFINITIONS.

549. A **Progression** consists of a collection of numbers arranged in the order of their magnitudes, each of which is greater or less than the preceding one, either by a constant *difference*, or in a constant *ratio*.

Thus, 1, 3, 5, 7, 9, 11, 13, 15,

is a progression in which each number exceeds the preceding one by 2, and

1, 3, 9, 27, 81, 243, 729,

is a progression in which each number is *three* times the preceding one.

550. The **Terms** of a progression are the numbers of which it consists.

551. The **Extremes** of a progression are the first and last terms; the other terms are the **Means.**

552. An **Increasing Progression** is a progression in which each term is *greater* than the preceding one.

Thus, 2, 4, 6, 8, 10, 12, is an increasing progression.

553. A **Decreasing Progression** is a progression in which each term is *less* than the preceding one.

Thus, 11, 9, 7, 5, 3, 1, is a decreasing progression.

554. An **Arithmetical Progression** is a progression in which the difference between any two consecutive terms is *constant*, and this constant difference is called the **Common Difference.**

Thus, 1, 4, 7, 10, 13, 16, is an arithmetical progression, whose *common difference* is 3.

The initials A. P. are used for *Arithmetical Progression.*

555. A **Geometrical Progression** is a progression in which the ratio of any term to the preceding term is *constant*, and this constant ratio is called **The Ratio** of the progression.

Thus, 1, 3, 9, 27, 81, 243, is a geometrical progression whose *ratio* is 3.

The initials G. P. are used for *Geometrical Progression.*

ARITHMETICAL PROGRESSION.

WRITTEN EXERCISES.

556. ***To find one of the extremes of an A. P., when the other extreme, the common difference, and the number of terms are given.***

1. Find the last term of an increasing A. P., the first term of which is 3, the common difference 2, and the number of terms 7.

OPERATION.

$$3 + 6 \times 2 = 15$$

EXPLANATION.—The second term is $3 + 2$; the third term is $3+2+2$, or $3+2\times 2$; the fourth term is $3 + 2 + 2 + 2$, or $3 + 3 \times 2$; and so on; hence, the seventh or *last term* is $3+6\times 2$, or 15.

2. Find the first term of an increasing A. P., the last term of which is 15, the common difference 2, and the number of terms 7.

EXPLANATION.—It follows, from the 1st example, that the 1st term must be a number, to which if 6×2 be *added*, the sum will be 15; hence, if 6×2 be *subtracted* from 15, the remainder 3 will be the *1st term*.

OPERATION.
$15 - 6 \times 2 = 3$

3. Find the last term of a decreasing A. P., the first term of which is 17, the common difference 2, and the number of terms 9.

EXPLANATION.—The second term is $17-2$; the third term is $17-2 \times 2$; the fourth term is $17-3 \times 2$; and so on; hence, the ninth or *last term* is $17-8 \times 2$, or 1.

OPERATION.
$17 - 8 \times 2 = 1$

4. Find the first term of a decreasing A. P., the last term of which is 1, the common difference 2, and the number of terms 9.

EXPLANATION.—It follows, from the 3d example, that the 1st term must be a number, from which if 8×2 be *subtracted*, the remainder will be 1; hence, if 8×2 be *added* to 1, the sum 17 will be the *1st term*.

OPERATION.
$1 + 8 \times 2 = 17$

FORMULAS:

In Increasing A. P.:

1. *Last term* = *First term* + *No. of terms less 1* × *Common Difference.*
2. *First term* = *Last term* — *No. of terms less 1* × *Common Difference.*

In Decreasing A. P.:

3. *Last term* = *First term* — *No. of terms less 1* × *Common Difference.*
4. *First term* = *Last term* + *No. of terms less 1* × *Common Difference.*

5. Find the last term of an increasing A. P., in which the first term is 5, the com. diff. 4, and the number of terms 10.

6. Find the 50th term of an increasing A. P., in which the first term is 2, and the common difference 3.

7. Find the 32d term of a decreasing A. P., in which the first term is 100, and the common difference 3.

8. Find the last term of an increasing A. P., in which the first term is $\frac{3}{4}$, the com. diff. $\frac{3}{8}$, and the number of terms 25.

557. ***To find the common difference, when the extremes and the number of terms are given.***

1. The extremes of an A. P. are 15 and 3, and the number of terms 7; what is the common difference?

OPERATION.

$$(15 - 3) \div (7 - 1) = 2.$$

EXPLANATION. — The difference between the extremes is equal to the product of the common difference by the number of terms less 1 (**556**, Form. 1); hence, the *common difference* is equal to $(15 - 3) \div 6 = 2$.

FORMULA: $$\textit{Common Difference} = \frac{\textit{Diff. of Extremes}}{\textit{No. of terms less 1}}.$$

2. Find the common difference in an A. P. of 8 terms, whose extremes are 3 and 18.

3. The extremes of an A. P. are 1 and 51, and the number of terms 26; what is the common difference?

4. The extremes are 0 and $2\frac{1}{2}$, and the number of terms is 18; what is the common difference?

558. ***To find the sum of the terms of an A. P., when the extremes and the number of terms are given.***

1. The extremes of an A. P. are 5 and 20, and the number of terms 6; what is the sum of the terms?

OPERATION.

$$\begin{aligned}
\text{Sum} &= 5 + 8 + 11 + 14 + 17 + 20; \\
\text{Sum} &= 20 + 17 + 14 + 11 + 8 + 5; \\
\hline
\text{Sum} \times 2 &= 25 + 25 + 25 + 25 + 25 + 25; \\
\text{Sum} \times 2 &= (5 + 20) \times 6; \\
\text{Sum} &= (5 + 20) \times \tfrac{6}{2} = 75.
\end{aligned}$$

EXPLANATION.—The common difference is 3 (**557**); hence, the sum $= 5 + 8 + 11$, etc., or the sum $= 20 + 17 + 14$, etc. Adding the corresponding terms of these two progressions, we have the sum $\times 2 = 25 \times 6 = (5 + 20) \times 6$; hence, the sum $= (5 + 20) \times \frac{6}{2} = 75$, the *sum of the terms.*

FORMULA *Sum = Sum of Extremes × ½ the No. of terms.*

2. The extremes of an A. P. are 8 and 80, and the number of terms 7; what is the sum of the terms?

3. The extremes of an A. P. are 2 and 2000, and the number of terms 1000; what is the sum of the terms?

4. A man traveled 14 miles the first day, 18 the second, 22 the third, and so on; how far did he travel in 12 days?

GEOMETRICAL PROGRESSION.

559. ***To find one of the extremes of a G. P., when the other extreme, the ratio, and the number of terms are given.***

1. The first term of a G. P. is 3, the ratio 2, and the number of terms 10; what is the last term?

OPERATION.

$$3 \times 2^9 = 1536.$$

EXPLANATION.—The second term is 3×2; the third term is $3 \times 2 \times 2$, or 3×2^2; the fourth term is $3 \times 2^2 \times 2$, or 3×2^3; and so on; hence, the *tenth* term is 3×2^9, or 1536.

2. The last term of a G. P. is 1536, the ratio 2, and the number of terms 10; what is the first term?

OPERATION.

$$1536 \div 2^9 = 3.$$

EXPLANATION.—It follows, from the first example, that the first term must be a number by which, if 2^9 be *multiplied*, the product will be 1536; hence, if 1536 be *divided* by 2^9, the quotient 3 will be the *first term.*

FORMULAS:

1. *Last term = First term × Ratio raised to the power whose exponent is the No. of terms less 1.*
2. *First term = Last term ÷ Ratio raised to the power whose exponent is the No. of terms less 1.*

3. The first term of a G. P. is 7, and the ratio 4; what is the 8th term?

4. The first term of a G. P. is 1024, and the ratio $\frac{1}{4}$; what is the 10th term?

5. The last term of a G. P. is 192, the ratio 2, and the number of terms 9; what is the first term?

6. A man traveled 2 miles the first day, 4 miles the second, 8 miles the third, and so on; how far did he travel on the 7th day?

7. Find the amount of $500 for 10 years, at 10%, compound interest.

The first term is $500, the ratio 1.10, and the number of terms 11.

560. ***To find the sum of the terms of a G. P., when the extremes and ratio are given.***

1. The extremes of a G. P. are 3 and 192, and the ratio 4; what is the sum of the terms?

OPERATION.

$$\text{Sum} \times 4 = 3 \times 4 + 12 \times 4 + 48 \times 4 + 192 \times 4;$$
$$\text{Sum} = 3 + 12 + 48 + 192;$$
$$\text{Sum} \times 3 = 192 \times 4 - 3;$$
$$\text{Sum} = \frac{192 \times 4 - 3}{3} = 255.$$

EXPLANATION.—Subtracting the sum from the sum × 4, the remainder is 192 × 4 − 3; hence, the *sum of the terms* $= \frac{192 \times 4 - 3}{3} = 255.$

2. The extremes of a G. P. are $\frac{1}{2}$ and $\frac{1}{54}$, and the ratio $\frac{1}{3}$; what is the sum of the terms?

OPERATION.

$$\text{Sum} = \tfrac{1}{2}+\tfrac{1}{6}+\tfrac{1}{18}+\tfrac{1}{54};$$
$$\text{Sum} \times \tfrac{1}{3} = \tfrac{1}{2} \times \tfrac{1}{3} + \tfrac{1}{6} \times \tfrac{1}{3} + \tfrac{1}{18} \times \tfrac{1}{3} + \tfrac{1}{54} \times \tfrac{1}{3};$$
$$\text{Sum} \times \tfrac{2}{3} = \tfrac{1}{2} - \tfrac{1}{54} \times \tfrac{1}{3};$$
$$\text{Sum} = (\tfrac{1}{2} - \tfrac{1}{54} \times \tfrac{1}{3}) \div \tfrac{2}{3} = \tfrac{20}{27}.$$

EXPLANATION.—Subtracting the sum $\times \frac{1}{3}$ from the sum, the remainder is $\frac{1}{2}-\frac{1}{54}\times\frac{1}{3}$; hence, the *sum of the terms* $= (\frac{1}{2}-\frac{1}{54}\times\frac{1}{3})\div\frac{2}{3} = \frac{20}{27}$.

FORMULA: *Sum = Difference between 1st term and product of the last term by the ratio ÷ Difference between the ratio and 1.*

3. The extremes of a G. P. are 1 and 128, and the ratio 2; what is the sum of the terms?

4. The extremes of a G. P. are $2\frac{2}{5}$ and $\frac{4}{405}$, and the ratio $\frac{1}{3}$; what is the sum of the terms?

5. Find the sum of the first 10 terms of the progression 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, etc.

6. The first term of a G. P. is 5, the ratio 2, and the number of terms 8; what is the sum of the terms?

7. What is the sum of the first 7 terms of the progression 2, 6, 18, etc.?

8. A man performed a journey in 7 days, traveling 5 mi. the first day, 10 mi. the second, 20 mi. the third, and so on; what was the length of the journey?

9. If a man were to buy 20 mules, paying 1 cent for the first, 3 cents for the second, 9 cents for the third, and so on; what would he pay for the whole?

10. A man agreed to labor 10 days upon the following terms: 1 mill for the first day, 1 cent for the second, 1 dime for the third, and so on in geometrical progression; what should he receive for his 10 days' labor?

ANNUITIES.

DEFINITIONS.

561. An **Annuity** is a sum of money payable at the end of any equal intervals of time.

562. A **Certain Annuity** is an annuity which continues for a stated time.

563. A **Perpetual Annuity,** or a **Perpetuity,** is an annuity which is to continue forever.

An annuity is said to be *forborne,* or *in arrears,* if the payments were not made when due.

564. The **Amount,** or **Final Value,** of an annuity is the sum of all the payments and the interest on them from the time they become due until the annuity ceases.

565. The **Present Worth** of an annuity is the sum of money which will, in the given time, and at the given rate per cent, amount to the final value of the annuity.

ANNUITIES AT SIMPLE INTEREST.

566. The formulas of Arithmetical Progression and of Simple Interest are applicable to problems in annuities at simple interest.

WRITTEN EXERCISES.

1. What is the amount of an annuity of $100 a year, for 5 years, at 7% simple interest?

OPERATION.

$$(\$100 + \$128) \times \tfrac{5}{2} = \$570$$

EXPLANATION.—The 5th payment bears no interest, because it is not due till the end of the 5th year; the 4th payment bears interest 1 yr.; hence, the amount of this payment is \$107; the amount of the 3d payment is \$114; and so on. These sums form an A. P in which the first term is the annuity, the common difference is the interest on the annuity for 1 yr., and the number of terms is the number of years. The sum of the terms of this progression is \$570 (**558**).

2. What is the present worth of an annuity of \$300 a year, for 5 years, at 6%, simple interest?

OPERATION.

$$(\$300 + \$372) \times \tfrac{5}{2} = \$1680;$$
$$\frac{\$1680}{1.30} = \$1292.30+.$$

EXPLANATION.—The amount of the annuity is \$1680, and the present worth of this amount is \$1292.30 (**452**, Form. 1).

3. What is the amount of an annuity of \$50 a quarter, for $4\frac{1}{2}$ years, at 2% per quarter?

4. What is the amount of an annuity of \$1000 a year, payable semi-annually, for 30 yr., at 6% per annum, simple interest?

5. What is the present worth of an annuity of \$500 a year, for 10 years, at 10%, simple interest?

6. What is the present worth of a perpetuity of \$600 a year, money being worth 8%, simple interest?

7. What is the amount of an annuity of \$1200 a year, payable monthly, for 16 yr., at 5% per annum, simple interest?

8. An annuity of \$500 a year is in arrears for 10 years; what is the amount at 6%, simple interest?

9. A farm is rented for 20 years, at \$500 a year, payable annually. What is the present value of the rent, at 6% simple interest?

ALLIGATION.

567. Alligation is a process of computation based upon the mixing of several ingredients of different values, so as to form a compound.

The term *alligation*, signifying *binding*, is used because in the calculation it is usual to link together the prices of different simples, according to a certain principle or rule.

568. Alligation Medial is the process of finding the value of each unit of a compound formed by mixing together simples, or ingredients of different values.

569. Alligation Alternate is the process of finding what proportion of several ingredients of different values are required to form a compound of a particular value.

570. The **Mean Value** is the average value of a unit of the compound.

WRITTEN EXERCISES.

571. ***To find the mean value of a mixture consisting of several ingredients of different values.***

1. If a grocer mix 50 lb. of sugar worth 7 cts. a pound, with 30 lb. worth 8 cts., and 40 lb. worth $9\frac{1}{2}$ cts., what is the value of the mixture prepared?

EXPLANATION.—The values of the ingredients are respectively 350 cts., 240 cts., and 380 cts.; and the value of the whole compound of 120 lb. is 970 cts. Hence, 1 lb. is worth $\frac{1}{120}$ of 970 cts., or $8\frac{1}{12}$ cts.

OPERATION.

$$\begin{array}{rl} 50 \times 7 & = 350 \\ 30 \times 8 & = 240 \\ 40 \times 9\frac{1}{2} & = 380 \\ \hline 120 &)\,970 \\ & 8\frac{1}{12} \text{ cts.} \end{array}$$

FORMULA: $\textit{Mean Value} = \dfrac{\textit{Whole value of mixture}}{\textit{Total quantity of ingredients}}.$

2. If 20 lb. of tea, worth 40 cts. a pound, 30 lb. worth 60 cts., 40 lb. worth $32\frac{1}{2}$ cts., and 60 lb. worth 38 cts., be mixed together, what will the mixture be worth a pound?

3. A miller mixes 25 bu. of wheat, at \$1.20, with 30 bu. at \$.95, and 40 bu. at \$1.05; and sells the whole at \$1 per bushel. What does he lose per bushel?

4. A person mixed 36 gal. of brandy at \$2.40, 24 gal. at \$2.75, and 30 gal. at \$3.10, with 35 gal. of water. What is the mixture worth per gallon?

572. ***To find the proportional quantities of ingredients of different values required to form a mixture of a given mean value.***

1. What proportion of teas, worth respectively 30, 40, 70, and 90 cents a pound, must be mixed together to form a compound worth 50 cents a pound?

OPERATION.

50	30	40	4	120
	40	20	2	80
	70	10	1	70
	90	20	2	180
			9	) 450
				50

50	30	20	2	60
	40	40	4	160
	70	20	2	140
	90	10	1	90
			9	) 450
				50

EXPLANATION.—By linking together *any two* prices of which one is *greater* and the other *less* than the mean value, we are enabled to ascertain how much of the one is needed to offset a certain quantity of the other; that is, the proportion of each required in the mixture.

Thus, if we take 1 lb. at 30 cts. we gain 20 cts. in a mixture sold at 50 cts.; and if we take 1 lb. at 90 cts., we lose 40 cts.; hence, the quantities of these to be taken must be in the ratio of 40 to 20, or 4 to 2; and in like manner, the quantities at 40 cents and 70 cts., must be as 20 to 10, or 2 to 1. Consequently, the quantities required will be as 4, 2, 1, and 2.

By linking 30 with 70, and 40 with 90, the proportional quantities will be 70—50, or 20 ; 50—30, or 20, for that couplet ; and 90—50, or 40 ; and 50—40, or 10, for the other couplet ; or 20, 40, 20, and 10, which are to each other as 2, 4, 2, 1.

When it is necessary to link one price with two or more, the several differences between the latter, and the mean value, must be added together to find the quantity of that to which they are linked. The *proof* is by an application of the first case.

2. A farmer has sheep worth $3½, $5, $6, and $8 a head. What number of each should he sell to obtain an average price of $5½ a head ?

3. What proportion of sugars, worth respectively 7 cents, 9 cents, 10 cents, 15 cents, and 20 cents per pound, must a grocer mix to be able to sell the mixture at 12 cents a pound ?

4. A dry goods merchant has muslins worth, respectively, 15 cts., 17 cts., 18 cts., 19 cts., and 20 cts. a yard. How many yards of each must he sell so as to realize an average price of 17½ cts. per yard ?

573. Connecticut Rule for Partial Payments.

If, at the time of a payment, a year's interest or more has accrued, and always in case of the last payment, follow the United States Rule.

If, at the time of payment, LESS *than a year's interest has accrued, except it be the last payment, find the difference between the amount of the principal for an* ENTIRE *year, and the amount of the payment for the remainder of a year after it is made, and this difference will form the new principal.*

If, at the time of payment, the interest accrued exceeds the payment, compute interest on the principal only.

574. VERMONT RULE FOR PARTIAL PAYMENTS.

The Statutes of Vermont make the following provisions for computing interest on notes when partial payments have been made:

On all notes or other similar obligations, whether made payable on demand or at a specified time, WITH INTEREST, *when payments are made, such payments shall be applied:*

1st. To liquidate the interest that has accrued at the time of such payments;

2d. To the extinguishment of the principal.

On all notes, bills, or other similar obligations, whether made payable on demand or at a specified time, WITH INTEREST ANNUALLY, *the annual interests that remain unpaid shall be subject to simple interest from the time they become due to the time of final settlement; but if in any year, reckoning from the time such annual interest began to accrue, payments have been made, the amount of such payments at the end of such year, with interest thereon from the date of payment, shall be applied:*

1st. To liquidate the simple interest that has accrued upon the unpaid annual interests;

2d. To liquidate the annual interests that have become due;

3d. To the extinguishment of the principal.

575. THE NEW HAMPSHIRE RULE.

Is the same as the Vermont Rule, except as follows:

1st. Payments made expressly on account of interest accruing but not yet due, do not draw interest.

2d. Such payments, at the end of the year, must be applied to the payment of the interest then due.

METRIC SYSTEM

576. The **Metric System** is a system of weights and measures based upon the *decimal notation.*

577. The **Meter** is the *unit* of length. It is also the fundamental unit, since from it every other unit of measure or weight is derived; hence the name *Metric System.*

578. The following are the *Standard* units, from which all the other units of the system are derived:

Units.	Names.		Derivation.		Signification.
LENGTH.	Mē´ter,	from the Greek	Μέτρον,	signifying	Measure.
SURFACE.	Är,	" Latin	*Area,*	"	Surface.
VOLUME.	Stēr,	" Greek	Στερεός,	"	Solid.
CAPACITY.	Lï´ter,	" "	Λίτρα,	"	Pound.
WEIGHT.	Grăm,	" "	Γράμμα,	"	Small weight.

Units of the higher or lower orders in the decimal scale are formed by prefixing to the standard unit,

579. 1st. For **Multiples**, or higher orders, the Greek numerals:

Děk´a,	from	Δέκα,	signifying	Ten,	10.
Hěk´to,	"	Ἑκατον,	"	One Hundred,	100.
Ki´lo,	"	Χίλιον,	"	One Thousand,	1000.
Mўr´ia,	"	Μύριοι,	"	Ten Thousand,	10000.

580. 2d. For **Sub-multiples**, or lower denominations, the Latin ordinals:

Děc´i,	from	*Decimus,*	signifying	One-Tenth,	$\frac{1}{10}$,	.1.
Cěnt´i,	"	*Centesimus,*	"	One-Hundredth,	$\frac{1}{100}$,	.01.
Mĭl´li,	"	*Millesimus,*	"	One-Thousandth,	$\frac{1}{1000}$,	.001.

These prefixes furnish the *key* to the whole system, since the *name* at once shows whether the unit is *greater* or *less* than the *standard* unit, and also *how many times* greater or less.

Thus, 1 deka-meter denotes 10 meters; 1 hekto-gram denotes 100 grams; 1 kilo-liter denotes 1000 liters, etc. Also, 1 deci-meter denotes $\frac{1}{10}$ (.1) of a meter; 1 centi-meter, $\frac{1}{100}$ (.01) of a meter; 1 milli-gram, $\frac{1}{1000}$ (.001) of a gram, etc.

581. The *Denominations* of the *Metric System* correspond to the *orders* of the Arabic Notation, and are written like decimals, or U. S. Money, the lowest denomination at the right.

The names, *mill, cent,* and *dime,* used in U. S. Money, correspond to *mil'li, cen'ti,* and *dec'i,* in the Metric System.

Hence, the eagle might be called a dek'a-dollar, since it is 10 dollars; the dime, a dec'i-dollar, since it is $\frac{1}{10}$ of a dollar, etc.

MEASURES OF EXTENSION.

582. The **Mē'ter**, which is the standard *unit of length*, is the *one ten-millionth* part of the distance from the equator to the north pole, measured on the surface of the earth, and is 39.37079 in., or 1.09 + yd.

Metric Denominations.		Symbols.	Metric Values.	U. S. Values.
	1 Mĭl'li-mē'ter.	*mm.*	.001 m.	.03937 in.
10 Mĭl'li-mē'ters	= 1 Cĕn'ti-mē'ter.	*cm.*	.01 m.	.3937 "
10 Cĕn'ti-mē'ters	= 1 Dĕc'i-mē'ter.	*dm.*	.1 m.	3.937 "
10 Dĕc'i-mē'ters	= 1 Mē'ter.	*m.*	1 m.	39.37 "
10 Mē'ters	= 1 Dĕk'a-mē'ter.	*Dm.*	10 m.	32.81 ft.
10 Dĕk'a-mē'ters	= 1 Hĕk'to-mē'ter.	*Hm.*	100 m.	19.93 rd.
10 Hĕk'to-mē'ters	= 1 Kĭ'lo-mē'ter.	*Km.*	1000 m.	.6214 mi.
10 Kĭ'lo-mē'ters	= 1 Mўr'ia-mē'ter.	*Mm.*	10000 m.	6.214 mi.

583. The **Square Meter, or Centar,** is the *unit* of ordinary *surface* measures, and is 1.196 sq. yd., or 10.764 square feet.

	1 Sq. Mĭl′li-mē′ter.	*sq. mm.*	.00155 sq. in.
100 Sq. Mĭl′li-mē′ters	= 1 Sq. Cĕn′ti-mē′ter.	*sq. cm.*	.155 sq. in.
100 Sq. Cĕn′ti-mē′ters	= 1 Sq. Dĕc′i-mē′ter.	*sq. dm.*	15.5 sq. in.
100 Sq. Dĕc′i-mē′ters	= 1 **Sq. Me′ter.**	*sq. m.*	1.196 sq. yd.

584. The **Ar** is the *unit* of land measures, and is a square whose side is 10 meters, and contains 100 sq. meters.

1 Cĕn′tar (*ca.*)	= 1 Sq. Mē′ter	*sq. m.*	1.196 sq. yd.
100 Cĕn′tars	= 1 **Ar.**	*a.*	119.603 sq. yd.
100 Ars	= 1 Hĕk′tar.	*Ha.*	2.471 acres.

1. The *Meter* is used in measuring cloths and *short* distances.

2. The *Kilo-meter*, equal to $\frac{5}{8}$ of a mile, nearly, is used for measuring *long* distances.

3. The *Dĕc′i-är* is $\frac{1}{10}$ of an *är*; the *Dĕk′-är* is 10 a.; an acre is 40 ärs, nearly.

585. The **Cubic Meter** is the *unit* of ordinary *solid* measures.

	1 Cu. Mĭl′li-mē′ter.	*cu. mm.*	.000061 cu. in.
1000 Cu. Mĭl′li-mē′ters	= 1 Cu. Cĕn′ti-mē′ter.	*cu. cm.*	.061 cu. in.
1000 Cu. Cĕn′ti-mē′ters	= 1 Cu. Dĕc′i-mē′ter.	*cu. dm.*	.0353 cu. ft.
1000 Cu. Dĕc′i-mē′ters	= 1 **Cu. Me′ter.**	*cu. m.*	35.3166 cu. ft.

586. The **Ster** is the *unit* of *wood* or *stone* measures, and is a *cubic meter,* equal to 35.3166 cu. ft., or .2759 cord.

	1 Dĕc′i-stēr	*ds.*	3.53 cu. ft.
10 Dĕc′i-stērs	= 1 **Ster.**	*s.*	35.316 cu. ft.
10 Sters	= 1 Dĕk′a-stēr.	*Ds.*	2.759 cord.

1. The square and cubic measures are only the squares and cubes of the measures of length.

2. The *cubic deci-meter* is called a *lī′ter* when used as a unit of liquid or of dry measures.

3. The *cubic cen′ti-me′ter* and *cubic mil′li-me′ter* are used for measuring minute bodies.

4. The *common cord* is nearly the same as 3.6 *sters*, or 36 *deci-sters*.

MEASURES OF CAPACITY.

587. The **Li′ter** is the unit of *capacity* or *volume* for both liquid and dry measures, and is 1 *cu. deci-meter*, or 1.0567 qt. liquid measure, or .908 qt. dry measure.

					Dry M.	Liq. M.
		1 Mĭl′li-lī′ter.	*ml.*	.001 *l.*	.061 cu. in.	.27 fl. dr.
10 Mĭl′li-lī′ters	=	1 Cĕn′ti-lī′ter.	*cl.*	.01 *l.*	.6102 "	.338 fl.oz.
10 Cĕn′ti-lī′ter	=	1 Dĕc′i-lī′ter.	*dl.*	.1 *l.*	6.102 "	.845 gi.
10 Dĕc′i-lī′ters	=	1 Li′ter.	*l.*	1 *l.*	.908 qt.	1.0567 qt.
10 Lī′ters	=	1 Dĕk′a-lī′ter.	*Dl.*	10 *l.*	9.08 "	2.64 gal.
10 Dĕk′a-lī′ters	=	1 Hĕk′to-lī′ter.	*Hl.*	100 *l.*	2.837 bu.	26.4 "
10 Hĕk′to-lī′ters	=	1 Kĭl′o-lī′ter.	*Kl.*	1000 *l.*	28.375 "	264.18 "
10 Kĭl′o-lī′ters	=	1 Mȳr′ia-lī′ter.	*Ml.*	10000 *l.*	283.72 "	2641.8 "

1. The *Liter* is generally used for measuring liquids in moderate quantities; the *Deka-liter*, in large quantities.
2. The *Hekto-liter* is used for measuring grains, etc.
3. Four *liters* are a little more than 1 gal.; 35 *liters*, very nearly 1 bu.
4. The *Kilo-liter* equals 1 *cubic meter*, or 1 *ster*, or 35.316 cubic feet.

MEASURES OF WEIGHT.

588. The **Gram** is the *unit of weight*, and is the weight of 1 *cu. centi-meter* of distilled water at its maximum density, equal to 15.432 grains, or .03527 oz. Avoir.

		1 Mĭl′li-grăm.	*mg.*	.001 g.	.01543 gr.
10 Mĭl′li-grăms	=	1 Cĕn′ti-grăm.	*cg.*	.01 g.	.1543 "
10 Cĕn′ti-grăms	=	1 Dĕc′i-grăm.	*dg.*	.1 g.	1.543 "
10 Dĕc′i-grăms	=	1 Gram.	*g.*	1 g.	15.43 "
10 Grams	=	1 Dĕk′a-grăm.	*Dg.*	10 g.	.3527 oz. Av.
10 Dĕk′a-grăms	=	1 Hĕk′to-grăm.	*Hg.*	100 g.	3.527 "
10 Hĕk′to-grăms	=	1 Kĭl′o-grăm.	*Kg.*	1000 g.	2.2046 lb. Av.
10 Kĭl′o-grăms	=	1 Mȳr′ia-grăm.	*Mg.*	10000 g.	22.046 "
10 Mȳr′ia-grăms 100 Kĭl′os	=	1 Quĭn′tal.	*Q.*	100000 g.	220.46 "
1000 Kĭl′os	=	1 Ton.	*T.*	1000000 g.	2204.6 "

1. The *Kilo-gram,* or *Kilo,* is the unit of common weight in trade, and is a trifle greater than $2\frac{1}{5}$ lb. Avoir.

2. 1 *Kilo-gram* = 1 *cubic decimeter* = 1 *liter* = $\begin{cases} \text{2.6792 lb. Troy.} \\ \text{2.2046 lb. Avoir.} \end{cases}$

3. 15 *grams* are taken as $\frac{1}{2}$ an *ounce,* at the U. S. Post Office.

589. Units of *length,* of *capacity,* and of *weight* form a scale of *tens;* hence, in writing metric numbers of these denominations, each order of units will occupy *one place.*

Thus, 7642 mm. may be written 764.2 cm., or 76.42 dm., or 7.642 m., or .7742 Dm., or .07642 Hm.

590. Units of *square measures* form a scale of *hundreds;* hence, *two places* must be allowed to each order.

Thus, 12 sq. m. 6 sq. dm. 3 sq. cm. may be written 12.0603 sq. m., or 1206.03 sq. dm., or 120603 sq. cm.; and 3 Ha. 4 a. 2 ca. are written 3.0402 Ha., or 304.02 a., or 30402 ca.

591. Units of *solid measures* form a scale of *thousands;* hence, *three places* must be allowed to each order.

Thus, 36 cu. m. 27 cu. dm. 4 cu. cm. may be written 36.027004 cu. m., or 36027.004 cu. dm., or 36027004 cu. cm.

592. ***To add, subtract, multiply, and divide Metric Numbers.***

1. Add 24.72 Dm., 8.65 Hm., 1436 cm., and 28.3 m.

OPERATION.—247.2 m. + 865 m. + 14.36 m. + 28.3 m. = 1154.86 m.

2. From 12.4 Hm. subtract 5.25 m.

OPERATION.—1240 m. − 5.25 m. = 1234.75 m., *Ans.*

3. Multiply 3.425 Hm. by 6.

OPERATION.—3.425 Hm. × 6 = 20.550 Hm., or 2055 m., *Ans.*

4. Divide 2.16 Km. by 3.6 m.

OPERATION.—2160 m. ÷ 3.6 m. = 600, *Ans.*

All computations in metric numbers are similar to those in integers and decimals, or in U. S. Money.

5. Add 27.6 m., 362 dm., 16.32 Dm., and 9.1 Km.

6. Express in kilometers the sum of 475 dm., 125.3 m., and 17.341 m.

7. From 12.9 Km. take 2574.2 m., multiply the result by 6.5.

8. The distance around a park is 5.125 Km. How many *meters* will a man travel, who walks 5 times around it?

9. If the distance from Washington to Boston is 619.598 Km., in what time will an engine make the distance, if it runs, on an average, 44.257 Km. an hour?

10. From a piece of silk containing 75.125 m., 4 dresses were cut, each containing 15.6 m. How much remained?

11. How long will it take a man to walk from Philadelphia to New York, if the distance is 160 Km., and he walks 800 Dm. an hour?

12. When corn is worth $2.24 per hektoliter, what is it worth per liter?

13. A car weighing 7.832 T., contains 136 bbl. of flour, each weighing 96.16 Kg. Find the weight of the car and contents, and their difference.

14. Find the cost of 29.5 Kg., at 16 cents per gram.

15. From 500 Kg. of sugar are taken 250 packages, each containing 1.25 Kg. How much remained?

16. A merchant resides 1456 m. from his store. How many kilometers will he travel in a week of 6 days, in making two journeys a day, to and from his place of business?

17. A tailor cut 5 suits of clothes, each suit containing 7.5 m., from a piece of cloth containing 45.75 m. How much remained?

18. How many vessels, each containing 3.5 l., can be filled from 42.70 Dl. of wine?

REDUCTION.

593. To change units of the Metric System to units of the Common System, use the equivalents in the preceding tables.

594. To change units of the Common System to units of the Metric System, use the equivalents in the following

595. TABLE.

LINEAR MEASURES.

1 Inch	= 2.54 *Centi-meters.*
1 Foot	= .3048 *Meter.*
1 Yard	= .9144 *Meter.*
1 Rod	= 5.029 *Meters.*
1 Mile	= 1.6093 *Kilo-meters.*

SQUARE MEASURES.

1 Sq. Inch	= 6.4528 *Sq. Centi-met.*
1 Sq. Foot	= .0929 *Sq. Meter.*
1 Sq. Yard	= .8361 *Sq. Meter.*
1 Sq. Rod	= 25.293 *Centars.*
1 Acre	= 40.47 *Ars.*
1 Sq. Mile	= 259 *Hektars.*

SOLID MEASURE.

1 Cu. Inch	= 16.387 *Cu. Centi-met.*
1 Cu. Foot	= 28.317 *Cu. Deci-met.*
1 Cu. Yard	= .7645 *Cu. Meter.*
1 Cord	= 3.6243 *Sters.*

MEASURES OF CAPACITY.

1 Fl. Ounce	= .02958 *Liter.*
1 Liq. Quart	= .9465 *Liter.*
1 Gallon	= 3.785 *Liters.*
1 Dry Quart	= 1.101 *Liters.*
1 Bushel	= .35243 *Hektoliter.*

MEASURES OF WEIGHT.

1 Grain Troy	= .0648 *Gram.*
1 Ounce Troy	= 31.324 *Grams.*
1 Pound Troy	= .37324 *Kilo.*
1 Ounce Avoir.	= 28.35 *Grams.*
1 Pound Avoir.	= .4536 *Kilo.*
1 Ton	= .907 *Tonneau.*

WRITTEN EXERCISES.

LINEAR MEASURES.

596. 1. How many feet in 12.25 m.? In 135 m.? In 76.75 m.?

2. How many yards in 68.27 m.? In 321.4 dm.?

3. How many meters in 90 yd.? In 12 yd. 1 ft. 6 in.?

4. How many miles in 8000 m.? In 450 Km.?

5. The distance around a lake is 5.16 Km.? How many meters will a man walk who goes 3 times around it? How many miles?

6. A lady bought 25.5 m. of silk for a dress, at $3.84 per meter. What did it cost her?

7. How many meters between two places 75 miles apart?

8. A merchant bought 300 m. of silk at $2½ a meter, and sold it at $2¼ a yard. Did he gain or lose, and how much?

Square Measures.

9. How many ars in 150 sq. yards? Acres in 1011 ars?

10. Change 336971 sq. m. to ars; to hektars; to acres.

11. Find the area of a floor 12.8 m. long, and 10.4 m. wide?

12. At 10 cent a sq. meter, what will it cost to paint a surface 22.5 m. long, and 12.4 m. wide?

13. How much carpeting, .8 m. wide, will cover a floor 6.24 m. long, and 5.5 m. wide, and what will be its cost, at $1.50 a yard?

Solid Measures.

14. How many cu. feet in 92 sters? How many cords?

15. How many sters in 256 cu. ft.? In 5.75 cords?

16. How many loads of earth, each filling 2.25 cu. m., will fill a space 7 m. long, 6 m. wide, and 2.25 m. deep?

17. How many hektoliters of wheat will a bin contain that is 5 m. square, and 2.5 m. deep?

18. What will be the cost of a pile of wood, 15.7 m. long, 1.5 m. high, and 3.76 m. wide, at $1.50 a ster? At $6 a cord?

19. What will be the cost of excavating a cellar 14.4 m. long, 8.6 m. wide, and 2.8 m. deep, at $.75 a ster?

Measures of Capacity.

20. How many gallons in 72.16 l.? Liters in 12 gal.?

21. How many liters in 3 bu. 1 pk.? Bushels in 130.5 l.?

22. From a flask containing 2 Hl., 2.5 Dl. were drawn out. How many liters remained? How many quarts?

23. A cistern 3.6 m. long, 2.5 m. wide, and 1.7 m. deep, will hold how many liters of water?

24. If 2 Kl. of barley cost $56.75, what will 25 bu. cost?

25. If the price of wine is $.75 a liter, what is it worth a gallon?

26. 1 bought 251.50 liters of wine in France, at 5.25 francs a liter; paid $.75 a liter duty and freight, and sold it at $3.50 a liter. What was my gain?

Measures of Weight.

27. In 960 grams, how many pounds Troy? Avoir.?

28. How many grams in 960 gr.? In 8 lb. Troy.? In 6 lb. 8 oz. Avoir.?

29. A barrel of flour contains 196 lb. Express its weight in metric units?

30. Which is more profitable, and how much per cwt., to sell cheese at 12 cents a pound, or at 25 cts. per kilo?

31. If a kilo of coffee cost $.90, what is the cost of 75 lb., at the same rate?

32. What price per pound is equivalent to $25 per kilo?

33. A grocer buys butter at $.25 per lb., and sells it at $.60 per kilogram. Does he gain or lose, and what per cent?

34. A jeweler bought a gem in Geneva which weighed 375.50 grams, at 10.25 francs per gram, and the duty on it was $6½; at what price per gram must it be sold to clear $125?

MENSURATION.

DEFINITIONS.

597. Mensuration treats of the measurements of *lines, angles, surfaces,* and *solids.*

598. A Line is that which has length, but not width or thickness.

599. A Straight Line is a line which does not change its direction.

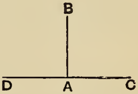
600. A Curved Line is a line which changes its direction at every point.

601. Parallel Lines are lines which have the same direction. They are equally distant from each other at every point.

602. An Angle is the opening between two straight lines, drawn from the same point.

Thus, the opening between the lines AB and AC is an angle. The lines AB and AC are the *sides* of the angle, and the point A, in which they meet, is the *vertex* of the angle.

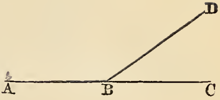
603. A Right Angle is the angle formed, when one straight line meets another so as to make the adjacent angles equal.



Thus, the adjacent angles BAC and BAD are equal, therefore, *right angles.*

The lines BA and CD are said to be perpendicular to each other.

604. An **Obtuse Angle** is an angle greater than a right angle; as DBA.



605. An **Acute Angle** is an angle less than a right angle; as DBC.

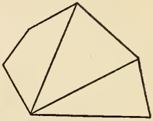
606. A **Surface** is that which has length and breadth, without thickness, and may be either *plane* or *curved.*

Thus, the surface of a table is *plane*, that of a ball is *curved.*

PLANE FIGURES.

607. A **Plane Figure** is a figure all parts of which are in the same plane.

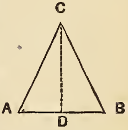
608. A **Polygon** is a portion of a plane bounded by straight lines.



The *Perimeter* of a polygon is the distance around it, and is equal to the sum of all its sides.

A polygon of three sides is called a *Triangle;* of four sides, a *Quadrilateral;* of five sides, a *Pentagon;* of six sides, a *Hexagon;* etc.

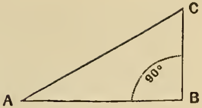
609. A **Triangle** is a polygon of three sides; as ABC.



The *Base* of a triangle is the side on which it is supposed to stand, AB.

The *Vertical Angle* is the angle opposite the base, as C.

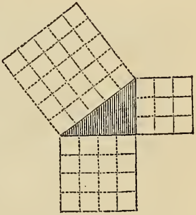
The *Altitude* is the perpendicular line drawn from the vertical angle to the base, as CD.



610. **A Right Triangle** is a triangle that has one right angle.

The side, AC, opposite the right angle is called the *Hypothenuse;* the side AB, the *Base;* and BC, the *Perpendicular.*

The relation of the squares described upon the sides of a right triangle is expressed in the following

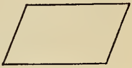


611. PRINCIPLES. I. *The square on the hypothenuse of a right triangle is equal to the sum of the squares on the other two sides.*

II. *The square on either of the two sides about the right angle is equal to the square on the hypothenuse, diminished by the square on the other side.*

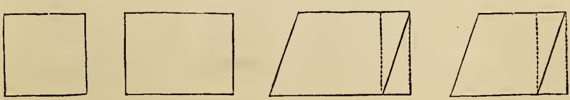
612. **A Quadrilateral** is a polygon of four sides.

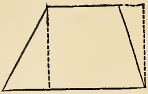
There are three kinds of quadrilaterals, the *Parallelogram,* the *Trapezoid,* and the *Trapezium.*



613. **A Parallelogram** is a quadrilateral whose opposite sides are parallel. The opposite sides are also equal.

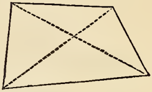
When a parallelogram is right-angled, it is called a *Rectangle;* when the four sides of a rectangle are equal, it is called a *Square;* when the parallelogram is not right-angled, it is called a *Rhomboid;* and when the sides of the rhomboid are equal, it is called a *Rhombus.*





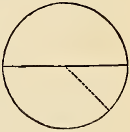
614. A **Trapezoid** is a quadrilateral, having two of its sides parallel.

615. A **Trapezium** is a quadrilateral, no two sides of which are parallel.



The *Altitude* of a parallelogram, or trapezoid, is the perpendicular distance between its parallel sides.

The *Diagonal* of any plane figure is a straight line joining the vertices of two of its angles not consecutive.



616. A **Circle** is a plane figure bounded by a curved line, called the *Circumference*, all points of which are equally distant from a point within, called the *Centre*.

The *Radius* of a circle is a straight line drawn from the centre to the circumference.

The *Diameter* of a circle is a straight line drawn through the centre, and terminated both ways by the circumference.

WRITTEN EXERCISES.

617. ***To find the area of a triangle.***

1st. When the base and altitude are given.

1. Find the area of a triangle whose base is 9 ft. and altitude 6 ft.

SOLUTION. $\frac{9 \times 6}{2} = 27$; hence, the *area* is 27 sq. ft.

EXPLANATION.—The area of a triangle is equal to one-half the product of its base and altitude.

FORMULA: *Area of triangle* $= \frac{1}{2}$ (*Base* $\times$ *Altitude*).

2. What is the area of a triangle whose base is 50 ft. and altitude 36 ft.?

3. Find the area of a triangular field whose base is 75 rods and altitude 64 rods.

4. How many square feet in the gable end of a house 44 ft. wide and 14 ft. 6 in. high?

2d. When the three sides are given.

5. What is the area of a triangle whose sides are 30, 40, and 50 feet?

SOLUTION.—$(30+40+50)\div 2 = 60 = \frac{1}{2}$ the sum of the sides.

$$60 \times \overline{60-30} \times \overline{60-40} \times \overline{60-50} = 60 \times 30 \times 20 \times 10 = 360000.$$

$\sqrt{360000} = 600$; hence the *area* is 600 sq. ft.

RULE.—*From the half sum of the three sides, subtract each side separately; then multiply the half sum and the three remainders together, and extract the square root of the product. The result will be the area.*

6. What is the area of a triangle, whose sides are 25, 36, and 49 inches?

7. How many acres in an equilateral triangular field, each of whose sides is 70 rods?

8. Find the area of an equilateral triangle whose sides are each 48 feet.

9. The base of a right triangle is 4, and its perpendicular is 3; what is the hypothenuse?

SOLUTION. $4^2+3^2=16+9=25$, the area of the square on the hypothenuse (**611**, 1). Therefore, the *square root* of 25 is the hypothenuse. $\sqrt{25}=5$, the *hypothenuse.* Hence,

FORMULA: *Hypothenuse* $= \sqrt{\mathit{Base}^2 + \mathit{Perpendicular}^2}$.

10. What is the hypothenuse of a right triangle, whose base is 30 ft., and its perpendicular 16 ft.?

11. Two rafters, each 20 ft. long, meet at the ridge of a roof 12 ft. above the base of the gable. What is the width of the house?

12. The triangular gable of a building has a base of 30 ft., and a perpendicular height of 8 ft.; how long must the rafters be that they may project over the sides 2 feet?

13. The rafters of a house are 17 ft. long, and the width of the gable is 30 ft.; what is the height of the gable?

SOLUTION. $\frac{1}{2}$ of 30 ft. = 15 ft.; $17^2-15^2=64$; $\sqrt{64}=8$; hence, the *height* of the gable is 8 ft. (**611**, II).

FORMULA: *Perpendicular* = $\sqrt{Hypothenuse^2 - Base^2}$.

14. The rafters of a house are 17 ft. long, and the height of the gable 8 ft.; what is the width of the building?

SOLUTION. $17^2-8^2=225$; $\sqrt{225}=15$, *half* the width; hence the *width* is 30 ft.

FORMULA: *Base* = $\sqrt{Hypothenuse^2 - Perpendicular^2}$.

15. A room is 40 ft. long and 30 ft. wide; what is the distance from one corner to the opposite corner?

16. A ladder 50 ft. long, whose foot is 20 ft. from a building, reaches to a window; how high is the window from the ground?

17. A tree was broken 39 ft. from the bottom, and fell so that the end struck 52 ft. from the foot; what was the length of the tree?

18. Two vessels sail from the same port, one sailing directly south 3 miles an hour, the other east 4 miles an hour; how far apart are they at the end of 24 hours?

19. A park 100 rd. long and 92 rd. wide, has a walk running through it from opposite corners in a straight line. How long is the walk?

20. A ladder 82 ft. long is placed against a house so that its foot is 18 ft. from the building. How high does it reach?

618. ***To find the area of a parallelogram.***

1. Find the area of a parallelogram 18 ft. by 12 ft. wide.

SOLUTION. 18 ft. $\times 12 = 216$ sq. ft $= 24$ sq. yd., *Ans.*

EXPLANATION.—The area of a parallelogram is equal to the area of a rectangle having the same base and altitude. Hence,

FORMULA: *Area of a parallelogram = Base × Altitude.*

2. How many square feet in a lawn 64 ft. long and $32\frac{1}{2}$ ft. wide?

3. How many acres in a field 80 rods square?

4. How many acres in a field in the form of a rhombus, each side measuring 40 rods, and the perpendicular distance between two sides being 18 rods?

5. Find the area of a piece of land in the form of a rhomboid, the base being 17.5 rods and altitude 12 rods.

619. ***To find the area of a trapezoid.***

1. What is the area of a trapezoid whose bases are 12 ft. and 10 ft., and altitude 5 ft.?

SOLUTION. $\frac{12+10}{2} \times 5 = 55$; hence the *area* is 55 sq. ft.

FORMULA: *Area of a trapezoid* = $\frac{1}{2}$ *Sum of bases* × *Altitude.*

2. What is the area of a trapezoid whose bases are 178 ft. and 124 ft., and altitude 11 ft.?

3. How many acres in a field in the form of a trapezoid, whose parallel sides are 38 rods and 26 rods, and the distance between them 10 rods?

4. How much land in a field, the two parallel sides being 75 and 60 rods respectively, and the perpendicular distance between them being 45 rods?

620. ***To find the circumference of a circle when the diameter is given.*** Conversely,

To find the diameter of a circle when the circumference is given.

1. Find the circumference of a circle whose diameter is 20 ft.?

OPERATION. 20 ft. $\times$ 3.1416 $=$ 62.832 ft., *Circumference.*

2. Find the diameter of a circle whose circumference is 18 ft.

OPERATION. 18 ft. $\div$ 3.1416 $=$ 5.7 ft.+, *Diameter.* Hence,

$$\text{FORMULAS:}\begin{cases} 1.\ \textit{Circumference} = \textit{Diameter} \times 3.1416. \\ 2.\ \textit{Diameter} = \textit{Circumference} \div 3.1416. \end{cases}$$

3. What is the diameter of a wheel whose circumference is 48 ft.?
4. What is the circumference of a tree whose diameter is 3 ft. 6 in.?
5. Find the radius of a circle whose circum. is 75 ft.

621. ***To find the area of a circle when the radius is given.*** Conversely,

To find the radius of a circle when the area is given.

1. Find the area of a circle whose diameter is 20 ft.

OPERATION. $\frac{20 \text{ ft.}}{2} = 10$ ft., the *radius;* $10^2 \times 3.1416 = 314.16$; hence, *area* $= 314.16$ sq. ft.

2. Find the radius of a circle whose area is 314.16 sq. ft.

OPERATION. $\sqrt{314.16 \div 3.1416} = 10$; *radius* $= 10$ ft. Hence,

$$\text{FORMULAS:}\begin{cases} 1.\ \textit{Area} = \textit{Radius}^2 \times 3.1416. \\ 2.\ \textit{Radius} = \sqrt{\textit{Area} \div 3.1416}. \end{cases}$$

3. Find the area of a circle whose radius is 12 ft.

4. Find the area of a circle whose circumference is 150 ft.

5. How many acres in a circular park whose circumference is 1 mile?

6. How long a rope will it take to fasten a horse to a post, so that he may graze over $1\frac{1}{4}$ acres of grass?

SOLIDS

622. A **Solid** or **Body** has three dimensions, *length, breadth,* and *thickness.*

623. A **Prism** is a solid, two faces of which are equal and parallel polygons, and the other faces parallelograms.

The *Bases* of a prism are the equal and parallel polygons.

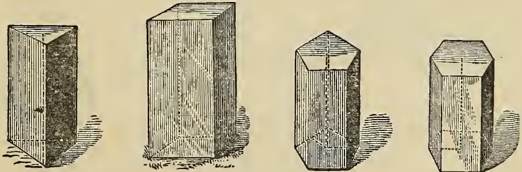
The *Lateral Faces* of a prism are all the faces except the bases.

The *Lateral* or *Convex Surface* of a prism is the sum of its lateral faces.

The *Lateral Edges* of a prism are the straight lines in which the lateral faces meet.

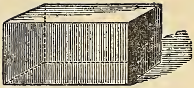
The *Altitude* of a prism is the perpendicular distance between the planes of its bases.

A Prism is *triangular, quadrangular, pentagonal, hexagonal,* etc., according as its bases are *triangles, quadrilaterals, pentagons, hexagons,* etc.

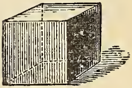


624. A **Right Prism** is a prism whose lateral faces are rectangles. Hence, its lateral edges are perpendicular to its bases.

625. A **Parallelopipedon** is a prism whose bases are parallelograms.

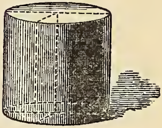


626. A **Rectangular Parallelopipedon** is a parallelopipedon, all the faces of which are rectangles.



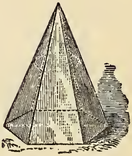
627. A **Cube** is a parallelopipedon, all the faces of which are squares (**251**).

628. A **Cylinder** is a solid bounded by a uniformly curved surface, with two bases which are equal and parallel circles.



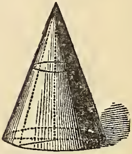
The *Altitude* of a cylinder is the perpendicular distance between its bases.

629. A **Pyramid** is a solid, one face of which is a polygon, and the other faces triangles which have a common vertex.

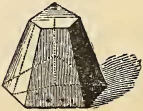


630. A **Cone** is a solid having one base, which is a circle, and a uniformly curved surface terminating in a point, called the vertex.

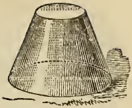
The *Altitude* of a pyramid or cone is the perpendicular distance from the vertex to the plane of the base.



The *Slant Height* of a regular pyramid is the straight line drawn from its vertex perpendicular to one side of the base; the *slant height* of a *cone* is a straight line from the vertex to the circumference of the base.



631. The **Frustum** of a pyramid, or of a cone is that part which remains after cutting off the top, by a plane parallel to the base.



632. A **Sphere** is a solid bounded by a curved surface, all points of which are equally distant from a point within, called the *Centre*.

The *Diameter* of a sphere is a straight line drawn through the centre, and terminated both ways by the surface.

The *Radius* of a sphere is a straight line drawn from the centre to the surface.

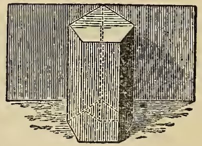
633. The **Volume** of a solid is the amount of space it contains.

634. The **Convex Surface** of a solid is all its surface, except its base or bases.

WRITTEN EXERCISES.

635. ***To find the convex surface of a right prism or cylinder.***

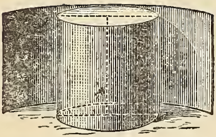
1. Find the convex surface of a prism whose base is a pentagon, each side of which is 5 ft., and its altitude 8 ft.



OPERATION. 5 ft. × 5 = 25 ft., *perimeter*; 25 × 8 = 200; 200 sq. ft., *convex surface.* Hence,

FORMULA: *Convex Surface of a Right Prism*
= *Perimeter of Base* × *Altitude.*

2. What is the convex surface of a prism whose altitude is 12 ft., and the base a triangle, each side of which is 5 ft. 6 in.?



3. Find the convex surface of a cylinder the altitude of which is 3 ft. 6 in., and the circumference 6 ft. 3 in.

OPERATION. $6.25 \times 3.5 = 21.875$; 21.875 sq. feet, *convex surface*. Hence,

FORMULA: *Convex Surface of a Cylinder*
$=$ *Circumference of Base* $\times$ *Altitude.*

4. The altitude of a cylinder is 4 ft. and the radius of its base 18 in. What is the area of the convex surface?

The *entire* surface of a prism or cylinder is found by adding the areas of the bases to the convex surface.

5. Find the entire surface of a cube whose edge is 6 ft.

6. Find the convex surface of a right prism whose altitude is 9 ft. and the base a regular hexagon, each side of which is 2 ft.

7. What is the entire surface of a parallelopipedon 8 ft. 6 in. long, 4 ft. wide, and 3 ft. high?

8. Find the entire surface of a cylinder, the altitude of which is 30 in. and the diameter 18 in.

636. ***To find the volume of a prism or cylinder.***

1. Find the volume of a square prism, the altitude of which is 9 ft. and each side of the base 4 ft.

OPERATION. $4 \times 4 \times 9 = 144$; 144 cu. ft., *Volume*.

2. Find the volume of a cylinder, the altitude of which is $7\frac{1}{2}$ ft. and the radius of the base 2 ft.

OPERATION. $2^2 \times 3.1416 \times 7\frac{1}{2} = 92.248$; 92.248 cubic feet, *Volume*. Hence,

FORMULA: *Volume of Prism or Cylinder*
$=$ *Area of Base* $\times$ *Altitude.*

3. What is the volume of a cube whose edges are 4 ft. 3 in.?

The area of the *base* of a cube is the square of its edge. Hence,

FORMULA: *Volume of a Cube = Cube of its Edge.*

4. Find the cost of a piece of timber 32 ft. long and 18 in. square at each end, at 15 cents a cubic foot.

5. What is the volume of a cylinder whose altitude is 8 ft. and diameter 3 ft. 6 in.?

6. What is the value of an oak log 20 ft. long, its average circumference being 8 ft., at $.30 a cubic foot?

637. ***To find the convex surface of a pyramid or cone.***

1. What is the convex surface of a square pyramid whose base is 12 ft. 6 in., and its slant height 18 ft.?

OPERATION. 12.5 ft. $\times 4 = 50$ ft., *perimeter of base;*
$50 \times \frac{18}{2} = 450$; 450 sq. ft., *convex surface.* Hence,

FORMULA: *Convex Surface of a Pyramid*
= Perimeter of Base × ½ Slant Height.

2. What is the convex surface of a cone, the slant height of which is 15 ft. and the circumference of the base 20 ft.?

OPERATION. $20 \times \frac{15}{2} = 150$; 150 sq. ft., *convex surface.* Hence,

FORMULA: *Convex Surface of a Cone*
= Circum. of Base × ½ Slant Height.

The entire surface of a pyramid or cone may be found by adding the area of the base to the area of the convex surface.

3. Find the entire surface of a cone whose slant height is 10 ft. and the radius of its base 3 ft. 6 in.

4. Find the entire surface of a pyramid, the base of which is 16 ft. square and the slant height 32 ft.

638. *To find the volume of a pyramid or cone.*

1. What is the volume of a square pyramid, the altitude of which is 21 ft. and its base 8 ft. square?

OPERATION. $8 \times 8 \times \frac{21}{3}$, or $8^2 \times \frac{21}{3} = 448$; 448 cu. ft., *Volume.*

2. Find the volume of a cone, the altitude of which is 12 ft. and the radius of the base 6 ft.

OPERATION. $6^2 \times 3.1416 \times \frac{12}{3} = 452.39$; 452.39 cu. ft., *Volume.* Hence,

FORMULA: *Volume of Pyramid or Cone*
$= $ *Area of Base* $\times \frac{1}{3}$ *Altitude.*

3. Find the volume of a pyramid whose altitude is 30 in., and the base a triangle 14 in. on each side.

4. Find the volume of a cone whose altitude is 48 ft. and the circumference of the base 78.54 ft.

5. What is the volume of a pyramid whose altitude is 60 ft., and its base a rectangle 15 ft. by 18 ft.?

6. Find the solid contents, in feet, of a square stick of timber terminating in a point, 12 by 18 inches at the base, and the length 36 feet.

639. *To find the surface of a sphere.*

1. What is the surface of a sphere whose diameter is 18 inches?

OPERATION. $18^2 \times 3.1416 = 1017.8784$; 1017.8784 sq. in. = 7.0686 sq. ft., *Surface.* Hence,

FORMULA: *Surface of a Sphere* = *Diameter*$^2 \times 3.1416$.

2. Find the surface of a globe 2 ft. 6 in. in diameter.

3. What is the surface of a ball whose radius is 18 in.?

4. The earth is 7912 miles in diameter. How many square miles on its surface?

640. ***To find the volume of a sphere.***

1. Find the volume of a sphere whose diameter is 3 ft.

OPERATION. $3^3 \times .5236 = 14.1372$; 14.1372 cu. ft., *Volume.* Hence,

FORMULA: *Volume of a Sphere* $=$ *Diameter*$^3 \times .5236$.

2. Find the volume of a globe whose diameter is 40 in.
3. Find the volume of a sphere whose radius is 6 inches.
4. How many cubic feet in a globe 18 in. in diameter?

641. ***To gauge or measure the capacity of a cask.***

1. How many wine gallons in a cask whose head diameter is 22 in., bung diameter 28 in., and its length 36 in.?

SOLUTION. $\overline{22 + 28}$ in. $\div 2 = 25$ in., *mean diameter.*
$25^2 \times 36 \times .0034 = 76.5$; 76.5 gal., *capacity.*

RULE.—*Multiply the square of the mean diameter in inches by the length in inches, and this product by .0034. The result will show the capacity in wine gallons.*

When the cask is not full stand it on the end, find the mean diameter of the part occupied by the liquid, multiply its square by the height of the liquid, and this product by .0034.

2. What is the capacity of a cask whose bung diameter is 30 in., head diameter 24 in., and its length 40 in.?
3. How many gallons in a cask whose head diameter is 22 in., bung diameter 26 in., length 36 in., and when standing on the end the height of the liquid is 24 in.?
4. What is the number of gallons in a cask whose bung diameter is 38 in., head diameter 32 in., and length 42 inches?

DUODECIMALS.

642. Duodecimals are denominate fractions of a linear, square, or cubic foot, formed by successively dividing by 12.

Duodecimals are used chiefly by artificers in the measurement of lines, surfaces, and solids.

The *foot* is the *unit* of measure, and is divided into 12 equal parts, called *primes* (′); each prime, into 12 equal parts, called *seconds* (″); each second, into 12 *thirds* (‴); and each third, into 12 *fourths* (⁗), etc. These marks used to denote the different denominations are called **Indices.**

TABLE OF UNITS.

$$1' = \tfrac{1}{12} \text{ of a foot.}$$
$$1'' = \tfrac{1}{12} \text{ of } 1' = \tfrac{1}{12} \text{ of } \tfrac{1}{12} \text{ of 1 ft.} = \tfrac{1}{144} \text{ of a foot.}$$
$$1''' = \tfrac{1}{12} \text{ of } 1'' = \tfrac{1}{12} \text{ of } \tfrac{1}{144} \text{ of 1 ft.} = \tfrac{1}{1728} \text{ of a foot.}$$
$$1'''' = \tfrac{1}{12} \text{ of } 1''' = \tfrac{1}{12} \text{ of } \tfrac{1}{1728} \text{ of 1 ft.} = \tfrac{1}{20736} \text{ of a foot.}$$

643. Duodecimals are added and subtracted in the same manner as other Compound Denominate Numbers.

644. Multiplication of Duodecimals is similar to that of Compound Numbers.

The denomination of the product of two or more factors is indicated by the *sum* of their indices.

WRITTEN EXERCISES.

645. 1. Multiply 18 ft. 6′ by 9 ft. 3′.

EXPLANATION.— $6' \times 3' = 18'' = 1'\ 6''$. Write the 6″ one place to the right, and add the 1′ to the next product. $18 \times 3' + 1' = 55' = 4$ ft. 7′, which write in their order. Next, $9 \times 6' = 54' = 4$ ft. 6′. Write the 6′ in its place, and add the 4 to the next product. $18 \times 9 + 4 = 166$ ft. The sum of the partial products is 171 ft. 1′ 6″.

18 ft.	6′		
9 ft.	3′		
4 ft.	7′	6′	
166 ft.	6′		
171 ft.	1′	6″	

Multiply and explain the following:

2. 16 ft. 9′ by 7 ft. 8′.	5. 12 ft. 4′ 9″ by 9 ft. 8′ 6″.
3. 14 ft. 10′ 8″ by 2 ft. 6′ 4″.	6. 45 ft. 0′ 10″ by 9′ 8″.
4. 10 ft. 9′ 8″ by 10′ 7″.	7. 24 ft. 6′ 2″ by 6 ft. 7′ 3″.

8. What are the contents of a block of granite 7 ft. 8′ 6″ by 3 ft. 4′ 2″ by 2 ft. 2′ 4″?

9. What is the area of a marble slab 5 ft. 6′ 4″ long and 2 ft. 3′ 6″ wide?

GOVERNMENT LANDS

646. In the survey and the division of a new territory of land, the first thing done is to run a line north and south, marking its course by stones, posts, marked trees, and other means, throughout its entire length. Such a line is called the **Principal Meridian.**

647. In like manner a line is run east and west, which is called the principal **Base Line.**

648. Next, lines are run 6 miles apart, and parallel to the *Principal Meridian;* then, other lines 6 mi. apart, and parallel to the *Base Line,* forming squares 6 mi. square, each containing 36 sq. miles, or 23040 acres. These divisions are called **Townships,** and are designated by their number, north or south of the Base Line.

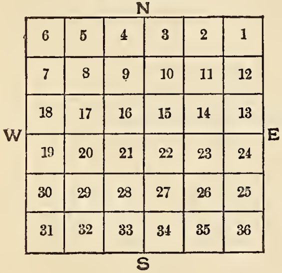
649. A **Range** is a line of townships running north and south, and is known by its number east or west of the *Principal Meridian.*

650. A **Township** is divided into 36 equal squares, called **Sections,** each 1 mile square, and containing 640 A.

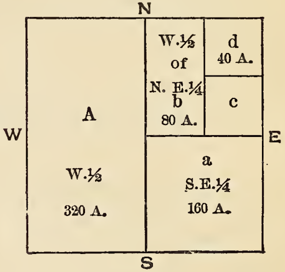
651. A *Section* is variously divided for purposes of sale. The U. S. Land Office recognizes the following divisions:

Half-Section	$= 1$ mi.	$\times \frac{1}{2}$ mi.	$= \frac{1}{2}$	sq. mi.	$= 320$	Acres.
Quarter-Section	$= \frac{1}{2}$ "	$\times \frac{1}{2}$ "	$= \frac{1}{4}$	"	$= 160$	"
Half-Quarter-Section	$= \frac{1}{2}$ "	$\times \frac{1}{4}$ "	$= \frac{1}{8}$	"	$= 80$	"
Quarter-Quarter-Section	$= \frac{1}{4}$ "	$\times \frac{1}{4}$ "	$= \frac{1}{16}$	"	$= 40$	"

A TOWNSHIP



A SECTION ENLARGED



652. The above diagrams show the division of a township into sections, and the subdivision of a section enlarged. The parts into which a section is divided and designated by letters A, a, b, c, and d, are named as follows:

A,	W $\frac{1}{2}$ of Section	$= 320$	Acres.
a,	S. E. $\frac{1}{4}$ of Section	$= 160$	"
b,	W. $\frac{1}{2}$ of N. E. $\frac{1}{4}$ of Section	$= 80$	"
c,	S. E. $\frac{1}{4}$ of N. E. $\frac{1}{4}$ of Section	$= 40$	"
d,	N. E. $\frac{1}{4}$ of N. E. $\frac{1}{4}$ of Section	$= 40$	"

Written Exercises.

653. 1. Thirty families of emigrants bought half a township of land, at Government price ($1.25 an acre), and divided it equally. How many acres did each receive? What part of a section? What was the cost of the whole, and of each part?

2. A man bought the E. $\frac{1}{2}$ of the N. E. $\frac{1}{4}$ of a section of land, at 2\frac{1}{2}$ an acre. What did it cost him?

3. He afterward purchased the N. E. $\frac{1}{4}$ of the S. E. $\frac{1}{4}$ of the same section. How many acres did he then own? Show by a diagram how it lay.

4. If I buy the N. W. $\frac{1}{4}$ and the N. E. $\frac{1}{4}$ of the N. E. $\frac{1}{4}$ of a section of land, how many acres do I buy? What part of a whole section? Locate the parts by diagram.

5. James Ray bought of the N. Pacific R. R. Co. Sect. 10, Township 6 N, Range 4 E, at \$2 an acre. He afterward sold the S. W. $\frac{1}{4}$, at \$2$\frac{1}{2}$ an acre; the E. $\frac{1}{2}$ of the S. E. $\frac{1}{4}$, at \$3 an acre; the W. $\frac{1}{2}$ of the N. W. $\frac{1}{4}$, at \$2.75 an acre; and the N. E. $\frac{1}{4}$, at \$2$\frac{1}{4}$ an acre. How many acres had he left? How much more or less than the cost of the whole did he receive for what he sold? Locate by a diagram.

6. What will be the cost of the material for fencing a quarter-quarter-section of land, with a board fence, allowing 2 posts and 4 boards to the rod, the boards being 12, 10, 8, and 8 inches wide, respectively; the posts costing \$10 per C. and the boards \$16 per M.?

Miscellaneous.

654. The Old *French Linear* and *Land Measure* is still partially used in Louisiana, and in other *French* settlements of the United States.

Table of Units.

12 Lines = 1 Inch.	6 Feet = 1 Toise.
12 Inches = 1 Foot.	32 Toises = 1 Arpent.

1024 Sq. Toises = 1 Sq. Arpent.

The *French Foot* equals 12.79 English inches.

The Arpent is the old French name for Acre, and contains nearly $\frac{5}{6}$ of an English Acre.

655. The following are still used to some extent in Texas, New Mexico, and other *Spanish* settlements of the United States:

TABLE OF UNITS.

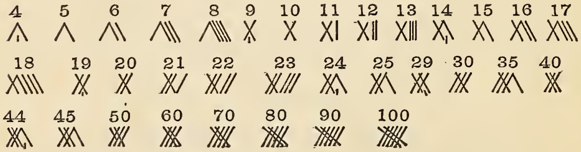
1000000 Sq. Varas = 1 Labor = 177.136 Acres (American).
25 Labors = 1 League = 4428.4 " "
The Spanish Foot = 11.11 in. (Am.); 1 Vara = $33\frac{1}{3}$ in. (Am.).
108 Varas = 100 yd.; and 1900.8 Varas = 1 Mile.

OTHER DENOMINATIONS IN USE.

5000	Varas Square	=	1	Square League.			
1000	Varas Square	=	1	Labor, or $\frac{1}{25}$ League.			
5645.376	Square Varas	=	4848	Sq. Yards	=	1	Acre.
23.76	Square Varas	=	1	Square Chain	=	$\frac{1}{10}$	Acre.
1900.8	Varas Square	=	1	Section	=	640	Acres.

LUMBERMEN'S NOTATION.

656. Lumbermen sometimes, in marking lumber, employ a modification of the Roman Method of Notation. The first three characters are like the Roman. The others are as follows:



ANSWERS.

The answers to many of the introductory and numerical examples and also to the more simple applied examples of many of the articles have been omitted. The pupil may be required to verify such, at the option of the teacher.

Art. 43. *1.* 6456. *2.* 7456. *3.* 8104. *4.* 18224. *5.* 31336. *6.* 111752.

Art. 44. *1.* $250.65. *2.* $386.45. *3.* $353.13. *4.* $411.22. *5.* $488.17. *6.* $690.38. *7.* $132.51. *8.* $166.37. *9.* $181.98. *10.* $306.43. *11.* $411.95. *12.* 7732 acres. *13.* 5032 yd. *14.* 6076 men. *15.* $614.36. *16.* 2165 bu.; $4678.70. *17.* $14010.50. *18.* 8057 lb. *19.* $15085.50. *20.* $18701.15. *21.* $9687.15. *22.* 3220 Cd. *23.* $7775, C's; $15550, *all.* *24.* 2778 sheep. *25.* $39725. *26.* 11574 mi. *27.* $30975. *28.* $4308735.24. *29.* $1362.23. *30.* $452.65. *31.* $571.79. *32.* $865.88. *33.* $1028.30. *34.* $961.47. *35.* $728. *36.* 7174716. *37.* 13778591. *38.* 6026729. *39.* 1433655. *40.* 1002991. *41.* 7109260. *42.* $1293.26. *43.* $119.82. *44.* $1584.80. *45.* $471.39. *46.* $2501.05. *47.* 10954973. *48.* 12891994. *49.* 9223449. *50.* 257388. *51.* 3198138.

Art. 56. *39.* 13102. *40.* 621977. *41.* 90858. *42.* 164040. *43.* 640073. *44.* $180.47. *45.* $47243.50. *46.* $533.82. *47.* 14916 ft. *48.* 30081 votes. *49.* 21452. *50.* $458.79.

Art. 57. *3.* 15867. *4.* 24217. *5.* $248.77. *6.* 6730.

Art. 58. *1.* 3747. *2.* 1553. *3.* $1534.25. *4.* 929496 sq. mi. *5.* 143563 sq. mi. *6.* $2577.25. *7.* $289.25. *8.* 85375 sq. mi.; 38219 sq. mi. *9.* $4307.40. *10.* $2211.35. *11.* $11390. *12.* $16064.70. *13.* 7415 bu. *14.* $7389. *15.* 797 bu. *16.* $7802.50, C's, $8539.25.

Art. 77. *2.* 49056, 65700, 111252, 212868. *3.* 530620, 799632, 360328, 656488. *4.* 7394760, 9955872, 11110176, 15727392. *5.* $46465.92, $69698.88, $42593.76, $169729.68. *6.* $578928. *7.* $49476. *8.* $4593.75. *9.* $15858.25. *10.* $28017, $3279. *11.* 5472076 feet. *12.* 1383242 days. *13.* $428716.08. *14.* 17013152 cts. *15.* 3909682 lb. *16.* 46385452 qt. *17.* 45968904. *18.* $370451.04. *19.* 1356967228. *20.* 646520814. *21.* 459886110. *23.* 62581459114.

Art. 82. *3.* 3700, 37000, 22200, 92500. *4.* 190400, 952000, 12240000, 9112000. *5.* 492000, 1180800, 12300000, 6150000. *6.* 3152100000, 17021340000, 19164768000. *7.* 1313160000. *8.* \$4950, \$6600. *9.* \$1059.84. *10.* \$940.80, \$1008. *11.* \$20700, \$30705. *12.* 84000 lb., 126000 lb. *13.* 112560 yd.; \$15758.40. *14.* 451200. *15.* 1804200. *16.* 2758720. *17.* \$25926.48. *18.* \$24210. *19.* \$17920000. *20.* 12600000. *21.* 87935040. *22.* 307660800. *23.* 80. *24.* 12. *25.* 15. *26.* 8. *27.* 4. *28.* 4. *29.* 5. *30.* 3. *31.* 10. *32.* 9.

Art. 83. *1.* 434, 88367608. *2.* \$60480900. *3.* \$53617.55. *4.* 41906656. *5.* \$922.56. *6.* \$825. *7.* \$1650. *8.* \$669.60. *9.* \$3003.78, \$5006.30. *10.* \$348. *11.* \$2145070. *12.* 223792 lb.; \$38044.64. *13.* \$1356264. *14.* \$1119.70. *15.* \$24250, \$13600, \$8175. *16.* \$109. *17.* \$11470. *18.* \$50.40. *19.* \$186.80. *20.* 179014770. *21.* 12353376. *22.* 1177920. *23.* 6008800. *24.* \$1453022.75. *25.* \$22727.40. *26.* 1105900.

Art. 101. *2.* $1931\frac{4}{18}$, 1337, $914\frac{30}{38}$, $755\frac{32}{46}$, $643\frac{40}{54}$. *3.* $168\frac{5}{75}$, $121\frac{21}{104}$, $108\frac{77}{116}$, $137\frac{1}{92}$. *4.* $303\frac{39}{241}$, $532\frac{143}{162}$, $1691\frac{2}{109}$. *5.* $1239\frac{14}{28}$, $1051\frac{23}{33}$, $723\frac{2}{48}$, $642\frac{38}{54}$, $525\frac{56}{66}$. *6.* $445\frac{79}{116}$, $254\frac{137}{203}$, $164\frac{203}{314}$, $474\frac{33}{109}$, $549\frac{93}{94}$. *7.* $21949\frac{109}{204}$, $663\frac{39}{156}$, $1472\frac{29}{231}$. *8.* \$38.64. *9.* \$4.16$\frac{8}{44}$. *10.* \$6.32. *11.* 2735. *12.* 826. *13.* 125. *14.* 31416. *15.* $10020\frac{5}{72}$. *16.* 1283 men. *17.* $162\frac{22}{29}$. *18.* $276\frac{10}{38}$. *19.* $9709\frac{52}{378}$. *20.* $5816\frac{5}{126}$. *21.* $700\frac{98}{263}$. *22.* $1188\frac{368}{414}$. *23.* $643\frac{102}{134}$. *24.* $482\frac{24}{168}$. *25.* $75\frac{5420}{12600}$ lb. *26.* \$125. *27.* 857 lb. *28.* 456 cows. *29.* $196\frac{350}{475}$ bales. *30.* 1500, 984, 1968. *31.* $73\frac{752}{1248}$ loads. *32.* \$1256. *33.* \$787.50, \$2.10. *34.* $4014\frac{49}{412}$, $3255\frac{277}{508}$, $2109\frac{361}{784}$. *35.* $2311\frac{898}{1326}$, $1243\frac{1289}{2465}$, $986\frac{796}{3108}$. *36.* $21712\frac{414}{4036}$, $16936\frac{3182}{5174}$, $5367\frac{13771}{16325}$. *37.* $3888\frac{34791}{53021}$, $1264\frac{95351}{163042}$.

Arts. 103, 104. *2.* 59. *3.* 573. *4.* 1354. *6.* 24. *7.* 19. *8.* 105. *9.* 143. *3.* 13475, $1347\frac{50}{100}$, $134\frac{750}{1000}$, $13\frac{4750}{10000}$. *4.* 70112, 35056, $4674\frac{1200}{9000}$, $1752\frac{19200}{24000}$. *5.* $3450\frac{3700}{9100}$, $2512\frac{3700}{12500}$, $8464\frac{2260}{3710}$, $1207\frac{21700}{26000}$. *6.* 64, 115. *7.* 83 lots. *8.* \$$776\frac{640}{960}$. *9.* $20\frac{3460}{3600}$. *10.* $103\frac{1254}{1320}$. *11.* $338\frac{1000}{2500}$. *12.* $338\frac{240}{600}$. *13.* $1358\frac{2701}{3170}$. *14.* \$$102\frac{200}{1280}$. *15.* $17\frac{60}{200}$. *16.* $294\frac{70}{320}$. *17.* 53100. *18.* $45\frac{700}{14000}$.

Art. 111. *4.* 3. *5.* 16. *6.* 12. *7.* $9\frac{2}{7}$. *8.* 61. *9.* $41\frac{1}{7}$. *10.* $1\frac{1}{98}$. *12.* \$$2\frac{1}{2}$. *14.* \$$88\frac{1}{5}$. *15.* \$13.50. *17.* $18\frac{3}{4}$ bu. *18.* 24. *20.* 25 bu. *21.* 36 cts. *22.* \$112. *23.* 48. *24.* $60\frac{3}{4}$ lb. *25.* $289\frac{1}{3}$. *26.* 7. *27.* 98.

Art. 116. *2.* \$11.52. *3.* \$36.00, \$93.75. *4.* \$158.12, \$276.04. *5.* \$1103.76. *6.* \$105.84. *7.* \$372, \$386.88, \$406.72.

Art. 117. *2.* \$.75. *3.* \$16, \$14.25, \$17.60. *4.* \$.09, \$.12. *5.* \$6.25, \$87.50, \$231.25. *6.* \$16.83, \$29.92, \$54.23, \$100.98.

Arts. 118, 120. *2.* 900, 720, $654\frac{6}{11}$, 600. *3.* 18, 40, 57, 12. *4.* 15, 20, 37. *2.* \$9, \$10.80. *3.* \$15, \$10. *4.* \$25, \$18.75. *5.* \$3.60, \$3.

6. \$18.50. *7.* $84\frac{20}{45}$ bu. *9.* \$168, \$180, \$192, \$216, \$324. *10.* \$264, \$375.

Art. 121. *2.* 4, 36, 23. *3.* 576, 432, 360, 288. *4.* 252, 315, 378, 504. *5.* 6, 3, 2.

Art. 122. *1.* $2733534\frac{7}{28}$, $1195921\frac{15}{64}$, $797280\frac{79}{96}$, $708694\frac{7}{108}$, $579840\frac{79}{132}$. *2.* 1887378. *3.* 67, rem. 999. *4.* 109, 278. *5.* 11798. *6.* 67 times. *7.* 13. *8.* 29535. *9.* 216 lb. *10.* 7198. *11.* 5102300. *12.* 6 tubs. *13.* 2199077. *14.* 307680, 19230. *15.* \$2120. *16.* \$1779.75. *17.* 66 bu. *18.* \$2.25. *19.* 95 bu. *20.* \$10922, \$20296, \$6880. *21.* 210 days. *22.* 122. *23.* 46. *24.* \$1560. *25.* \$.12. *26.* \$28.50. *28.* 51250. *29.* Lot, \$4930 ; house, \$7570. *30.* \$3167, 1st ; \$4220, 2d. *31.* 1781, 2008. *32.* \$754. *33.* \$.60. *34.* 96 ; \$51. *35.* \$3704. *36.* 21. *37.* 105399. *38.* 14912. *39.* \$996. *40.* \$2330.

Art. 142. *2.* 2, 2, 3, 3. *3.* 2, 2, 3, 7. *4.* 2. *5.* 2, 2, 3. *6.* 3, 7. *7.* 2, 2, 2, 3. *8.* 2, 2, 3. *9.* 2, 2, 2. *10.* 5.

Art. 143. *2.* 4. *3.* 2. *4.* 17. *5.* 14. *6.* 42. *7.* 26. *8.* 8. *9.* 12. *10.* 28. *12.* 15. *13.* 13. *14.* 16. *15.* 31. *16.* 7. *17.* 101. *18.* 595. *19.* 15. *20.* 11. *21.* 137. *22.* 1. *23.* 2. *24.* 2. *25.* 84. *26.* 132.

Art. 144. *2.* 240. *3.* 432. *4.* 2520. *5.* 630. *6.* 140. *7.* 19656. *8.* 2520. *9.* 780. *10.* 468. *11.* 1600. *12.* 2772. *13.* 44100.

Art. 146. *1.* 2, 2, 3, 7 ; 2, 3, 5, 5, 7 ; 3, 5, 7. *2.* 56 ft.; 1 ton ; \$2, \$.27. *3.* 14 ft. *4.* 60. *5.* 3240 bu. *6.* 2337. *7.* 35. *8.* \$37 ; 20, 27, 31. *9.* \$1190. *10.* 10626.

Art. 178. *24.* $30\frac{31}{120}$. *25.* $85\frac{7}{8}$ lb. *26.* $183\frac{17}{60}$ acres. *27.* \$$10\frac{33}{40}$. *28.* $212\frac{37}{42}$. *29.* \$$241\frac{13}{24}$. *30.* \$$2940\frac{1}{4}$, \$$3701\frac{1}{12}$. *31.* $\frac{5}{14}$. *32.* $297\frac{29}{32}$ bu. *33.* $278\frac{7}{12}$ tons. *34.* $\frac{31}{165}$. *35.* $17\frac{5}{9}$. *36.* $15\frac{43}{48}$. *37.* $26\frac{4}{5}$.

Art. 181. *7.* 49. *8.* 64. *9.* $39\frac{1}{5}$. *10.* $\frac{3}{20}$. *11.* $6\frac{4}{5}$. *12.* $2\frac{10}{27}$. *13.* $38\frac{5}{18}$. *14.* 2. *15.* $36\frac{2}{3}$. *16.* 4274, $2735\frac{1}{4}$, 889. *17.* \$100. *18.* \$$113\frac{1}{4}$. *19.* \$21. *20.* \$$\frac{63}{80}$. *21.* \$45. *22.* \$$50\frac{3}{5}$. *23.* \$$20.40\frac{1}{8}$. *24.* \$$214\frac{11}{28}$. *25.* \$$.95\frac{5}{16}$. *26.* \$73. *27.* \$$1.25\frac{9}{32}$. *28.* \$$1758\frac{3}{4}$. *29.* \$$128\frac{11}{12}$. *30.* \$810. *31.* \$$95\frac{17}{20}$. *32.* \$38. *33.* \$$54\frac{2}{5}$. *34.* \$$91\frac{59}{64}$, \$$30\frac{41}{64}$. *35.* 31 tons. *36.* $444\frac{21}{32}$. *37.* $\frac{55}{72}$. *38.* $\frac{12}{29}$. *39.* 17. *40.* $61\frac{4}{9}$. *41.* $19\frac{7}{27}$. *42.* $\frac{9}{56}$. *43.* 90. *44.* $5\frac{1}{7}$. *45.* $6\frac{31}{36}$. *46.* $10\frac{1}{16}$. *47.* $106\frac{2}{3}$. *48.* $11\frac{2}{3}$. *49.* $1\frac{79}{105}$. *50.* $6\frac{59}{66}$. *51.* $\frac{76}{315}$. *52.* $1\frac{12}{35}$. *53.* 14. *54.* $13\frac{1139}{2400}$. *55.* $22\frac{2}{175}$. *56.* $3\frac{17}{140}$. *57.* $7\frac{137}{140}$. *58.* $37\frac{53}{63}$. *59.* $26\frac{3}{10}$. *60.* $18\frac{161}{180}$.

Art. 184. *4.* $\frac{4}{51}$. *5.* $\frac{7}{108}$. *6.* 54. *7.* $2\frac{1}{4}$. *8.* $3\frac{1}{9}$. *9.* $\frac{7}{51}$. *10.* $1\frac{3}{28}$. *11.* $\frac{9}{38}$. *12.* $2\frac{17}{26}$. *13.* $3\frac{25}{32}$. *14.* $\frac{8}{39}$. *15.* $\frac{3}{32}$. *16.* $168\frac{3}{4}$. *17.* $348\frac{4}{9}$. *18.* $21\frac{27}{28}$. *19.* $\frac{153}{608}$. *20.* 153. *21.* $\frac{14}{15}$. *22.* $8\frac{1}{3}$. *23.* $9\frac{4}{5}$. *24.* $\frac{6}{23}$. *25.* $4\frac{4}{5}$. *26.* $\frac{12}{95}$. *27.* 36. *28.* $\frac{21}{50}$. *29.* $\frac{3}{7}$. *30.* $\frac{320}{289}$. *31.* 100. *32.* 121. *33.* $\frac{45}{101}$. *34.* 75 bu. *35.* 50 lb., $38\frac{1}{6}$ lb., $54\frac{2}{5}$ lb. *36.* $3\frac{3}{4}$ bu., 8 bu.,

$16\frac{1}{4}$ bu. *37.* 9, 25. *38.* $3\frac{19}{27}$. *39.* $56\frac{5}{8}$. *40.* $120\frac{8}{9}$, $113\frac{1}{3}$, $56\frac{2}{3}$, $145\frac{1}{15}$. *41* \$2100. *42.* $12\frac{20}{39}$. *43.* $16\frac{2}{3}$, $25\frac{25}{27}$, $28\frac{679}{757}$. *44.* 68, $133\frac{1}{3}$, $335\frac{11}{15}$, 200. *45* $83\frac{1}{3}$, $60\frac{4}{45}$. *46.* \$$10516\frac{2}{3}$. *47.* $\frac{21}{128}$, 18. *48.* $4\frac{1}{3}$, $\frac{3807}{5500}$. *50.* $1\frac{1}{4}$. *51.* $\frac{23}{27}$. *52.* $5\frac{11}{15}$. *53.* $1\frac{3}{13}$. *54.* $3\frac{3}{5}$. *55.* $\frac{45}{254}$. *56.* $1\frac{5}{19}$. *57.* $1\frac{43}{77}$.

Art, 192. *1.* 1400. *2.* $118\frac{4}{19}$, $780\frac{13}{18}$. *3.* \$$5376\frac{7}{10}$, \$$4032\frac{21}{40}$, \$$2016\frac{21}{80}$, \$$2150\frac{17}{25}$. *4.* $89\frac{2}{7}$ A. *5.* $\frac{15}{58}$. *6.* $2\frac{9}{40}$. *7.* $12\frac{1}{2}$ da. *8.* 1. *9.* $2\frac{7}{16}$, $4\frac{11}{16}$, $7\frac{1}{2}$. *10.* $\frac{81}{196}$. *11.* \$$28662\frac{2}{3}$. *12.* \$13.40. *13.* A, \$175 ; B, \$200. *14* \$4000, \$12000. *15.* $9\frac{1}{2}$, $4\frac{3}{4}$. *16.* $33\frac{4}{7}$, $20\frac{1}{7}$. *17.* \$3192. *18.* \$54. *19.* 90. *20.* \$1100. *21.* 80. *22.* 21. *23.* $29\frac{67}{72}$, $8\frac{59}{72}$, $204\frac{37}{72}$, $3\frac{50}{127}$. *24.* \$17491.50. *25.* $\frac{5}{6}$. *26.* Gained 25 cts. *27.* \$$948\frac{1}{8}$. *28.* 21. *29.* \$392. *30.* $6\frac{223}{305}$. *31.* $\frac{7}{22}$. *32.* $\frac{1}{2}$.

Art. 215. *1.* 57500 cts., 8432 cts. *2.* \$26.84, \$68.25. *3.* 4875 m., 926 m. *4.* \$8.42, 78.5 cts. *5.* \$30.256, 650 m. *6.* 3.5000, .2250, 4.0015. *7.* .40000, .00280, .00156. *8.* 42.04000, 125.12600, .00360, .10800, 5.00306. *9.* .60034, 325.06000, 4.00074, .08610, 25.80000. *11.* $\frac{1}{25}$. *12.* $\frac{3}{25}$ *13.* $\frac{4}{5}$. *14.* $\frac{9}{20}$. *15.* $\frac{12}{25}$. *16.* $\frac{16}{25}$. *17.* $\frac{7}{20}$. *18.* $\frac{1}{8}$. *19.* $\frac{17}{250}$. *20.* \$$\frac{3}{8}$. *21.* \$$\frac{7}{8}$. *22.* \$$\frac{3}{16}$. *23.* $\frac{13}{2000}$. *24.* $\frac{15}{16}$. *25.* $\frac{5}{16}$. *26.* $\frac{3}{1250}$. *27.* $\frac{1}{3125}$. *28.* $\frac{19}{32}$. *29.* $\frac{3}{5000}$. *30.* $\frac{11}{16}$. *32.* \$$\frac{1}{16}$, \$$\frac{1}{3}$. *33.* \$$\frac{1}{12}$, \$$\frac{3}{8}$. *34.* \$$\frac{1}{8}$, \$$\frac{5}{16}$. *35.* \$$\frac{5}{8}$; $\frac{5}{16}$. *36.* \$$\frac{3}{16}$; $\frac{5}{6}$. *37.* \$$\frac{7}{8}$; $\frac{31}{4000}$. *38.* $\frac{33}{40000}$. *39.* $\frac{1}{6}$. *40.* $\frac{1}{7}$. *42.* $15\frac{31}{40}$. *43.* \$$87\frac{4}{9}$. *44.* $150\frac{3}{8}$, $9\frac{31}{4000}$. *45.* $705\frac{5}{12}$, $26\frac{1}{18}$, *46.* $125\frac{1}{125}$. *47.* $8\frac{1}{12}$.

Art. 216. *2.* .625, .875. *3.* .0625, .1875. *4.* .025, .0625. *5.* .08, .7. *6.* .6875, .075. *7.* .56, .9375. *8.* .65625, .132. *9.* .7578125, .3216 *10.* .376, .03375. *11.* .0728, .05078125. *12.* .685, .005625. *13.* .8015, .60625.

Art. 221. *1.* .32, .1171875, .00352, .066, .013671875. *2.* $.\dot{6}$, $.\dot{4}$, $\dot{2}9\dot{7}$, $.\dot{9}512\dot{1}$, $.\dot{6}43\dot{5}$, $.\dot{5}3\dot{4}$. *3.* $.8\dot{3}$, $.4\dot{6}$, $.13\dot{8}$, $.45\dot{3}$, $.1\dot{7}\dot{8}$.

Art. 222. *2.* $\frac{1}{3}$, $\frac{7}{9}$. *3.* $\frac{1}{33}$, $\frac{3}{11}$. *4.* $\frac{31}{99}$, $\frac{5}{11}$. *5.* $\frac{2}{3}$, $\frac{1}{99}$. *6.* $\frac{5}{111}$, $\frac{27}{37}$. *7.* $\frac{36}{37}$, $\frac{12}{37}$. *8.* $\frac{42}{101}$. *9.* $\frac{39}{41}$. *12.* $\frac{7}{90}$, $\frac{103}{180}$. *13.* $\frac{1}{12}$, $5\frac{5}{18}$. *14.* $\frac{17}{55}$, $4\frac{17}{45}$. *15.* $2\frac{9}{110}$. *16.* $\frac{16}{27}$. *17.* $3\frac{137}{300}$. *18.* $3\frac{13}{198}$. *19.* $7\frac{25}{132}$. *20.* $\frac{1}{540}$. *21.* $\frac{1}{44}$. *22.* $\frac{13}{37}$. *23.* $\frac{1}{88}$.

Art. 225. *13.* 39.4024, 24.7224. *14.* 191.85156., 159.84844. *15.* 4.060514, 4.059886. *16.* 2484.729, 515.287. *17.* 508.75 tons, 291.25 tons. *18.* 145.825 rd., 95.775 rd. *19.* \$238, \$137.75. *20.* \$283.25, \$230.50. *21.* \$335.9375, \$264.0625. *22.* 227.25, 222.75. *23.* \$1700.875, \$699.125. *24.* 92.4, 75.6. *25.* 10.000006, 9.999994. *26.* .2853, .2397. *27.* \$6.27, \$5.13. *28.* $.67\dot{1}$ .54. *29.* 6, 3. *30.* \$1, \$.25. *31.* \$67.875, \$12.125. *32.* $.8\dot{8}6$, $.71\dot{3}$. *33.* \$58.3625. *34.* 199.0198. *35.* \$49.15. *36.* 2.46. *37.* \$361.2175. *38.* .773. *39.* \$.7525.

Art. 228. *36.* .0093. *37.* .0008892. *38.* 3870. *39.* 256.5. *40.* \$864506.25. *41.* .1027875. *42.* \$.00255. *43.* .210125. *44.* 224. *45.* .000081. *46.* \$1071.25, \$107.13, \$10712.50, \$1606.88. *47.* \$6274.02, \$9226.50, \$13932.02, \$24604. *48.* 14, 22. *49.* \$22.60. *50.* \$571.88, \$506.25. *51.* \$1245.56, \$1109.06. *52.* \$34.36, \$28.63. *53.* \$150.61, \$138.80. *54.* \$387.50, \$337.50. *55.* \$2875, \$3100. *56.* \$506.40. *57.* \$9375, \$6875. *58.* \$62.50. *59.* \$178.13. *60.* \$6963.84. *61.* \$.09, \$.06$\frac{1}{2}$. *62.* \$2.12$\frac{1}{2}$, \$1.38. *63.* \$33.26, \$26.75. *64.* \$12, \$12.50. *65.* \$.18$\frac{3}{4}$. *66.* \$66.44. *67.* \$1.50, \$2.88. *68.* \$30.66, \$40.88. *69.* \$87.21. *70.* 64, 320. *71.* 7.28, 7.904. *72.* 282.18$\frac{3}{4}$. *73.* \$2.06, \$9.63. *74.* \$2372.20. *75.* 46. *76.* 311.65. *77.* .065.

Art. 230. *2.* \$25.53. *3.* \$49.70. *4.* \$10.43. *5.* \$2.45, \$2.23, \$3.79, \$3.23. *6.* \$146.25, \$175.78. *7.* \$8.63, \$18.92.

Art. 235. *1.* \$91.76. *2.* \$105.95. *3.* \$712.32. *4.* \$2380.77. *5.* \$55.13. *6.* \$42.38. *7.* \$3455.40. *8.* \$21.37. *9.* \$82.86.

Art. 236. *1.* 36. *2.* 57.6, 86.4, 100.8, 201.6. *3.* \$27.66, \$.92. *4.* \$26.25. *5.* \$52.59. *6.* \$18.75. *7.* \$11.27. *8.* \$2.40. *9.* \$31.04. *10.* \$268.13. *11.* \$237.67. *12.* \$17.65. *13.* \$18.75. *14.* \$273.28. *15.* \$3840.84. *16.* 3432, 132. *17.* \$2.50. *18.* 127. *19.* 62.5 tons; \$289.06. *20.* \$15000. *21.* 219. *22.* \$27.56. *23.* \$198.53. *24.* 342$\frac{9}{14}$ sacks. *25.* \$1543.75. *26.* \$3403.05. *27.* \$1312.50. *28.* .0002938. *29.* 41.3337.

Art. 286. *2.* 85590 min. *3.* 19561 oz. *4.* 355 in. *5.* 477 gi. *6.* 3680 in. *7.* 2349d. *8.* 5174 pwt. *9.* 26660 l. *10.* gr. 20447. *11.* 10807$\frac{1}{2}$ yd. *12.* 5445000 sq. ft. *13.* 1156 gi. *14.* 284726 lb. *15.* 1606 oz. *16.* 16575 l. *17.* 1452.36 mi. *18.* 1872 in. *19.* 149 doz. *20.* 2624 sq. rd. *21.* 311040 cu. in. *22.* 1792 cu. ft. *23.* 888 qt. *24.* 6105 mi. *25.* 17520 hr. *26.* 245000 cts. *27.* 23760 in. *28.* f ʒ 6784. *29.* 213840 cu. in. *30.* 11092 yd. *31.* 8624 lb. *32.* 114 qr. *33.* 50 eighths. *34.* 59066″. *35.* 520 qr. *48.* \$218.99. *49.* \$80.78. *50.* \$73. *51.* \$28.95. *52.* \$17.89. *53.* \$132.86. *54.* \$1917. *55.* 610 bbl. *56.* \$64.80. *57.* 288 pp. *58.* \$497.28. *59.* 7273$\frac{5}{11}$ sacks. *60.* \$25.20. *61.* \$51.15, \$52.31. *62.* \$48.43. *63.* \$26.95, \$33.69. *64.* \$132. *65.* \$4.80, \$6.48. *66.* \$7.68.

Art. 289. *26.* \$7. *27.* \$3484.80. *28.* \$60. *29.* \$180. *30.* \$210. *31.* 2 bu. 1 pk. 6 qt. 1 pt. *32.* 45 A. *33.* \$31.25. *34.* 7 wk. 1 da. 17 hr. 30 min. *35.* 270 reams 16 qr. 16 sheets.

Art. 291. *2.* 7 oz. 10 pwt. *3.* 2 ft. 3 in. *4.* 6s. 6d. *5.* 2 rd. 5 l. 4$\frac{2}{5}$ in. *6.* 1 sq. ft. 18 sq. in. *7.* 5 sp. 4$\frac{1}{2}$ in. *8.* 3 da. 2 hr. 40 min.

9. 3 gal. 3 qt. 1 pt. 2 gi. *10.* ℥ 8 ʒ 2. *11.* 16 sec. *12.* 2 pk. 6 qt. $1\frac{5}{7}$ pt. *13.* $8\frac{4}{5}$ oz. *14.* 3 qt. 1 pt. *15.* 15 sheets. *16.* 1 ch. 1 rd. 3 l. *17.* 6 doz. $10\frac{2}{7}$ units. *18.* 60 ch. 2 rd. 10 l. *19.* 2 cu. ft. 89.856 cu. in. *20.* 16 gal. 3 qt. 1 pt. *21.* 2 lb. $13\frac{5}{7}$ oz. *22.* 48 lb. *23.* 24 lb. *24* 3 cd. ft. 8 cu. ft. *25.* 28 ch. 3 rd. 5 l. *26.* 14 lb. 11.2 oz. *27.* 48° 36′. *28.* 4 cwt. 33 lb. 8 oz.

Art. 293. *3.* $\frac{5}{8}$. *4.* $\frac{1039}{2500}$. *5.* $\frac{9}{10}$. *6.* $\frac{4}{9}$. *7.* $\frac{1}{4}$. *8.* $\frac{6}{13}$. *9.* $\frac{3}{10}$. *10.* $\frac{1}{32}$. *11.* $\frac{61}{128}$. *12.* $\frac{141}{500}$. *13.* $\frac{3}{5}$. *14.* $\frac{3}{40}$. *15.* $\frac{7}{64}$. *16.* $\frac{5}{9}$. *17.* $\frac{7}{8}$. *18.* $\frac{127}{384}$. *19.* $\frac{31}{252}$. *20.* $\frac{5}{7}$. *21.* $\frac{15}{32}$. *22.* $\frac{193}{200}$. *23.* $\frac{44}{63}$. *24.* $\frac{11}{16}$. *25.* $\frac{1467}{1760}$. *26.* $\frac{125}{1182}$. *27.* $\frac{7}{10}$. *28.* $\frac{7}{16}$.

Art. 295. *3.* 28 T. 16 cwt. 69 lb. 6 oz. *4.* 16 wk. 2 da. 21 hr. 7 min. 12 sec. *5.* 16 Cd. 3 cd. ft. 7 cu. ft. *6.* 5 hhd. 6 gal. 2 qt. *7.* 75 A. 80 P. *8.* 2809 units. *9.* 24 lb. 4 oz. 1 pwt. 12 gr. *10.* 3 da. 7 hr. 58 min. 15 sec. *11.* 7 T. 65 lb. 10 oz. *12.* ℔ 19 ℥ 8 ʒ 0 ℈ 2 gr 15. *13.* 5 T. 9 cwt. 24 lb. *14.* 1 mi. 287 rd. 15 ft. $9\frac{3}{4}$ in. *15.* 11 bu. 3 pk. 4 qt. $1\frac{8}{15}$ pt. *16.* $80.78. *17.* $293.80. *18.* $107.25.

Art. 296. *3.* 2 T. 8 cwt. 8 lb. 10 oz. *4.* 223 bu. 2 pk. 6 qt. *5.* 17 gal. 2 qt. 1 pt. *6.* 9 mi. 29 rd. 14 ft. *7.* 24 A. 9 sq. ch. 7 P. *8.* 4 T. 13 cwt. 40 lb. *9.* 4 da. 5 hr. 48 min. *10.* 2 lb. 7 oz. 6 pwt. 16 gr. *11.* 212 rd. 10 ft. $5\frac{2}{5}$ in. *12.* ℔ 4 ℥ 10 ʒ 3. *13.* 22 min. 14 sec. *14.* 17° 50′ 22″. *15.* 6 ft. $10\frac{1}{2}$ in. *16.* 3 sq. ch. 12 P. 100 sq. l. *17.* 2 pk. 3 qt. $\frac{1}{5}$ pt. *18.* 6 doz. 10 units. *19.* 4 bbl. 11 gal. 1 qt. *20.* £71 15s. 11d. 2 far. *21.* 39 sq. rd. 14 sq. yd. 5 sq. ft. 84 sq. in. *22.* 54 bu. 3 pk. 4 qt. *23.* 4 doz. 2 pens. *24.* 38 Cd. 7 cd. ft. 8 cu. ft.

Art. 297. *2.* 12 yr. 19 da. *3.* 11 yr. 4 mo. 19 da. *4.* 4 yr. 4 mo. 15 da. *5.* 3 yr. 5 mo. 24 da. 15 hr. *6.* 1 yr. 11 mo. 17 da. *7.* 3 yr. 11 mo. 28 da. *8.* 4 yr. 3 mo. 22 da. *10.* 2 yr. 10 mo. 6 da. *11.* 1 yr. 9 mo. 2 da. 6 hr. *12.* 12 yr. 6 mo. 1 da.

Art. 298. *2.* 34 lb. 3 oz., 39 lb. 11 oz. 10 pwt., 45 lb. 8 oz. *3.* 60 T. 16 cwt. 84 lb., 68 T. 8 cwt. 94 lb. 8 oz., 53 T. 4 cwt. 73 lb. 8 oz. *4.* 464 bu. 1 pk. *5.* 7 lb. 3 oz. 3 pwt. *6.* 510 A. *7.* 172 T. 7 cwt. 4 lb. *8.* 2 T. 10 cwt. 24 lb.; $452.16. *9.* 94 A. *10.* 5 mi. *11.* 48 bu. 3 pk. *12.* 223 lb. 2 oz. 8 pwt. *13.* 7 T. 19 cwt. 12 lb. *14.* 14 wk. 2 da. 14 hr 9 min. 36 sec. *15.* 11 cwt. 40 lb. *16.* 69 hhd. 23 gal. 2 qt. 1 pt. *17* 145 wk. 4 da. 4 hr. 48 min. *18.* 26 bu. 4 qt.

Art. 299. *2.* 3 lb. 11 oz. 8 pwt. $4\frac{1}{4}$ gr., 2 lb. 7 oz. 12 pwt. $2\frac{5}{6}$ gr., 1 lb. 11 oz. 14 pwt. $2\frac{1}{8}$ gr. *3.* 1 T. 4 cwt. 4 lb. 15 oz., 8 cwt. 1 lb. $10\frac{1}{3}$ oz., 10 cwt. 30 lb. 11 oz. *4.* ℔ 7 ℥ 8 ʒ 5 ℈ 1 gr. $15\frac{1}{3}$, ℔ 17 ℥ 4 ʒ 4 ℈ 1 gr. $14\frac{1}{2}$, ℔ 5 ℥ 9 ʒ 4 gr. $11\frac{1}{2}$. *5.* 55 gal. 2 qt. 1 pt., 46 gal. 1 qt. $3\frac{1}{3}$ gi. *6.* 6 hhd. 13 gal. 2 qt. $1\frac{3}{5}$ gi. *7.* £9 10s. 8d. 2 far. *8.*

28 A. 12 P. *9.* 79 Cd. 3 cd. ft. 4 cu. ft. *10.* 2° 57′ 34″. *11.* 11 mi. 220 rd. 1 ft. 6 in. *12.* 5 cu. yd. 22 cu. ft. *13.* 1 wk. 23 hr. 43 min. $38\frac{2}{11}$ sec. *14.* 2 ft. 3 in. *15.* £4 2s. 4d. *16.* 1 oz. 17 pwt. 4 gr. *17.* 85 A 3 sq. ch. 5 P. $208\frac{1}{3}$ sq. l. *18.* 3 gal. 3 qt. $3\frac{5}{9}$ gi., 2 gal. 3 qt. 1 pt. $\frac{2}{3}$ gi., 2 gal. 1 qt. $2\frac{2}{15}$ gi. *19.* 5 cu. yd. *20.* 5. *21.* 15 da., $12\frac{6}{7}$ da.

Art. 303. *3.* 10 min. $45\frac{1}{5}$ sec. *4.* 3 hr. 13 min. $42\frac{1}{15}$ sec. *5.* 5 hr. 58 min. $5\frac{4}{5}$ sec. *6.* 4 hr. 55 min. $36\frac{3}{5}$ sec. *7.* 11 hr. 17 min. $45\frac{4}{15}$ sec. A. M. *8.* 8 hr. 58 min. $33\frac{2}{5}$ sec. A. M. *9.* 10 hr. 31 min. $45\frac{7}{15}$ sec. A. M. *10.* 5 hr. 17 min. $32\frac{2}{3}$ sec. P. M. *11.* 5 hr. 7 min. $52\frac{1}{15}$ sec. P. M. *12.* 52 min. $13\frac{14}{15}$ sec. A. M.

Art. 304. *2.* 71° 30′. *3.* 21° 15′. *4.* 36° 22′ 30″. *5.* 47° 36′ 15″. *6.* 20° W. *7.* 14° E. *8.* 37° 30′ E. *9.* 48° 45′ W.

Art. 305. *1.* \$80.12. *2.* 72 centals, $56\frac{1}{4}$ centals, 48.913+ centals. *3.* $518\frac{2}{5}$ bu., 322 bu. *4.* $6\frac{6}{19}$ bbl. *5.* 2171.624 mi. *6.* \$6.91. *7.* \$21.99. *8.* \$347.90. *9.* 3 hr. 5 min. $17\frac{1}{5}$ sec. P. M., 2 hr. 55 min. $36\frac{3}{5}$ sec. P. M., 8 hr. 55 min. $56\frac{3}{5}$ sec. A. M., 8 hr. 19 min. 30 sec. A. M. *10.* \$230. *11.* £308 4s. 7d. $\frac{5300}{9733}$ far. *12.* \$162.19. *13.* 1 wk. 3 da. 12 hr. *14.* 1602 volumes. *15.* 2205 bu. 3 pk. *16.* \$574.42. *17.* 1 lb. 3 oz. 8 pwt. 21 gr. *18.* \$551.70. *19.* 279 rd. 2 yd. 2 ft. 3 in. *20.* 14 wk. 1 da. 22 hr. 26 min. 24 sec. *21.* 29 gal. 3 qt. 1 pt. $3\frac{7}{13}$ gi. *22.* 5 qt. $\frac{38}{75}$ pt. *23.* 6 Cd. 3 cd. ft. 1 cu. ft. $1036\frac{4}{5}$ cu. in. *24.* 7 sq. ch. 8 P. *25.* \$30000. *26.* \$650. *27.* \$691.20. *28.* \$24.38. *29.* \$30.75. *30.* \$37.50. *31.* \$2580.48. *32.* \$180. *33.* \$112.50. *34.* \$80640. *35.* \$420. *36.* \$5.38. *37.* \$23277.38. *38.* \$93.71. *39.* \$560. *40.* \$67. *41.* \$328.32. *42.* \$225.14. *43.* \$60. *44.* \$87040. *45.* \$360. *46.* \$53.57. *47.* \$65.28. *48.* \$86.40. *49.* \$1.42. *50.* \$62.15. *51.* \$1377. *52.* \$3.75. *53.* \$108. *54.* \$11.33.

Art. 312. *1.* 3 sq. ft. 36 sq. in. *2.* 3 sq. rd. 7 sq. yd. 36 sq. in. *3.* 17 sq. yd. 3 sq. ft. 36 sq. in. *4.* 2 sq. rd. 3 sq. yd. 8 sq. ft. 90 sq. in. *5.* 4 A. 2 sq. ch. 12 sq. rd. *6.* 25 sq. yd. 4 sq. ft. 72 sq. in. *7.* 66 sq. ft. *8.* 184 sq. rd. 14 sq. yd. *9.* 65 sq. yd. 4 sq. ft. *10.* $18\frac{1}{2}$ ft. *11.* 37 ft. *12.* 33 yd.; \$29.70. *13.* $123\frac{3}{7}$ yd.; \$185.14. *14.* $24\frac{122}{135}$ yd.; \$45.82. *15.* $45\frac{1}{3}$ yd.; \$96.33. *16.* $103\frac{17}{81}$ yd.; \$283.83. *17.* \$64.60. *18.* \$218.16, \$76.44. *19.* 450 stones; \$312.50. *20.* 1050 A. *21.* 24 ch. 1 rd. 2 l. 1.9+ in. *22.* $\frac{5}{8}$. *23.* 32 rd. *24.* 105 A. 6 sq. ch. 4 sq. rd. *25.* 600. *26.* $40\frac{2}{3}$. *27.* $10\frac{2}{7}$ yd., $12\frac{6}{7}$ yd. *28.* \$157.50. *29.* \$30.88. *30.* 89 rd., 210 rd.

Art. 321. *1.* 48 cu. ft. *2.* $85\frac{1}{2}$ cu. ft. *3.* 390 cu. yd. *4.* 2850 cu. ft. *5.* $175\frac{25}{32}$ cu. ft. *6.* 195 cu. in. *7.* 4050 cu. ft. *8.* $380\frac{1}{4}$ cu. ft. *9.* 2 cu. yd. 8 cu. ft. 864 cu. in. *10.* 6 cu. yd. 4 cu. ft. 648 cu. in. *11.* 3 ft. *12.* 8 ft. 6 in. *13.* $\frac{1}{4}$ ft. *14.* \$173.33.

15. $284\frac{4}{33}$ Perch. *16.* $111\frac{3}{8}$ cu. ft. *17.* $13\frac{3}{4}$ Cd. *18.* \$3.94, \$5.25. *19.* $190\frac{10}{11}$ Perch. *20.* \$380.24. *21.* 22320. *22.* 76440; \$573.30.

Art. 322. *3.* $18\frac{2}{3}$ board ft. *4.* $22\frac{1}{2}$ board ft. *5.* 35 board ft. *6.* $34\frac{1}{2}$ board ft. *7.* 33 board ft. *8.* 17 board ft. *9.* 52 board ft. *10.* 19 board ft. *11.* 44 board ft. *12.* $19\frac{1}{2}$ board ft. *13.* \$11.55. *14.* \$5.04. *15.* \$7.29. *16.* \$201.60. *17.* $92\frac{4}{9}$ sq. ft. *18.* \$45.57. *19.* \$23.89.

Art. 324. *1.* $6218\frac{1}{10}$ cu. in. *2.* 1866 cu. ft. 1182 cu. in. *3.* 560 cu. ft. 1621.815 cu. in. *4.* 505 cu. ft. 540 cu. in. *5.* 631 cu. ft. 1107 cu. in. *6.* 467.2 cu. ft. *7.* 72 cu. ft. 307.2 cu. in. *8.* 32 hhd. *9.* 8 bbl. *10.* 241.0691 + bu. *11.* 184 cu. ft. 310.16 cu. in. *12.* 325.443 bu. barley; 260.354 bu. potatoes. *13.* $718\frac{10}{77}$ gal.; 3 T. *14.* 322.26 + bu. *15.* \$20.68, gain. *16.* 6.767 bu. +. *17.* 156.9375 T. *18.* \$983.56. *19.* 37.636 + bbl. *20.* $942\frac{6}{11}$ gal.; 101.249 bu. +. *21.* 68.7047 bu. + corn; 54.9638 bu. + apples. *22.* 8 ft. 11.52 + in. *23.* \$14.25. *24.* \$134.37½. *25.* $198\frac{33}{64}$ Cd.; \$694.80.

Art. 337. *10.* 30 bu.; \$2.25; 8.25 ft.; 12.48 T. *11.* \$34.65. *12.* 12 T., 8.1 T., 6.75 T. *13.* \$7968.75. *14.* \$32.10, \$609.90. *15.* 254.8 lb. *16.* \$2400. *17.* \$1139.06. *18.* $\frac{25}{64}$ left; \$42666.66⅔. *19.* $583\frac{1}{3}$ bu. *20.* \$1881.25, \$1618.75.

Art. 339. *12.* 35%. *13.* $15\frac{1}{2}$%. *14.* $62\frac{1}{2}$%. *15.* 25%. *16.* 25%. *17.* 44%. *18.* $62\frac{1}{2}$%.

Art. 341. *10.* 3200 bbl. *11.* \$10500. *12.* \$29600. *13.* 2000 bu. *14.* \$1371.43. *15.* 213 A. 120 sq. rd. *16.* \$50000. *17.* \$4290.

Art. 343. *3.* 125. *4.* 1256. *5* 93.75 mi. *6.* 800 mi. *7.* \$57. *8.* \$12000. *9.* \$160 loss. *10.* \$.0315 +. *11.* \$3.68. *12.* \$7.50.

Art. 344. *1.* \$524.19. *2.* \$5.56. *3.* \$18.37, \$131.63. *4.* $5\frac{61}{100}$%, 1%. *5.* $11\frac{14}{25}$%, 30%. *6.* \$3.25. *7.* \$3.50.

Art. 352. *2.* \$150 gain, \$900. *3.* \$54 loss, \$396. *4.* \$1.05 profit, \$5.25. *5.* $\frac{3}{4}$ ct. loss, $29\frac{1}{4}$ cts. *6.* \$1.14 profit, \$7.98. *7.* \$3.41, \$2.585. *8.* 9 cts., $9\frac{1}{5}$ cts. *9.* \$3.30, \$3.55½, \$3.93¾. *10.* 0.

Art. 353. *2.* $16\frac{2}{3}$% gain. *3.* $14\frac{2}{7}$% loss. *4.* $33\frac{1}{3}$% gain. *5.* 20% gain. *6.* 10% loss. *7.* $16\frac{2}{3}$% gain. *8.* $66\frac{2}{3}$% gain. *9.* 30% gain. *10.* $37\frac{1}{2}$%. *11.* $66\frac{2}{3}$%. *12.* $33\frac{1}{3}$%. *13.* 44%.

Art. 354. *2.* \$4.70. *3.* \$1. *4.* \$.68. *5.* \$5. *6.* \$22.50, \$18. *8.* \$3.50. *9.* \$1.68. *10.* \$282.35. *11.* \$5. *12.* \$.75. *13.* \$18750. *14.* \$.75. *15.* \$.12½. *17.* \$1.47. *18.* 15 cts. *19.* \$75. *20.* \$1.066 *21.* \$4.23. *22.* \$4.96.

Arts. 360, 361. *2.* $34.20. *3.* $9.30. *4.* $52.50. *5.* $351.78. *6.* $28.35. *7.* $907.20. *8.* $4729.35. *2.* $2\frac{1}{2}\%$. *3.* $\frac{1}{2}\%$. *4.* $4\frac{1}{2}\%$ *5.* $\frac{1}{3}\%$. *6.* $3\frac{7}{145}\%$. *7.* $1\frac{1}{2}\%$.

Arts. 362, 363. *1.* $14400. *2.* $19200. *3.* $1226. *4.* $1285. *5.* $4000. *6.* $2463.02. *7.* $9000. *8.* $2550. *9.* $5340. *10.* 2840 bu.; $\$5458.12\frac{1}{2}$. *1.* $560. *2.* $2750. *3.* $4680. *4.* $6419.75. *5.* 5750 lb. *6.* 500 shares. *7.* 3200 yds.

Art. 369. *1.* $72. *2.* $40.50, $30.75. *3.* $65.70. *4.* $8.75, $22.50. *5.* $210. *6.* $75. *7.* $100.63.

Arts. 370, 371. *1.* $1\frac{1}{2}\%$. *2.* $4\frac{1}{2}\%$. *3.* $\frac{3}{8}\%$. *4.* $\frac{3}{4}\%$. *5.* $1\frac{1}{4}\%$. *6.* $5\frac{1}{2}\%$. *7.* $1\frac{3}{4}\%$. *1.* $14000. *2.* $13600. *3.* $5276. *4.* $24500. *5.* $\$10.15\frac{37}{50}$ per bbl. *6.* $28000. *7.* $5025.64. *8.* $4010.

Art. 379. *1.* $4025. *2.* $263.50. *3.* $1\frac{1}{5}\%$. *4.* 3 mills, $12. *5.* $56000. *6.* $7100. *7.* $2800. *8.* $2500000. *10.* $18.88. *11.* $36.69.

Art. 390. *2.* $251.25. *3.* $162. *4.* $3037.50. *5.* $1612.80. *6.* $44.10. *7.* $261.49. *8.* $1640. *9.* $2760.92, $7659.22. *10.* $1\frac{43}{132}\%$. *11.* 3s. 8d. 2.8+ far. *12.* 40 gal. *13.* 8%. *14.* $1630.

Arts. 400, 401. *2.* $3726. *3.* $3937.50, $4685.63. *4.* $4400. *5.* $288. *2.* 12. *3.* 30. *4.* 24. *6.* 1000.

Arts. 402, 403. *2.* $14625. *3.* $42000. *2.* $1280. *3.* $192.771.

Arts. 404, 405. *2.* 8%, $6\frac{2}{13}\%$, $5\frac{5}{7}\%$. *3.* $11\frac{13}{17}\%$. *4.* Neither, N. Y. 6's at 84. *5.* $5\frac{115}{217}\%$. *2.* $75, $60. *3.* $\$133\frac{1}{3}$, $\$114\frac{2}{7}$, $80, $\$66\frac{2}{3}$. *4.* $\$83\frac{1}{3}$. *5.* $\$64\frac{2}{7}$.

Art. 406. *1.* 2 hr. 35 min. 18 sec. *2.* 78 A. 5 sq. ch. 3.2 P.; 1 T. 6 cwt. 26 lb. *3.* $8\frac{1}{3}\%$. *4.* $1500. *5.* 25%. *6.* $6153.25. *7.* $1.19. *8.* $1\frac{1}{4}\%$. *9.* $8290.16. *10.* $3750. *11.* $15490. *12.* $640. *13.* 2250. *14.* $78\frac{13}{54}\%$. *15.* $89\frac{2}{7}\%$. *16.* Nothing. *17.* $1500, $1690, $4000. *18.* $4640. *19.* 124. *20.* $118.75. *21.* $1\frac{97}{203}\%$. *22.* 400 bbl. *23.* $1.74; 28%. *24.* $1558.97. *25.* $70. *26.* $153687.50. *27.* $2275 asking price, $22.50 per A., $787.50 profit. *28.* $1432.08. *29.* 4%; $140. *30.* $8\frac{408}{449}\%$. *31.* An. Prem. $875; A pays $9174.31; B, $6880.73; C, $5504.59; D, $3440.37. *32.* $12953.37, $453.37. *33.* $80. *34.* $9600. *35.* 2.9636 cts., $447.04. *36.* $4\frac{1}{2}\%$. *37.* $2425.17. *38.* $4500. *39.* $.45. *40.* $\$1242\frac{4}{13}$. *41.* 60%. *42.* $1432.15. *43.* $11\frac{139}{351}\%$.

Art. 417. *2.* $18, $18.38, $21. *3.* $15.27, $14.84. *4.* $198.22. $177.89. *5.* $1221.58, $161.09. *6.* $315, $930. *7.* $93.44, $92.71,

\$91.98, \$96.36. *8.* \$647,17, \$713.06. *9.* \$514.50. *10.* \$17.72. *12.* \$25.65, \$29.93, \$34.20, \$51.30. *13.* \$35.53, \$33.94. *14.* \$98.63, \$142.46. *15.* \$53.44, \$33.04. *16.* \$158.59, \$49.30. *17.* \$21.34, \$70.90. *18.* \$287.50, \$155. *19.* \$53.78, \$428.38. *20.* \$472.50. *21.* \$442.55. *22.* \$6822.29.

Art. 420. *2.* \$35.70. *3.* \$6.28. *4.* \$19.98. *5.* \$70.11. *6.* \$5.88. *7.* \$186. *8.* \$26.11. *9.* \$26.25. *10.* \$1.92. *11.* \$8.66. *12.* \$40.94, \$57.31, \$65.50. *13.* \$8.97, \$12.56, \$14.36. *14.* \$3.21, \$4.49, \$5.13. *15.* \$51.33, \$71.86, \$82.12. *16.* \$69.44, \$97.22, \$111.11. *17.* \$26.05, \$36.47, \$41.68. *18.* \$3992.73.

Art. 423. *6.* \$68.60, \$78.40, \$98. *7.* \$3.46, \$3.17, \$2.30. *8.* \$188, \$70.50. *9.* \$2.04, \$2.92, \$1.75. *10.* \$4.62, \$2.02. *11.* \$36.95, \$22.76 *12.* \$242.55, \$151.20. *13.* \$87.50, \$81.25, \$26.25. *14.* \$4520. *15.* \$406.53, \$395. *16.* \$5070. *17.* \$2619.78. *18.* \$92.08. *19.* \$341.25. *20.* \$1011.60. *21.* \$33.83. *22.* \$126.46.

Art. 425. *2.* \$5.55. *3.* \$24.93. *4.* \$13.26. *5.* \$1.36. *6.* \$1.62. *7.* \$17.61. *8.* \$23.56, \$17.53. *9.* \$.06+. *10.* \$79.49. *11.* \$.013+. *12.* \$654.45. *13.* \$47.59. *14.* \$1226.83.

Art. 426. *3.* \$780, \$585. *4.* \$1260, \$1008. *5.* \$7855.97, \$13747.95. *6.* \$200. *7.* \$3032.90.

Art. 427. *2.* 5%. *3.* 7%−. *4.* 4%. *5.* 30%+. *6.* 1%. *7.* 12%. *8.* 12%, 8%. *9.* $2\frac{1}{2}$% per month. *10.* $6\frac{75}{556}$%. *11.* 9%.

Art. 428. *2.* 1 yr. 4 mo. *3.* 1 yr. 4 mo. 28 da.+. *4.* 2 yr. 3 mo. 15 da. *5.* 8 mo. 18 da.+. *6.* 16 yr., 12 yr. 6 mo., 14 yr. 3 mo. $12\frac{6}{7}$ da., 10 yr. *7.* 16 yr. 8 mo., 8 yr. 4 mo., 33 yr. 4 mo. *8.* 2 yr. 3 mo. 12 da. *9.* 2 yr. 3 mo. $12\frac{6}{7}$ da. *10.* 1 yr. 9 mo. 15 da.+.

Art. 429. *2.* \$57.63. *3.* \$296.78. *4.* \$12.82. *5.* \$428.16. *6.* \$466. *7.* \$106.60.

Art. 431. *2.* \$523.69, \$48.69, \$742.47, \$102.47. *3.* \$1225.04, \$225.04, \$1099.65, \$224.15. *4.* \$2963.64, \$463.64, \$3954.67, \$354.67. *5.* \$1018.72, \$268.72, \$5819.87, \$1319.87. *6.* \$695.56. *7.* \$40.48. *8.* \$71.37. *9.* \$407.29.

Art. 432. *2.* \$529.93, \$133.93. *3.* \$2075.47, \$600.47. *4.* \$1167.42, \$306.92. *5.* \$1819.58, \$319.58. *6.* \$1806.53, \$681.53. *7.* \$948.97, \$373.97. *8.* \$3445.87, \$945.87. *9.* \$1637.61, \$387.61. *10.* \$647.28.

Art. 434. *2.* \$411.20, \$1411.20. *3.* \$305.24, \$1070.24. *4.* \$643.90, \$3283.90. *5.* \$66.08, \$252.88. *6.* \$1.76. *7.* \$163.03. *8.* \$1373.97.

Art. 445. *3.* \$456.89. *4.* \$496.51. *5.* \$342.39. *6.* \$3309.26. *7.* \$3932.07. *8.* \$1194.26.

Art. 446. *2.* \$139.82. *3.* \$1492.52. *4.* \$1502.53.

Art. 452. *2.* \$398.74, \$76.76, \$455.02, \$20.48. *3.* \$520, \$15.60, \$502.13, \$33.47. *4.* \$846.25, \$13.75, \$814.52, \$45.48. *5.* \$1479.45, \$95.55, \$1444.40, \$130.60, \$1427.49, \$147.51. *6.* \$2.54. *7.* \$912.775, *8.* \$10.06. *9.* $19\frac{3}{13}$¢ better to buy for cash. *10.* \$454.19. *11.* \$600 cash is \$7.41 better than \$640 at 12 mo., and \$3.85 better than \$620 for 6 mo. *12.* \$901.50, \$1030.51. *13.* \$6059.19. *14.* \$510.22.

Art. 460. *2.* \$9.72, \$530.28. *3.* \$9.19, \$865.81, \$25.21, \$594.79. *4.* \$36.81, \$2107.69, \$49.09, \$2095.41. *5.* \$1.72, \$311.08, \$4.07, \$308.73. *6.* \$45.21, \$2454.79, \$12.08, \$1137.92. *7.* \$.75. *8.* \$37.46, \$1523.54. *9.* \$749.68. *10.* \$855.03. *11.* Nov. 4; 87 da.; \$459.29. *12.* Dec. 6; 42 da.; \$729.93. *13.* Nov. 17; 77 da.; \$974.68. *14.* June 11; 57 da.; \$298.85. *15.* Feb. 3; 33 da.; \$1253.16. *16.* \$1158.09. *17.* \$465.87; \$4788.30.

Art. 461. *2.* \$820.65, \$620. *3.* \$1487, \$2092.60. *4.* \$564.92, \$593.70. *5.* \$1641.31. *6.* \$900. *7.* \$1280.37. *8.* \$647.17. *9.* \$2488.51. *10.* \$1503.57.

Art. 473. *3.* \$663. *4.* \$1511.25. *5.* \$279.63. *6.* \$3223.80. *7.* \$493.75. *8.* \$392.58. *9.* \$379.24. *10.* \$2493.40. *11.* \$1435.68. *12.* \$2189.70. *13.* \$4687.51. *14.* \$1900.48.

Art. 474. *3.* \$933.66. *4.* \$740.66. *5.* \$373.26. *6.* \$490.52. *7.* \$1511.34. *8.* \$2581.30. *9.* \$606.32. *10.* \$3200. *11.* \$4006.72.

Art. 481. *4.* \$2540.15. *5.* \$698.39. *6.* \$1688.37. *7.* \$2717.14. *8.* \$788. *9.* \$3042.96. *10.* \$669.34. *11.* \$1302.82.

Art. 482. *3.* £1026 13s. 10d. 2 far.+. *5.* £1312 16s. 4d. $3\frac{2}{3}$ far.+. *6.* 22533 franc. *7.* $2491\frac{19}{67}$ guilders. *8.* \$10953.88. *9.* 8311.22 francs.

Art. 489. *2.* 3.77+ mo. *3.* March 20, 1881. *4.* $6\frac{1}{6}$ mo. *5.* 78 days; April 2, 1880. *6.* $13\frac{1}{3}$ mo. *8.* June 25th of the next year. *9.* In 6.625 mo. *10.* July 5th. *11.* 3 mo.

Art. 490. *2.* Feb. 10, 1881. *3.* Jan. 5, 1882. *4.* Nov. 29. *5.* \$5577.49. *6.* Nov. 23, 1880. *7.* Sept. 28, 1882. *8.* \$2235.87. *9.* \$1451.60.

Arts. 493, 494. *2.* Sept. 15, 1881. *3.* Feb. 17, 1881. *2* \$1737.21. *3.* \$1413.88. *4.* \$1830.60. *5.* July 5.

Art. 521. *3.* 400 bu. *4.* 144 yd. *5.* $52\frac{16}{17}$ mi. *6.* \$141.75. *7.* \$7000. *8.* $106\frac{2}{3}$ yd. *9.* 4 mo. *10.* \$48. *11.* 8 mo. *12.* $10\frac{2}{3}$ yd.

13. $106\frac{9}{13}$ ft. *14.* \$310.50. *15.* 32 da. *16.* \$$2.81\frac{1}{4}$. *17.* $61\frac{5}{7}$ yd. *18.* 10 hr. *19.* \$117. *20.* 980 bu. *21.* 350 men. *22.* \$.80. *23.* \$216.75. *24.* 55 mi. *25.* $17\frac{1}{2}$ A. *26.* 64 da. *27.* 240000 brick. *28.* 107250 lb.

Arts. 527, 528. *3.* A's \$$7466\frac{2}{3}$, B's \$$5066\frac{2}{3}$, C's \$$3466\frac{2}{3}$. *4.* A, \$$4242\frac{6}{7}$; B, \$$3535\frac{5}{7}$; C, \$$5185\frac{5}{7}$. *5.* W. Johnson, \$$29953\frac{1}{8}$; B. Johnson, \$$19968\frac{3}{4}$; C. Mott, \$$13978\frac{1}{8}$. *2.* A, \$$424\frac{88}{113}$; B, \$$778\frac{86}{113}$; C, \$$796\frac{52}{113}$. *3.* A, \$$5866\frac{2}{3}$; B, \$$4133\frac{1}{3}$.

Art. 534. *3.* 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144. *4.* 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, 1331, 1728. *5.* $\frac{9}{16}$, $\frac{16}{49}$, $\frac{49}{144}$, $6\frac{1}{4}$, $76\frac{9}{16}$. *6.* $\frac{1}{8}$, $\frac{27}{64}$. $\frac{125}{216}$, $11\frac{25}{64}$, $42\frac{7}{8}$. *7.* .01, .0004, 12.25, 3.4225, .000009. *8.* .008, .000027, 15.625, 1.157625, 1.003003001. *9.* 2250000, 3375000000, 5062500000000. *10.* 759375. *11.* 262144. *12.* 15625, 15625, 1953125. *13.* Nothing.

Art. 535. *2.* 1369, 3136, 2304, 4225. *3.* 8464, 15625, 17424.

Art. 537. *1.* 262144, 373248. *2.* 592704, 1953125. *4.* 15625, 59319, 91125, 132651. *5.* 884736, 1030301, 2000376, 2863288.

Art. 543. *5.* $\frac{8}{13}$. *6.* $\frac{12}{13}$. *7.* 111. *8.* 15. *9.* 17. *10.* 24. *11.* 38. *12.* 145. *13.* 416. *14.* 999. *15.* 989. *16.* 17.177+. *17.* .3789. *18.* 1.5367. *19.* 54.32. *20.* .4. *21.* 1.2649+. *22.* .1264+. *23.* 3.6. *24.* 1.02. *25.* .0284+. *26.* $\frac{13}{16}$. *27.* 15.4919+. *28.* .8819+. *29.* 4.21307. *30.* $2\frac{7}{8}$. *31.* .7155+. *32.* .09. *33.* 1400 ft. *34.* 1400 yd. *35.* 30 in. by 120 in. *36.* 13.5 rd., or 3 ch. 1.5 rd. *37.* \$720.

Art. 545. *5.* 44, 2.1. *6.* 95, 7.4. *7.* $\frac{17}{21}$, $\frac{52}{55}$. *8.* 48. *9.* 67. *10.* 134. *11.* 411. *12.* 258. *13.* 638. *14.* 975. *15.* 6031. *16.* 3.19. *17.* .097. *18.* .124. *19.* .029. *20.* 1.442+. *21.* .6694+. *22.* .3107+. *23.* $\frac{2}{3}$. *24.* .6616+. *25.* 3.546+. *26.* 7.4. *27.* 1.93+. *28.* .1972+. *29.* .315+. *30.* 3.382+. *31.* 10.21+. *32.* 3.47+, $\frac{11}{24}$, 5.35. *33.* $819\frac{1}{5}$, 6, *34.* 37 ft. *35.* 66 sq. ft. 96 sq. in. *36.* 2.884 ft. *37.* 441 sq. in. *38.* 32 in. *39.* 24 ft. *40.* 16 sq. ft.

Arts. 547, 548. *2.* 6. *3.* 36. *4.* 3.2. *5.* 8. *1.* \$166.60. *2.* $106\frac{2}{3}$ A. *3.* 964.276+ bu. *4.* \$546. *5.* $1813\frac{1}{3}$ A. *6.* \$10.16. *7.* $10\frac{10}{11}$ min. after 2 o'clock P. M. *8.* $308\frac{388}{539}$. *9.* \$139.57. *10.* 200. *11.* .625. *12.* Neither. *13.* \$450. *14.* .15. *15.* $26\frac{2}{3}$ cts. *16.* \$139.33. *17.* $2\frac{335}{924}$. *18.* \$2526.53. *19.* \$1.005. *20.* $31\frac{1}{2}$ lb. *21.* $20\frac{4}{7}$ ft. *22.* \$8.50. *23.* $5\frac{5}{6}$ ft. *24.* $74\frac{2}{3}$ cts. *25.* $7\frac{1}{2}$%. *26.* \$1154.45. *27.* $19\frac{1}{4}$ yd. *28.* \$144. *29.* \$200.84. *30.* 54 A. $104\frac{5}{6}$ P. *31.* 336 rd. *32.* \$1838.20. *33.* 81 men. *34.* 87 rd. 3 yd. 2 ft. 6 in. *35.* \$1518.60. *36.* 7350 sq. ft. *37.* Cash price : Credit price = 100 : 103. *38.* \$173.72. *39.* 60 A. 152+ P. *40.* \$1749.85. *41.* £164 7s. 9+d. *42.* 4 da. *43.* \$260.643. *44.* \$22500.

45. \$48 suit, \$60. *46*. 7%. *47*. \$5116.50, \$153.50. *48*. 192%; \$9600. *49*. 16° West. *50*. 8.53+ ft. each way. *51*. $9\frac{1}{2}$ mills. *52*. \$.$47\frac{13}{21}$. *53*. \$1248.32. *54*. 10.17+ ft. each way. *55*. \$60.60. *56*. 151.25. *57*. \$445.84 *58*. \$.50, \$.80, \$1.08. *59*. Nov. 21, equated time, \$917.10. *60*. $16\frac{2}{3}$ yr. *61*. \$176.40. *62*. \$482.05. *63*. 42 mi. *64*. $4097\frac{1}{2}$ yd. *65*. Nov. 10, 1882, date Mat.; 4 mo. 16 da., term dis.; \$1980.82, pro. *66*. 90 yd.; 63 cts. *67*. \$364.99. *68*. \$453.75. *69*. \$316.67. *70*. \$660 at sim. int., \$679.20 at an. int., \$680.24 at com. int. *71*. $\frac{29}{69}$. *72*. 45 bu. *73*. 75 da., term of cred.; June 26. *74*. 1 mi. 56 ch. 1 rd. 2 yd. 1 ft. 6 in. *75*. \$1334.50. *76*. \$$1225.12\frac{1}{2}$. *77*. Feb. 16. *78*. \$$261\frac{27}{43}$, \$$279\frac{3}{43}$, \$$209\frac{13}{43}$. *79*. $12\frac{1}{2}$%. *80*. Feb. 17. *81*. 12 bbl. at \$7, 28 bbl. at \$10. *82*. 8d. $1\frac{63}{65}$ far. *83*. A, \$$1333\frac{1}{3}$; B, \$$1639\frac{21}{61}$; C, \$$1027\frac{59}{183}$. *84*. 11 T. 11 cwt. 56 lb. 4 oz. *85*. \$398.03. *86*. 17 ft. *87*. \$7869.28. *88*. \$11951.25. *89*. \$429.67, \$430.66, \$430.67. *90*. \$480, \$360, \$420. *91*. 2 men. ***92***. 16560. *93*. \$2760. *94*. 2% better on money loaned.

Arts. 556, 557. *5*. 41. *6*. 149. *7*. 7. *8*. $9\frac{3}{4}$. *2*. $2\frac{1}{7}$. *3*. 2. *4*. $\frac{5}{34}$.

Arts. 558, 559. *2*. 308. *3*. 1001000. *4*. 432 mi. *3*. 114688. *4*. $\frac{1}{256}$. *5*. $\frac{3}{4}$. *6*. 128 mi. *7*. \$1296.87.

Arts. 560, 566. *3*. 255. *4*. $3\frac{241}{405}$. *5*. $1\frac{511}{512}$. *6*. 1275. *7*. 2186. *8*. 635. *9*. \$17433922. *10*. 1111111.111. *3*. \$1053. *4*. \$56550. *5*. \$3625. *6*. \$7500. *7*. \$26840. *8*. \$6350. *9*. \$7136.36.

Arts. 571, 572. *2*. $41\frac{1}{5}$ cts. *3*. $5\frac{15}{19}$ cts. *4*. \$$1.96\frac{8}{25}$. *2*. 5, 1, 1, 4. *3*. 8, 3, 3, 5, 5. *4*. 5, 4, 1, 1, 5.

Art. 592. *5*. 9327 m. *6*. .190141 Km. *7*. 67117.7 m. *8*. 25625 m. *9*. 14 hr. *10*. 12.725 m. *11*. 20 hr. *12*. \$.0224. *13*. 20.90976 T., 5.24576 T. *14*. \$4720. *15*. 187.5 Kg. *16*. 34.944 Km. *17*. 8.25 m *18*. 122.

Art. 596. *1*. 40.19+ ft., 442.91+ ft, 251.8+ ft. *2*. 74.66+ yd., 35.14+ yd. *3*. 82.296 m., 11.43 m. *4*. 4.97+ mi., 279.63 mi. *5*. 15480 m.; 9.618+ mi. *6*. \$97.92. *7*. 120697.5. *8*. Lost \$11.81. *9*. 1.254+ a., 24.98+ A. *10*. 3369.71 a., 33.6971 Ha., 83.265+ A. *11*. 133.12 sq. m. *12*. \$27.90. *13*. 46.91+ yd.; \$70.37. *14*. 3249.127 cu. ft., 25.38+ Cd. *15*. 7.2486 sters, 20.839725 sters. *16*. 42. *17*. 625 Hl. *18*. \$132.82, \$146.58. *19*. \$260.06. *20*. 19.062868 gal.; 45.4244+ l. *21*. 114.5374+ l.; 3.7029375 bu. *22*. 175 l.; 184.9225 qt. *23*. 15300 l. *24*. \$25. *25*. \$2.84. *26*. \$436.79. *27*. $2.57\frac{1}{6}$ lb. Troy, 2.1162 lb. Avoir. *28*. 62.216+ grams, 2986.3901+ grams, 2948.681+ grams.

29. 8 Mg. 8 Kg. 9 Hg. 5 g. 6 dg. *30.* 66 cts. more profitable at 12¢ a lb. *31.* \$30.62. *32.* \$11.33. *33.* $8\frac{4}{5}$% + gain. *34.* \$2.33.

Art. 617. *2.* 900 sq. ft. *3.* 15 A. *4.* 319. *6.* 433.705 + sq. in. *7.* 13.26 +. *8.* 997.66 + sq. ft. *10.* 34 ft. *11.* 32 ft. *12.* 19 ft. *15.* 50 ft. *16.* 45.82 + ft. *17.* 104 ft. *18.* 120 mi. *19.* 135.88 + rd. *20.* 80 ft.

Arts. 618, 619. *2.* 2080. *3.* 40. *4.* $4\frac{1}{2}$. *5.* 210 sq. rd. *2.* 1661 sq. ft. *3.* 2. *4.* 18.98 + A.

Arts. 620, 621. *3.* 15.278 ft. *4.* 10.9956 + ft. *5.* 11.936 + ft. *3.* 452.39 + sq. ft. *4.* 1790.488 + sq. ft. *5.* 50.929 + A. *6.* 7.97 + rd.

Arts. 635, 636. *2.* 198 sq. ft. *4.* 37.699 + sq. ft. *5.* 216 sq. ft. *6.* 108 sq. ft. *7.* 143 sq. ft. *8.* 15.31 + sq. ft. *3.* 76.76 + cu. ft. *4.* \$10.80. *5.* 76.969 + cu. ft. *6.* \$30.56.

Arts. 637, 638. *3.* 148.44 + sq. ft. *4.* 1280 sq. ft. *3.* 848.70 + cu. in. *4.* 7854 cu. ft. *5.* 5400 cu. ft. *6.* 18 cu. ft.

Art. 639. *2.* 19.635 sq. ft. *3.* 28.2744 sq. ft. *4.* 196663355.75 square miles.

Arts. 640, 641. *2.* 19.39 + cu. ft. *3.* .5236 cu. ft. *4.* 1.76715 cu. ft. *2.* 99.144 gal. *3.* 47 + gal. *4.* 174.93 gal.

Art. 645. *2.* 128 ft. 5′. *3.* 37 ft. 7′ 7″ 6‴ 8⁗. *4.* 9 ft. 6′ 4″ 3‴ 8⁗. *5.* 120 ft. 4′ 1″ 4‴ 6⁗. *6.* 36 ft. 3′ 8″ 0‴ 8⁗. *7.* 161 ft. 10′ 8″ 8‴ 6⁗. *8.* 56 ft. 7′ 5″ 3‴ 3⁗ 8′′′′′. *9.* 12 ft. 8′ 0″ 2‴.

Art. 653. *1.* 384 A.; $\frac{3}{5}$; \$14400, \$480. *2.* \$200. *3.* 120 A. *4.* 200 A.; $\frac{5}{16}$ sec. *5.* 160 A.; \$60 less. *6.* \$331.52.

DATE DUE SLIP

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